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Optimal execution of open-market stock repurchase programs

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Abstract

This paper formalizes the following intuition about open-market share repurchases. Firms do open-market share repurchases to return free cash, which would otherwise be wasted. However, when the firm goes to buy its own shares with this cash, it has inside information and hence the actual execution is characterized by adverse selection. The market knows that the firm has inside information, and consequently the ask price is high to compensate for this adverse selection problem. This implies that, all else equal, the greater the adverse selection problem compared to the cash waste problem, the higher the ask price, and, therefore, the wider the bid–ask spread and the lower the share repurchase completion rate. We test this implication on a sample of U.S. firms and report evidence consistent with the model.

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1. Introduction

Over the last two decades, announcements of open-market stock repurchase programs (henceforth, "open-market programs") have become common practice (see, for example, Grullon and Michaely, 2002). Yet, empirical evidence suggests there is great variability associated with their execution. First, there is great variability documented about actual

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completion rates. In the United States, Stephens and Weisbach (1998) and Jagannathan et al. (2000) document that average actual repurchase rates are only 70–80%. Frequently only a small fraction of the quantity announced is actually repurchased, and many announcing firms do not repurchase at all. Chan et al. (2005) report similar results. Actual repurchase rates are even lower outside the U.S. Ikenberry et al. (2000) find average actual repurchase rates to be as low as 28% in Canada, and Rau and Vermaelen (2002) find average actual repurchase rates of only 37% in the United Kingdom.

Empirical studies also disagree on the effect of open-market program announcements on liquidity, as measured by the bid–ask spread. In the U.S., Barclay and Smith (1988) find widening of the spread. Miller and McConnell (1995) find no widening of the bid–ask spread, while Wiggins (1994), Franz et al. (1995), and Cook et al. (2004) actually find narrowing of the spread during periods of actual repurchases. Outside the U.S., Brockman and Chung (2001) and Ginglinger and Hamon (2007) find widening of the spread during actual repurchase periods in Hong Kong and France, respectively.¹

Why is there variability in actual repurchase and in the bid-ask spread during the repurchase period across firms and across studies? What is the optimal way to execute an open-market repurchase program? The purpose of this paper is to develop a theoretical framework with which to answer these questions.

Unlike most earlier theoretical investigations of open-market programs, we build on the motivation to distribute free cash in order to avoid its waste. Growing empirical evidence suggests that the availability of free cash, and the need to avoid wasting it, play an important role in decisions to execute open-market programs. For example, Stephens and Weisbach (1998) and Oswald and Young (2004) find that actual repurchases depend on the availability of free cash. Nohel and Tarhan (1998) find that repurchasing helps firms that are likely to overinvest improve their performance.²

We take the program announcement as given in order to focus on the execution.³ Assuming uncertainty and asymmetric information about firm value, we show that the execution is the solution to an optimization problem over waste-prevention benefits from paying out free cash and gains (or losses) from the informed trade of the firm. Specifically, if the firm learns that it does not have free cash, it refrains from executing the program so as not to hurt investment. If, instead, the firm learns that it does have free cash, it will always execute the repurchase when the stock is undervalued, because in this case it benefits from preventing the waste of free cash, and it also accrues trading gains from the (informed) repurchase trade. When the stock is overvalued, however, the firm is less likely to execute the repurchase because in this case it faces a tradeoff between waste prevention gains and trading losses. The greater the overvaluation, the less likely the execution. Thus, open-market programs enhance value to shareholders by distributing free cash, but also result in wealth transfers among shareholders because of the informed/strategic trade of the firm.

¹Most of the above studies also report findings on market depth consistent with their findings about bid–ask spread. That is, studies that find narrowing of the spread also find an increase in market depth measured by the price impact on order imbalances, and studies that find widening of the spread find a reduction in market depth.

 $^{^{2}}$ On the agency costs of free cash flow see, for example, Jensen (1986).

³Extensive literature investigates the motivation to announce a repurchase program and the choice between a repurchase program and other payout methods such as a self-tender offer repurchase and a dividend (for a review of this literature, see, for example, Allen and Michaely, 2003). We acknowledge that these questions are also important, but we do not model them in this paper.

The model provides predictions about actual repurchase rates and the bid-ask spread that might explain the discrepancies among the empirical studies on completion rates and spreads: When uncertainty about the firm value is relatively high, actual repurchases are driven by the motivation to take advantage of information through strategic trading, and, hence, open-market programs are characterized by low completion rates and wide bid-ask spreads. In this case, expected wealth transfers among shareholders (expropriations) because of the firm's informed trade are more significant than expected value enhancement through the disbursement of free cash. In contrast, when uncertainty about the firm value is relatively low, actual repurchases are driven by the motivation to distribute free cash in order to avoid its waste, and open-market programs are thus characterized by higher completion rates and narrower bid-ask spreads. In this case, expected value enhancement through the disbursement of free cash is more significant than expected wealth expropriations. All else equal, the more severe the waste problem is, the greater the incentive to distribute the free cash, and consequently, the higher the completion rate and the narrower the bid-ask spread. These results naturally generate testable predictions about how the execution will depend on firm characteristics, such as growth opportunities and size. One novel prediction of our theory is that the program completion rate and the bid-ask spread are negatively correlated. Namely, low program completion rates are associated with wide bid-ask spreads. While there exists significant empirical literature that investigates completion rates of repurchase programs and the bid-ask spread during the repurchase period, we are unaware of any research that investigates the correlation between them. We test this implication on a sample of U.S. firms and report evidence that supports the model.

The model highlights two important properties of open-market programs so far largely ignored in the theoretical literature that might explain their increasing popularity over other payout mechanisms (e.g., self-tender offers and dividends). First, the model suggests that open-market programs give the firm financial flexibility, that is, the firm retains the option not to eventually repurchase, should the availability of free cash change. Most firms have considerable amounts of cash on their balance sheet at the time they announce a repurchase program. Our thrust is that, at the time they make the announcement, whether this cash is free or not is yet to be determined.⁴ Second, while the trading gains associated with the (informed) repurchase trade are generally viewed as a negative property of open-market programs (e.g., Barclay and Smith, 1988), the model here suggests that these trading gains do not represent a zero sum game: uninformed shareholders' benefits from waste prevention may outweigh their trading losses.⁵

Most earlier theoretical investigations of open-market repurchase programs focus on signaling undervaluation motivation. A signaling motivation, however, seems inconsistent with the noncommitting nature of open-market programs confirmed by the low actual repurchase rates, and does not explain the mixed evidence on the bid–ask spread. Among the signaling papers, very few consider the optionality of the programs (i.e., distinguish between announcement and actual repurchase). The later group includes Ikenberry and

⁴Supporting evidence on the flexibility of open-market programs is provided in Guay and Harford (2000), Jagannathan et al. (2000), and Brav et al. (2005).

⁵Outside our model, dividends both prevent the waste of free cash and avoid the wealth redistribution problem as they are pro-rata. Yet dividends are also tax disadvantageous; once declared, they are not optional, and have been shown to commit the firm to future payouts.

Vermaelen (1996), Bhattacharya and Dittmar (2003), and Oded (2005). Brennan and Thakor (1990) also consider the optionality of open-market programs, but focus on the wealth transfers associated with the execution rather than on signaling undervaluation. Interestingly, the agency costs of free cash are largely ignored in theoretical work about repurchases in general and for open-market repurchase programs in particular. To our knowledge, the only theoretical papers that consider free cash distribution as a motivation in repurchase policy are Chowdhry and Nanda (1994), and Lucas and McDonald (1998). These studies do not distinguish between announcement and actual repurchase, and thus apply more to tender offers than to open-market programs.

The rest of this paper is organized as follows. The assumptions are set up and discussed in Section 2. Section 2 also demonstrates the main idea using a numerical example. A general formulation and a solution are given in Section 3. In Section 4, the prediction that bid–ask spread and program completion rate are negatively correlated is tested. Section 5 concludes.

2. Assumptions and example

There are three dates indexed by t = 0, 1, 2. All agents are risk neutral, the interest rate is zero, and there are no taxes or transaction costs. Consider an equity-financed firm. At t = 0, the firm owns a project and some cash, where it is unclear what portion of the cash will be needed to finance the project and what portion of the cash is free cash. At t = 1, the firm generates assets in place with value of $\tilde{A} \in \{A - X, A + X\}$ with equal probability, where 0 < X < A, and realizes free cash $\tilde{C} \in \{0, C\}$ with equal probability, where 0 < C, and \tilde{A} and \tilde{C} are independent.⁶ Thus, there are four equally likely outcomes for the firm value V at t = 1: $V_1 \in \{A - X, A - X + C, A + X, A + X + C\}$. We will generally omit the time index for t = 1, as most of the action happens on this date. At t = 2, the firm is sold or dismantled, and investors get the value of their shares. The firm is run by a manager who maximizes the terminal value per share.⁷ Information is symmetric at t = 0. However, at t = 1 only the manager observes the realization of \tilde{A} and \tilde{C} , while all other agents know only the distribution of these variables. The practical interpretation here is that the manager observes the realized value of the firm's project, as well as what portion of the firm's cash is actually needed for the project; the rest becomes free cash. The shareholders and the market do not observe this information yet. At t = 2, all information is publicly known.

There are N shares outstanding at t = 0, and we normalize the values of A, X, C, and V to be values per share using lowercase letters a, x, c, and v, respectively. At t = 0, the firm announces a repurchase program that it may execute at t = 1. The firm can buy back shares at t = 1 only with free cash (otherwise, the value of assets in place is severely damaged).⁸ If the firm does have free cash but does not distribute it with a repurchase at

⁶For simplicity, we have assumed a cash distribution of $\{0, C\}$. Assuming two positive values of cash instead does not change the qualitative results.

 $^{^{7}}$ It will be shown in Section 3.3.2 that, in our set up, this objective function is equivalent to maximizing the expected wealth of the original shareholders.

⁸Since our focus is optimal execution of the program, we take it as given that the firm has a program it can execute at t = 1. We consider the case without a program only as a benchmark. In our setup, for the firm, announcing always dominates not announcing, and, since at t = 0 all information is symmetric, the announcement has no signaling content. In practice, firms must make their programs publicly known (announce)



Fig. 1. Time line.

t = 1, a portion $(1 - \delta)$ of the free cash is lost, where $\delta \in (0, 1]$ is publicly known. We take the waste problem, $\delta < 1$, as a given, as we want to focus on the execution itself. Thus, we refrain from modeling the reasons for the waste and assume no private benefits from waste for the manager.⁹ Without loss of generality, we assume that the firm will repurchase whenever indifferent. As in most payout policy models, we assume that neither borrowing nor stock issues are possible.

At t = 1, there is a market for the stock. Liquidity traders place quantity bids $Q_A < N/3$ and $Q_B < N/3$ that they want to buy and sell, respectively.¹⁰ The market-maker sets prices p_A , p_B in the buy and sell markets, respectively, before investors place their quantity bids (anticipating Q_A , Q_B and the possibility of informed trade from the firm side) to earn zero expected profit.¹¹ The timing is described in Fig. 1.

Given the assumptions that no one benefits from the waste and that the market-maker has zero expected profit, we define social wealth as the sum of the wealth of liquidity buyers, liquidity sellers, and non-trading shareholders. The following example demonstrates a central aspect of the model: how uncertainty in the value of assets in place and

⁽footnote continued)

beforehand. In the U.S., the requirement to announce comes from the stock exchange, whereas in most other countries firms are required to announce by law.

⁹Models that assume the manager does not benefit from the waste of free cash include Chowdhry and Nanda (1994). In practice, keeping free cash in the firm is costly to shareholders even without private benefits to the manager from waste. This is either because the cash will be invested in negative-NPV projects, or because the manager cannot prevent waste at lower management levels. In could also be because investors have better investment opportunities for this cash outside the firm. We exclude $\delta = 0$ to simplify the analysis. The results would hold also for $\delta = 0$.

¹⁰We focus on t = 1 because this is when the repurchase takes place, but it could be assumed that the market opens also at t = 0 and 2. The restriction on liquidity trade is without loss of generality in order to limit the discussion to the feasible range of the results.

¹¹Prices are thus independent of the actual order flow, as in Glosten and Milgrom (1985), Rock (1986), and Noe (2002). At the cost of significantly complicating the analysis, a market mechanism in which the market-maker observes the order flow before posting prices (as in Kyle, 1989) could be employed. The qualitative results should still hold. Furthermore, in the U.S. there is no reporting requirement of actual repurchases other than in the financial statements. Hence the public learns about actual repurchases only long after they occur, if at all (see Stephens and Weisbach, 1998; Cook et al., 2004).



Fig. 2. Graphs depicting firms F and G in Example 1. (a) Firm F—a firm for which uncertainty about assets in place is high relative to uncertainty of free cash. A = 9.5, X = 2.5, C = 2, N = 10, Q = 3, $\delta = 0.8$. (b) Firm G—a firm for which uncertainty about assets in place is low relative to uncertainty of free cash. A = 9.5, X = 0.5, C = 2, N = 10, Q = 3, $\delta = 0.8$.

uncertainty of free cash interact to determine the program execution. While we will generally take the program announcement as given, the example also considers the case without a program as a benchmark.

Example 1. Consider a case with high uncertainty of the value of assets in place and relatively low uncertainty of free cash (shown in Fig. 2a). At t = 0, firm F has N = 10 shares. At t = 1, the value of assets in place is realized to be either 7 or 12 with equal probability, and free cash is realized to be either 2 or 0 with equal probability. Thus, a = 0.95, x = 0.25, c = 0.2, and there are four possible states, as described in Fig. 2a. Assume that at t = 1, liquidity buyers place quantity orders $Q_A = Q_B = 3$, and suppose further that if the free cash is not distributed at t = 1, then $\delta = 0.8$, i.e., the waste rate is $1 - \delta = 0.2$. Suppose that the firm has not announced a repurchase program and hence cannot repurchase. Then the expected firm value at t = 2 is

$$0.5(7 + 12 + 0.8 \times 2) = 10.3.$$

Since there is no informed trade at t = 1, this is the price the market-maker will sell and buy for at t = 1. That is, without a repurchase program, $p_A = p_B = 1.03$. If, instead, the firm has announced a program (at t = 0), at t = 1 it will buy shares only in the upper state in Fig. 2a. To earn zero expected profit, the market-maker must set p_A , such that

$$3((p_A - 0.7) + (p_A - (0.7 + 0.8 \times 0.2)) + (p_A - 1.2)) + \left(p_A - \frac{7+5}{10-\frac{2}{p_A}}\right) \left(3 + \frac{2}{p_A}\right) = 0,$$

which upon solution implies $p_A = 1.1093$. The implied average terminal stock value at t = 2 is

$$0.25\left(0.7 + 0.86 + 1.2 + \frac{7+5}{10-\frac{2}{1.1093}}\right) = 1.056.$$

This is also the price at which the market-maker buys for at t = 1 (no adverse selection on sell market), that is, $p_B = E[p_2] = 1.056$.¹² In comparison to the case where the firm does not have a program, liquidity buyers lose

$$Q_B(p_A - E[p_2]) = 3(1.1093 - 1.056) = 0.16.$$

Original shareholders gain a total of

$$N(p_A - 1.03) = 10(1.056 - 1.03) = 0.26$$

(i.e., 0.026 per share regardless of when they sell). Social wealth increases because of the repurchase by

$$0.26 - 0.16 = 0.1.$$

Now, consider instead, a case with low uncertainty of the value of assets in place relative to the uncertainty of free cash (shown in Fig. 2b). Specifically, consider firm G, for which everything is the same as for firm F, except that at t = 1 the value of the assets in place is realized to be either 9 or 10 with equal probability (the free cash is still either 2 or 0 with equal probability). Thus, for firm G, a = 0.95, x = 0.05, and c = 0.2. There are four possible states as described in Fig. 2b. If the firm has not announced a repurchase program and hence cannot repurchase, the expected firm value at t = 2 is

$$0.5(9 + 10 + 0.8 \times 2) = 10.3,$$

and $p_A = p_B = 1.03$ (the same as for firm *F*). However, if firm *G* has announced a program (at t = 0), at t = 1 it will buy shares not only in the upper state in Fig. 2b, but rather in both states in which it has free cash. To earn zero expected profit, the market-maker must set p_A , such that

$$3((p_A - 0.9) + (p_A - 1)) + \left(p_A - \frac{9}{10 - \frac{2}{p_A}}\right) \left(3 + \frac{2}{p_A}\right) + \left(p_A - \frac{9 + 1}{10 - \frac{2}{p_A}}\right) \left(3 + \frac{2}{p_A}\right) = 0,$$

¹²One can verify that the firm will not buy in the state with low asset value with free cash: If it does not repurchase, terminal value per share would be $0.7 + (0.8 \times 0.2) = 0.86$. If it does repurchase, the terminal value per share would only be 7/(10 - (2/1.1093)) = 0.8540.

which upon solution implies $p_A = 1.0829$. The implied average terminal stock value at t = 2 is

$$0.25\left(0.9+1.0+\frac{9}{10-\frac{2}{1.0829}}+\frac{9+1}{10-\frac{2}{1.0829}}\right) = 1.0576.$$

This is also the price at which the market-maker buys for at t = 1 (no adverse selection on sell market), that is, $p_B = E[p_2] = 1.0576.^{13}$ In comparison to the case where the firm does not have a program, liquidity buyers lose

 $Q_B(p_A - E[p_2]) = 3(1.0829 - 1.0576) = 0.0760.$

Original shareholders gain a total

 $N(p_A - 1.03) = 10(1.0576 - 1.03) = 0.2760$

(i.e., 0.0276 per share regardless of when they sell). Social wealth increases because of the repurchase by

0.2760 - 0.0760 = 0.2.

Table 1 highlights the differences between firm F and G in the example.

Example 1 demonstrates that, when uncertainty in the value of assets in place relative to the waste of free cash is low (firm G in Table 1), an open-market program will result in a higher completion rate and a narrower bid-ask spread in comparison to the case in which uncertainty in the value of assets in place relative to the waste of free cash is high (firm F in Table 1). Furthermore, when uncertainty about the value of assets in place is low, the increase in social wealth is greater and there is also less wealth transfer from liquidity/ outside investors to insiders. For both firms, the inherited flexibility of open-market programs leads to informed trade from the firm side. Managers repurchase to enhance the value of terminal shares. This value enhancement comes partly at the expense of new shareholders and partly because the repurchase prevents the waste of free cash.

3. The formal model

Example 1 illustrates the effect of uncertainty about asset value and the waste of free cash on the program execution for specific parameter values. It does not illustrate the effect of waste (both firms in the example have the same degree of waste, $\delta = 0.8$). This section provides a general analysis of the manner in which uncertainty and waste and the interaction between them shape the optimal execution of a repurchase program.

Because informed trade is possible only in the buy market, we focus on this market and denote $Q_A \equiv Q$, $p_A \equiv p$. Given the assumptions set out in Section 2, the market-maker's zero-expected-profit condition is

$$\sum_{j} [\Pr\{j\}(p - v_{2|v_j, r_j})(Q + r_j|_p)] = 0,$$
(1)

¹³One can verify that the firm will indeed buy in the state with low asset value with free cash: if it does not repurchase, the terminal value per share would be $0.9 + (0.8 \times 0.2) = 1.06$. If it does repurchase, terminal value per share would only be 9/(10 - (2/1.0829)) = 1.1039.

	Firm F	Firm G
A	9.5	9.5
X	2.5	0.5
С	2	2
Ν	10	10
Q_A	3	3
δ	0.8	0.8
Stock price without a repurchase program $(p_A = p_B = E[p_2])$	1.03	1.03
Completion rate = probability that repurchase is executed	25%	50%
Ask price with repurchase (p_A)	1.1093	1.0829
Bid price with repurchase p_B = average terminal value	1.0560	1.0760
Bid-ask spread = $p_A - p_B$	0.0533	0.0253
Original shareholders' wealth—increase relative to without repurchase	0.260	0.2760
Liquidity buyers' loss—relative to without repurchase	0.160	0.0760
Social wealth increase—relative to without repurchase	0.1	0.2

Table 1 Comparison of firm F and firm G in Example 1.

where *j* indicates the four possible outcomes (states) of the firm value *V* at t = 1, r_j is the number of shares the firm repurchases at t = 1 in state *j*, and $v_{2|v_j,r_j}$ is the value of each share at t = 2 depending on v_i (the value per share in state *j* realized at t = 1) and on r_j .

Definition 1. Equilibrium is a set $(\{r_j\}, p)$ consisting of a repurchase strategy $\{r_j\} \in (0, C/p)$ set by the manager given $\{v_j\}, p$ to maximize the terminal value per share v_2 , and a price p set by the market-maker, such that condition (1) is satisfied.

It is immediate to show that if the firm does not have a repurchase program, $p = E[v_2] = a + \delta c/2$. Henceforth, we take it as a given that the firm announces a repurchase program at t = 0, and we focus on the optimal execution.

Lemma 1. In any equilibrium, the firm never repurchases in the states v = a - x and v = a + x, and it always repurchases with all available cash in the state v = a + x + c.

Proofs of all lemmas and propositions appear in the Appendix. Accordingly, we can write the market-maker's zero-expected-profit condition (1) as

$$(p - a - x)Q + (p - v_{2|v=a-x+c})(Q + r|_{p,v=a-x+c}) + (p - (a + x))Q + \left(p - \left(\frac{A + X}{N - \frac{C}{p}}\right)\right)\left(Q + \frac{C}{p}\right) = 0.$$
(2)

Condition (2) essentially requires that the average of the differences between the price that the market-maker is willing to sell for and the terminal value of a share, weighted by the quantity sold in each state, be equal to zero. The first and the third terms correspond to the states with low and high asset values, respectively, where the firm has no cash and therefore does not repurchase. The last term corresponds to the state with high asset value and cash (v = a + x + c). By Lemma 1, in this state the firm will always repurchase. In this state the terminal value per share is (A + X)/(N - C/p), and the market-maker sells Q + C/p shares. The second term corresponds to the interesting state with low asset value and with

cash (v = a - x + c) and in which the decision to repurchase depends on the model parameters. In this term, the value of r (repurchase) is either C/p or 0, depending on whether the firm repurchases in this state, and $v_{2|v=a-x+c}$ is either (A - X)/(N - C/p) or $a - x + \delta c$, depending on whether the firm repurchases in this state. An important feature of repurchases under asymmetric information reflected in (2) is the nonlinearity in value introduced through the firm's trade. Specifically, when the firm does repurchase to take advantage of its private information, the per-share value increases not only because trading gains are added to the value of the terminal shares, but also because these trading gains are shared by a reduced number of shares.

Definition 2. A full repurchase equilibrium is an equilibrium in which the firm repurchases at t = 1 whenever it has free cash, i.e., in both states v = a - x + c and v = a + x + c. A partial repurchase equilibrium is an equilibrium in which the firm repurchases at t = 1 only when it has free cash and the asset value is high, i.e., only in state v = a + x + c.

In any full repurchase equilibrium, condition (2) becomes

$$(p - (a - x))Q + \left(p - \frac{A - X}{N - \frac{C}{p}}\right)\left(Q + \frac{C}{p}\right) + (p - (a + x))Q + \left(p - \left(\frac{A + X}{N - \frac{C}{p}}\right)\right)\left(Q + \frac{C}{p}\right) = 0,$$
(3)

whereas in any partial repurchase equilibrium, condition (2) becomes

$$(p - (a - x))Q + (p - (a - x + \delta c))Q + (p - (a + x))Q + \left(p - \left(\frac{A + X}{N - \frac{C}{p}}\right)\right)\left(Q + \frac{C}{p}\right) = 0.$$
(4)

Lemma 2 presents the solution for the price p of (3) and (4) in a full repurchase equilibrium and in a partial repurchase equilibrium, respectively.

Lemma 2. In any full repurchase equilibrium, the price p at which the market-maker sells at t = 1 is

$$p = \frac{a + c - \frac{Nc}{2Q} + \sqrt{\left((a + c) - \frac{Nc}{2Q}\right)^2 + 2c\left[(a + c)\frac{N}{Q} - a\right]}}{2}.$$
(5)

In any partial repurchase equilibrium, the price p at which the market-maker sells at t = 1 is

$$p = \frac{a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c + \sqrt{\left(a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c\right)^2 + c\left((a + x + c)\frac{N}{Q} - (3a - x + \delta c)\right)}}{2}.$$
(6)

The firm's decision about whether to repurchase in the state v = a - x + c depends on the one hand on how deep the undervaluation is and on the other hand on how severe the waste is. Specifically, since by assumption the manager maximizes the value of the terminal shares, she will not buy if the terminal stock value without repurchase is higher than the terminal stock value with the repurchase, that is, if

$$a - x + \delta c > \frac{Na}{N - \frac{C}{p}},$$

which after rearrangement is equivalent to

$$p > \frac{a-x}{\delta} + c. \tag{7}$$

Otherwise (that is, if $p \le (a - x)/\delta + c$), the firm will always repurchase (recall that without loss of generality, we have assumed that the firm will repurchase whenever indifferent). The important and nonintuitive insight reflected in (7) is that when deciding whether or not to repurchase, the manager does not compare the value to the price, but rather compares the projected terminal values under each alternative.¹⁴

Lemma 3 combines condition (5) with the requirement that (7) does *not* hold to give a necessary and sufficient condition for a full repurchase equilibrium; it also combines conditions (6) and (7) to give a necessary and sufficient condition for a partial repurchase equilibrium.

Lemma 3. A necessary and sufficient condition for a full repurchase equilibrium is

$$\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right] \le c\left[\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{2N}{Q}+4\right)-2\left(\frac{N}{Q}+1\right)\right].$$
(8)

A necessary and sufficient condition for a partial repurchase equilibrium is

$$\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right] > c\left(\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{N}{Q}+4\right)-2-\left(1+\frac{x}{a}\right)\frac{N}{Q}\right).$$
(9)

Conditions (8) and (9) are the basis for our results.

3.1. Special cases

3.1.1. Special case 1: $\delta = 1$

Suppose there is no free cash waste, whether the firm repurchases or not, i.e., $\delta = 1$. In this case, the sole purpose of repurchasing is to achieve trading gains based on asymmetric information. The repurchase also does not increase social wealth.

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¹⁴Note that $p > a - x + \delta c$ is not enough to assure no repurchase. That is, to assure no repurchase it is not enough that the price is higher than the terminal stock value without a repurchase. That assurance requires the stronger restriction, reflected in (7), that the terminal stock value without a repurchase be higher than the terminal stock value without a repurchase.

Proposition 1. Suppose $\delta = 1$. If both

$$\frac{x}{a} < \frac{1}{\frac{N}{Q} + 2} \tag{10}$$

and

$$\frac{2x\left(1-\frac{x}{a}\right)}{1-\frac{x}{a}\left(\frac{N}{Q}+2\right)} \le c \tag{11}$$

hold, the outcome is a full repurchase equilibrium. Otherwise, the outcome is a partial repurchase equilibrium.

A full repurchase equilibrium requires that the firm buy in the state with a low value of assets in place and free cash (v = a - x + c). Otherwise, a partial repurchase equilibrium prevails (the firm always buys in the other state with free cash, v = a + x + c). When $\delta = 1$, for the firm to repurchase in the state with a low value of assets in place and free cash, the price must not be higher than the stock value in this state. This is because when $\delta = 1$, condition (7) becomes p < a - x + c. The equilibrium price, in turn, must provide the market-maker with zero expected profit and hence reflects the expected value, pushed somewhat higher to reflect the level of adverse selection associated with the repurchase. Adverse selection, however, is positively correlated with both variability in the value of assets in place, x/a, and with the level of free cash when the firm does have cash, c. When x/a is significant, the price that the market-maker sets to earn zero expected profit will always be too high for the firm to repurchase in the state with low asset value and with cash (v = a - x + c), no matter what the value of c is. However, when x/a is low enough, its effect on the price becomes less significant, so that when c is high enough, the stock value with free cash will be higher than the price that gives the market-maker zero expected profit even if the value of assets in place is realized to be low and a partial repurchase equilibrium cannot hold. Thus a full repurchase equilibrium will prevail.

Fig. 3 graphs the results in Proposition 1. It illustrates how the decision on whether to repurchase in the state v = a - x + c depends on the variability in the value of assets in place, x/a, and the level of free cash when the firm does have free cash, c. The vertical dashed line indicates where condition (10) holds with equality. To the right of this line, the variability in the firm value, introduced through the variability in assets in place, is too high so that it pushes the stock price too high for a full repurchase equilibrium to exist. Thus a partial repurchase equilibrium prevails. To the left of the dashed line, the type of equilibrium depends on the level of free cash. Specifically, the solid curved line indicates where condition (11) holds with equality. Below this curved line, the level of free cash when the firm does have free cash is too low (c is too low), so that in the state with low asset value and cash, the firm will not repurchase, and hence partial repurchase equilibrium prevails. Above this line (c is sufficiently high), a full repurchase equilibrium prevails. Note that a deeper market (larger Q/N) means that the dashed line gets pushed to the right, and the solid curved line gets pushed down, so that the region where a full equilibrium prevails is wider and the region where a partial equilibrium prevails is narrower. Also, in the region where x/a < 1/(N/Q + 2), the smaller x/a is, the lower the required level of c for a full repurchase equilibrium to exist.



Fig. 3. Existence of a partial repurchase equilibrium and a full repurchase equilibrium when $\delta = 1$. This figure demonstrates the results in Proposition 1. The figure illustrates how the decision of whether or not to repurchase in the state v = a - x + c depends on the variability in the value of assets in place, x/a, and the level of free cash when the firm does have cash, c. The vertical dashed line indicates where condition (10) holds with equality. To the right of this line the variability in the firm value, introduced through the variability in assets in place, is too high so that it pushes the stock price too high for a full repurchase equilibrium to exist, and therefore a partial repurchase equilibrium prevails. To the left of the dashed line, equilibrium type depends on the level of free cash. Specifically, the solid curved line indicates where condition (11) holds with equality. Below this curved line, the level of free cash, when the firm does have free cash is too low (c is too low), so that in the state with low asset value and cash the firm will not repurchase and hence a partial repurchase equilibrium prevails. Above this line (c is sufficiently high), a full repurchase equilibrium prevails.

3.1.2. Special case 2: x = 0

Suppose there is no variability in the value of assets in place, i.e., x = 0.

Proposition 2. When x = 0, a full repurchase equilibrium always exists, and a partial repurchase equilibrium never exists.

Intuitively, when there is no variability in the value of assets in place, only free cash determines the variability in the firm value. Because the market-maker sets a price to earn zero expected profit, the firm will be undervalued whenever it has free cash. Thus, regardless of the waste rate, the firm will always repurchase when it has free cash. If, in addition, $\delta < 1$, the repurchase completely prevents the waste of free cash.

3.2. The general case

In the general case, a full repurchase equilibrium and a partial repurchase equilibrium are not mutually exclusive and consequently must be analyzed separately. We first investigate the existence of a full repurchase equilibrium based on condition (8) and then investigate the existence of a partial repurchase equilibrium based on condition (9).

3.2.1. Full repurchase equilibrium

Proposition 3. Existence of full repurchase equilibrium. A full repurchase equilibrium always exists if

$$\delta \le 1 - \frac{x}{a},\tag{12}$$

and never exists if

$$\left(1 - \frac{x}{a}\right)\frac{\frac{N}{Q} + 2}{\frac{N}{Q} + 1} \le \delta.$$
(13)

In the range

$$\left(1-\frac{x}{a}\right) < \delta < \left(1-\frac{x}{a}\right)\frac{\frac{N}{Q}+2}{\frac{N}{Q}+1},\tag{14}$$

a full repurchase equilibrium exists if $c \ge c^F$, where

$$c^{F}\left(a,x,\delta,\frac{N}{Q}\right) = \frac{\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right]}{\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{2N}{Q}+4\right)-2\left(\frac{N}{Q}+1\right)},\tag{15}$$

and does not exist otherwise.

Proposition 3 suggests that the existence of a full repurchase equilibrium depends primarily on the relation between δ and x/a. Only when δ and x/a meet particular joint conditions does the relation between these variables and c also matter. Specifically, when both δ and x/a are low, a full repurchase equilibrium always holds, and when both δ and x/a are high, a full repurchase equilibrium never holds. Otherwise, if both are neither too low nor too high, existence will depend on the level of free cash when the firm does have cash c, where there is some level of c, namely, c^F , defined in (15), above which a full repurchase equilibrium exists and below which it does not. Thus, in this region, for a high enough c, a full repurchase equilibrium will exist.

Fig. 4 demonstrates the results of Proposition 3 by means of a graph. The figure illustrates how the existence of a full repurchase equilibrium depends on the variability in the value of assets in place, x/a, and the waste rate captured by δ . The solid line indicates where condition (12) holds with equality. The dotted line indicates where condition (13) holds with equality. In the area above the dotted line, a full repurchase equilibrium never exists. In the area below the solid line, a full repurchase equilibrium always exists. In the area captured between the lines (where (14) holds), a full repurchase equilibrium exists if $c > c^F$, where c^F is defined in (15).

In the area above the dotted line in Fig. 4, δ is high (free cash waste is not significant) and hence the intuition in Proposition 1 for the case with high x/a still goes through. That is, if δ and x/a are high (up and to the right of the dotted line in the figure), the waste of free cash is not material, whereas the adverse selection introduced though the variability in



Fig. 4. Existence of a full repurchase equilibrium. This figure demonstrates the results of Proposition 3. The figure illustrates how the existence of a full repurchase equilibrium depends on the variability in the value of assets in place, x/a, and the waste rate captured by δ . The solid (curved) line indicates where condition (12) holds with equality. The dotted (curved) line indicates where condition (13) holds with equality. In the area above the dotted line, a full repurchase equilibrium never exists. In the area below the solid line, a full repurchase equilibrium always exists. In the area captured between the lines where (14) holds, a full repurchase equilibrium exits if $c > c^F$, where c^F is defined in (15).

value of assets in place is strong, so that the ask price that the market-maker sets is very high. Consequently, in the state in which the value of assets in place is realized to be low, the firm is better off not repurchasing. Although in this case some free cash is lost, thereby reducing shareholders' value, the alternative of paying too much for the shares would hurt share value even more. Between the lines, the effect of free cash waste becomes significant, so that the intuition of Proposition 1 no longer holds. Specifically, in this region, variability in the value of assets in place still motivates no repurchase in the state with low asset value (with free cash) but the potential benefit from preventing free cash loss is now significant and motivates a repurchase. Consequently, in this region, existence of a full repurchase equilibrium depends on the level of free cash when the firm does have cash, c. A higher level of free cash amplifies the benefit from waste prevention more than it magnifies the loss from paying a higher price set by the market-maker to compensate for higher adverse selection. This is not only because the benefits from waste prevention are higher, but also because these benefits are shared by a reduced number of shares. Thus, in this region, there is some level of c, namely, c^F , above which a full repurchase equilibrium exists and below which it does not. Below the solid line, a full repurchase equilibrium always exists. In this region, the waste rate is so high relative to the variability of asset value that the firm is willing to buy back shares even if the price is very high. This is because the alternative is losing most of the cash and severely damaging firm value.

Finally, note that higher N/Q does not affect the solid line in Fig. 4, but does push the dotted line down, making a full repurchase equilibrium less probably. This is because lower liquidity increases the effect of adverse selection on price (pushes it up) so that, other things equal, when the existence of a full repurchase equilibrium does depend on adverse selection

(i.e., when the waste does not dominate), a full repurchase equilibrium is less likely to prevail.

3.2.2. Partial repurchase equilibrium

Proposition 4. Existence of partial repurchase equilibrium.

Case 1: Suppose

$$\frac{x}{a} < \frac{2Q}{N},$$

then

$$1 - \frac{x}{a} < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}}.$$

In this case, a partial repurchase equilibrium never exists if

$$\delta \le 1 - \frac{x}{a} \tag{16}$$

and always exists if

$$\left(1 - \frac{x}{a}\right)\frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}} \le \delta.$$
(17)

In the range

$$1 - \frac{x}{a} < \delta < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}},\tag{18}$$

a partial repurchase equilibrium exists if $c < c^{P}$, where

$$c^{P}\left(a,x,\delta,\frac{N}{Q}\right) \equiv \frac{\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right]}{\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{N}{Q}+4\right)-2-\left(1+\frac{x}{a}\right)\frac{N}{Q}},\tag{19}$$

and does not exist otherwise. Case 2: Suppose

$$\frac{x}{a} > \frac{2Q}{N},$$

then

$$\left(1-\frac{x}{a}\right)\frac{\left(\frac{N}{Q}+4\right)}{2+\left(1+\frac{x}{a}\right)\frac{N}{Q}} < 1-\frac{x}{a}$$

In this case, a partial repurchase equilibrium never exists if

$$\delta \le \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}},\tag{20}$$

and always exists if

$$1 - \frac{x}{a} \le \delta. \tag{21}$$

In the range

$$\left(1-\frac{x}{a}\right)\frac{\left(\frac{N}{Q}+4\right)}{2+\left(1+\frac{x}{a}\right)\frac{N}{Q}} <\delta < 1-\frac{x}{a},$$
(22)

a partial repurchase equilibrium exists if $c > c^{P}$, where c^{P} is defined in (19) and does not exist otherwise.

Case 3: Suppose

$$\frac{x}{a} = \frac{2Q}{N},$$

then

$$\left(1-\frac{x}{a}\right)\frac{\left(\frac{N}{Q}+4\right)}{2+\left(1+\frac{x}{a}\right)\frac{N}{Q}} = 1-\frac{x}{a}.$$

In this case a partial repurchase equilibrium never exists if

$$\delta \le 1 - \frac{x}{a},$$

and always exists otherwise.

Proposition 4 seems complex because the relation between the restrictions on δ changes with x/a, and three separate cases must thus be considered. However, the same results are obtained in all three cases: existence of a partial repurchase equilibrium depends primarily on the relation between δ and x/a. Only in a certain region does the relation between these variables and c also matter. Specifically, when both δ and x/a are low, a partial repurchase equilibrium never holds, and when both δ and x/a are high, a partial repurchase equilibrium always holds. Otherwise, the existence of a partial repurchase equilibrium depends on the level of free cash c as follows. If δ is high but x/a is low, there is some level

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Fig. 5. Existence of a partial repurchase equilibrium. This figure demonstrates the results of Proposition 4. The figure illustrates how the existence of partial repurchase equilibrium depends on the variability in the value of assets in place, x/a, and the waste rate captured by δ (high waste rate is low δ). As in Fig. 4, the solid (curved) line indicates where condition (16) holds with equality. The dashed (curved) line indicates where condition (17) holds with equality. In the area above both lines, a partial repurchase equilibrium always exists. In the area below both lines, a partial repurchase equilibrium never exists. In the area captured between the lines, to the left of their crossing point, a partial repurchase equilibrium exists if $c < c^P$, where c^P is defined in (19), whereas in the area captured between these lines to the right of their crossing point, a partial repurchase equilibrium exists if $c > c^P$. (A partial repurchase equilibrium does not exist at the crossing point.)

of c, namely, c_P , given in (19), below which a partial repurchase equilibrium exists and above which it does not exist; if x/a is high but δ is low, a partial repurchase equilibrium exists above that same level of c and does not exist below that level.

Fig. 5 demonstrates the results of Proposition 4. The figure illustrates how the existence of partial repurchase equilibrium depends on the variability in the value of assets in place, x/a, and the waste rate captured by δ (high waste rate is low δ). As in Fig. 3, the solid line indicates where condition (16) holds with equality. The dashed curved line indicates where condition (17) holds with equality. In the area above both lines, a partial repurchase equilibrium always exists. In the area below both lines, a partial repurchase equilibrium never exists. In the area captured between the lines, to the left of their crossing point, a partial repurchase equilibrium exists if $c < c^P$, whereas in the area captured between these lines to the right of their crossing point, a partial repurchase equilibrium exists if $c > c^P$. (A partial repurchase equilibrium does not exist at the crossing point.)

In the area above the solid line in Fig. 5, δ is relatively high (the waste of cash is not significant), so that the intuition of Proposition 1 still holds. That is, if x/a and δ are high (to the right and above both curved lines in Fig. 5), the effect of the variability in value of assets in place on the price is strong enough to deter the firm from repurchasing if the value of assets in place is realized to be low, whatever the realized level of free cash. Above the solid line and to the left of the dashed line (between the two lines and left of their crossing

point), the variability of assets in place is low enough to make the variability in the level of cash important. If there is little free cash (when $c < c^P$), a partial equilibrium will exist. Otherwise, if free cash is high (when $c \ge c^P$), a partial repurchase equilibrium cannot exist, because in the states in which the firm does have free cash, given the price set by the market-maker to earn zero expected profit, repurchase results in higher terminal stock value even if the value of assets in place is realized to be low.

The situation is different than in that of Proposition 1 when δ becomes significantly low (below the solid line), because in this region the waste of free cash becomes significant so that the firm will repurchase regardless of the price and therefore a partial repurchase equilibrium never holds. This is in turn because the unappealing alternative is to watch the free cash disappear without contributing to the firm's value. A partial repurchase equilibrium may still hold, though, even if the waste rate is high (δ low) if there is enough variability in the value of assets in place (between the lines and to the right of their crossing point). In this region, because there is considerable variability in the value of assets in place, a higher level of free cash magnifies wealth expropriations through adverse selection more than it magnifies benefits from waste prevention. As a result, the price that assures zero expected profit to the market-maker, assuming that the firm buys only in the high state, increases very quickly in *c*. Consequently, in this region, there is some level of *c*, namely, c^P , below which a partial repurchase equilibrium cannot exist but above which it can.

Last, note that higher N/Q (lower liquidity) does not affect the solid line in Fig. 5 but does push the dashed line down and to the right, thus making a partial repurchase equilibrium more likely. This is because lower liquidity amplifies the effect of adverse selection on price (increases the price), so that other things equal, a partial repurchase equilibrium is more likely to prevail.

3.2.3. Coexistence of full and partial repurchase equilibrium

Fig. 6 combines Figs. 4 and 5 to demonstrate the ranges of existence for both partial and full repurchase equilibria, depending on x/a and δ . In the area below both the solid and the dashed lines (to the left of their crossing point and below the solid line), both x/a and δ are low, hence only a full repurchase equilibrium exists. However, in the area below the solid line above the dashed line (down and right of their crossing point between the lines), δ is low but x/a is now relatively high, so a partial repurchase equilibrium can also exist if $c < c^P$. In the area above the dotted line, both δ and x/a are high, hence only a partial repurchase equilibrium exists. However, in the area below the dotted line but above both the dashed and the solid lines, x/a is high but δ is relatively not as high so that a full repurchase equilibrium can also exist if $c > c^F$.

In the area above the solid line but below the dashed line (up and left of their crossing point between the lines in Fig. 6), the existence of both equilibria depends on the level of c. If c is low enough that $c < c^F$, only a partial repurchase equilibrium exists, and if c is high enough that $c > c^P$, only a full repurchase equilibrium exists. If $c^F < c < c^P$, that is, if

$$\frac{\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right]}{\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{2N}{Q}+4\right)-2\left(\frac{N}{Q}+1\right)} < c < \frac{\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right]}{\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{N}{Q}+4\right)-2-\left(1+\frac{x}{a}\right)\frac{N}{Q}}, \tag{23}$$



Fig. 6. Existence of partial repurchase equilibrium and full repurchase equilibrium. Fig. 6 combines Figs. 4 and 5 to demonstrate the ranges of existence for both partial and full repurchase equilibria depending on x/a and δ . In the area below both the solid and the dashed lines (i.e., to the left of their crossing point and below the solid line), both x/a and δ are low, hence only full repurchase equilibrium exists. However, in the area below the solid line above the dashed line (i.e., to the right of their crossing point between the lines), δ is low but x/a is now relatively high, so a partial repurchase equilibrium can also exist if $c < c^P$. In the area above the dotted line, both δ and x/a are high, hence only a partial repurchase equilibrium exists. However, in the area below the dotted line but above both the dashed and the solid lines, x/a is high but δ is relatively not as high so that a full repurchase equilibrium can also exist if $c > c^F$. In the area above the crossing point of these lines), the existence of both equilibria depends on the level of c. If c is sufficiently low to render $c < c^P$, only a partial repurchase equilibrium exists, and if c is sufficiently high to render $c > c^F$, only a full repurchase equilibrium exists. If $c^F < c < c^P$, both partial and full repurchase equilibria can exist.

both partial and full repurchase equilibria can exist. Consider condition (23). If $\delta = 1$, both limits on *c* are identical, and the range is thus empty. In this case, as in the special case $\delta = 1$, a full repurchase equilibrium and a partial repurchase equilibrium are mutually exclusive. For any $\delta < 1$, there is a range of values of *c* where both equilibria exist. Within the discussed area (the triangle shaped area in Fig. 6), this range of *c* indicated in (23) widens with the decrease in δ . Note that because this area is the only area where the existence of both full and partial repurchase equilibria depends on *c*, and because the range given in (23) is nonempty except when $\delta = 1$, an equilibrium always exists.¹⁵

The following proposition provides comparative statics results on c^F and c^P .

Proposition 5. When the existence of a full repurchase equilibrium depends on c, the value of c^F increases in x/a and δ . When the existence of a partial repurchase equilibrium depends

¹⁵It could be argued instead that when both equilibria coexist, only the full repurchase equilibrium would stand refinements (e.g., the intuitive criterion; see for example Kreps, 1990, p. 418).

on c, the value of c^{P} increases in x/a and δ for

$$\frac{x}{a} < \frac{2Q}{N},$$

and decreases with x/a and δ otherwise.

The results on c^F and c^P in Proposition 5 are consistent with the intuition we gave earlier for Propositions 3 and 4, respectively. Specifically, when the existence of a full repurchase equilibrium depends on c, the level of c above which a full repurchase equilibrium can hold, c^F , increases with uncertainty and declines with waste. This is consistent with the intuition we gave earlier for Proposition 3. Similarly, when the existence of a partial repurchase equilibrium depends on c, when uncertainty is low, the level of c below which a partial repurchase equilibrium can hold, c^P , increases with uncertainty and declines with waste; when uncertainty is high, the level of c above which a partial repurchase equilibrium can hold declines with uncertainty and increases with waste. This is consistent with the intuition we gave earlier for Proposition 4.

The following example demonstrates coexistence of partial and full repurchase equilibria.

Example 2. Suppose A = 12, X = 5, C = 5, N = 10, $Q_A = 1$, and $\delta = 0.55$. Then there exists a partial repurchase equilibrium for p = 1.808. There also exists a full repurchase equilibrium for p = 1.627. In Fig. 6, these equilibria are between the dashed and the solid lines and to the right of their crossing point.¹⁶ To understand the coexistence result, suppose the market-maker sets the price at p = 1.808. Condition (4) becomes

$$(1.808 - 0.7) + (1.808 - (0.7 + 0.55 \times 0.5)) + (1.808 - (0.7 + 1)) + \left(1.808 - \frac{12 + 5}{10 - \frac{5}{1.808}}\right) \left(1 + \frac{5}{1.808}\right) = 0.$$

The market-maker gains in the states $\{a - x, a + x, a - x + c\}$ and loses in the state $\{a + x + c\}$. He makes zero expected profit. The firm does not repurchase in the state a - x + c and will not deviate; if it did, the terminal value per share would be 7/(10 - 5/1.808) = 0.968, which is lower than 0.975 (the terminal value without a repurchase in that state). Thus, a partial repurchase equilibrium exists. Now suppose the market-maker sets a price p = 1.627. Condition (3) becomes

$$(1.627 - 0.7) + \left(1.627 - \frac{7}{10 - \frac{5}{1.627}}\right) \left(1 + \frac{5}{1.627}\right) + (1.627 - (0.7 + 1)) + \left(1.627 - \frac{12 + 5}{10 - \frac{5}{1.627}}\right) \left(1 + \frac{5}{1.627}\right) = 0.$$

¹⁶Namely, in the region where the existence of a full repurchase equilibrium does not depend on *c* and where the existence of a partial repurchase equilibrium requires $c > c^P$. Indeed, one can verify that for the chosen parameter set, condition (12) holds (0.55<0.583), condition (22) holds (0.505<0.583), and $c = 0.5 > c^P = 0.234$.

The market-maker gains in the states $\{a - x, a - x + c\}$ and loses in the state $\{a + x, a + x + c\}$. He makes zero expected profit. The firm repurchases whenever it has free cash. The firm does repurchase in the state $\{a - x + c\}$; if it does not, the terminal value per share would be 0.975, which is lower than 7/(10 - 5/1.627) = 1.014 (the terminal value with a repurchase in that state).

The intuition for the coexistence here is as follows. Going from the partial repurchase equilibrium to the full repurchase equilibrium, the market-maker reduces the price and therefore his gain in the state $\{a - x\}$ is reduced, he now loses in the state $\{a + x\}$, and his loss in the state $\{a + x + c\}$ is increased. With that lower price, however, it now pays the firm to repurchase in the state $\{a - x + c\}$. The market-maker now makes money in the state $\{a - x + c\}$, not only at the liquidity buyers expense, but also at the firm's expense. This additional gain in the state $\{a - x + c\}$ compensates the market-maker for lower gains in the other states, and again he ends up with zero expected profit. The firm pays 1.627 per share to realize only 1.0147 on the terminal date, but it is happy to do so; if it does not, $1 - \delta = 45\%$ of the cash will be lost, which is more than its trading loss $1 - \frac{1.014}{1.627} = 37.6\%$.

3.3. The good equilibrium and the bad equilibrium

In this section, we show that the full repurchase equilibrium is indeed better than the partial repurchase equilibrium, as Examples 1 and 2 suggest. We demonstrate that, in a full repurchase equilibrium, completion rate and social wealth are higher while bid–ask spread and wealth expropriation are lower. We first consider completion rate and social wealth and then bid–ask spread and wealth expropriation. In the analysis, we must make sure we do not compare apples to oranges. For example, it will not be correct to compare the social wealth improvement (relative to no repurchase) in a full repurchase equilibrium under a high δ to the social wealth improvement in a partial repurchase equilibrium under a low δ .

3.3.1. Completion rate and social wealth

By definition, the completion rate in a full repurchase equilibrium is higher than the completion rate in a partial repurchase equilibrium. Similarly, implications about social wealth are immediate from the analysis of full versus partial repurchase equilibrium and the value of δ . Specifically, the completion rate is 50% in a full repurchase equilibrium and only 25% in a partial repurchase equilibrium.¹⁷ In a full repurchase equilibrium, all the loss incurred in the case of no repurchase is saved. In a partial repurchase equilibrium, only half of this loss is saved. Expected social wealth increases by $0.5(1 - \delta)C$ with a full repurchase equilibrium (relative to no repurchase).

3.3.2. Prices, bid-ask spread, and wealth expropriation

We now show that the bid-ask spread and wealth expropriations (transfers from the liquidity buyers to the original shareholders) are both lower in a full repurchase

¹⁷Of course, this is because we have assumed four states with equal probabilities and also that the firm has cash available for repurchase only in two states. Assuming a different distribution of states would result in different completion rates, but would not alter the qualitative result.

equilibrium than in a partial repurchase equilibrium.¹⁸ Because here we need to consider the bid price, in this subsection we revert to explicitly indicating bid and ask prices p_A , p_B and quantities Q_A , Q_B at t = 1. (At t = 0, 2 or without a repurchase there is no information asymmetry/adverse selection rendering this notation irrelevant.)

Lemma 4. In any equilibrium, full or partial, $p_0 = p_B = E[p_2]$.

The following properties are helpful for the analysis. First, because at t = 2 all information is public, then $p_2 = v_2$, and accordingly maximizing v_2 is equivalent to maximizing p_2 . Second, since maximizing p_2 results in maximizing $E[p_2]$, and since by Lemma 4 $p_B = E[p_2]$, maximizing the value of terminal shares is equivalent to maximizing the value of original shareholders' wealth. In comparison to the situation without repurchase, original shareholders who sell at t = 1 gain the difference between the expected terminal price without repurchase and the expected terminal price with it, where $E[p_2] = p_B$. Original shareholders who sell at t = 2 gain the same. Third, since $E[p_2] = p_B$, and since the loss per share of liquidity buyers is $p_A - E[p_2]$, the bid-ask spread is equal to the loss per share of liquidity buyers. Accordingly, aggregate loss of liquidity buyers is

$$Q_A(p_A - E[p_2]).$$

The focus of our analysis here is thus on the difference, $p_A - E[p_2]$.

Note also that, since the market-maker gets zero expected profit, the expected gain of original shareholders can be calculated as the sum of the increase in social wealth and the loss of the liquidity buyers. We do not care when original shareholders sell, because there is no adverse selection in the sell market, and hence they are paid $E[p_2]$ per share whenever they sell. There is thus no need to distinguish between original shareholders groups (short-term shareholders who sell at t = 1 and long-term shareholders who sell at t = 2).

Propositions 6 and 7 generalize the results in Examples 2 and 1, respectively.

Proposition 6. Whenever a full repurchase equilibrium and a partial repurchase equilibrium coexist, the full repurchase equilibrium leads to a higher completion rate, greater social wealth, narrower bid–ask spread, and less wealth expropriation.

Proposition 7. Fix all parameters except x. Then, in a full repurchase equilibrium, the bid-ask spread and wealth transfers are independent of x/a, whereas in a partial repurchase equilibrium, they increase with x/a.

Propositions 6 and 7 together establish that a full repurchase equilibrium is better than a partial repurchase equilibrium in all four dimensions: completion rate, social wealth, bid–ask spread, and wealth expropriations. This is so not only when they coexist, but also when we move along horizontal lines in Fig. 6. We suggest that this is as far as a comparison can go without comparing apples to oranges.

¹⁸The bid–ask spread is one measure of liquidity. Another is market depth, which is the price impact of order imbalances (see O'Hara, 1995). Cook et al. (2004) consider also the number of trading days. In our model, we could measure average trade volume. Consequently, the expected increase in liquidity with a repurchase, in comparison to no repurchase, is C/4p in a partial repurchase equilibrium and C/2p in a full repurchase equilibrium. Thus, trade volume is higher in a full repurchase equilibrium. We could also measure depth with bid–ask spread/volume change. However because, in the model, bid–ask prices are set before investors place their bids, predictions about depth would be consistent with predictions about the bid–ask spread. Thus we use only the bid–ask spread to measure liquidity.

3.4. Implications

The model generates testable predictions about actual repurchase characteristics. Specifically, a full repurchase equilibrium means a higher completion rate, more social wealth, narrower bid–ask spread, and less wealth expropriation. A full repurchase equilibrium is more likely to prevail when x/a is low (low variability of assets in place), which could be interpreted as low uncertainty of assets value, i.e., in mature industries as opposed to growth industries, or in large firms as opposed to small firms. Low x/a could also be interpreted as low information asymmetry, which is associated with efficient and transparent financial markets.¹⁹ A full repurchase equilibrium is also more likely to prevail when δ is low (high potential for waste of free cash), which could be interpreted as a lack of positive NPV projects, low management efficiency, or poor governance quality. In general, the model also predicts that, when the values of x/a and δ are not extreme, a high uncertainty of free cash c will make a full repurchase equilibrium more likely.²⁰ Thus, the model predicts different levels of completion rates and different levels of widening of the spread depending on firm and industry characteristics.²¹

4. Empirical analysis of bid-ask spread and program completion

One testable prediction of the model is that the program completion rate and the bid-ask spread are negatively correlated. In this section, we use a sample of U.S. firms to test this prediction. The results of the empirical analysis strongly support it.

4.1. Data and methodology

4.1.1. Sample selection

The sample of repurchases is from the SDC's Mergers and Acquisitions database. We start with all open-market repurchase announcements with announcement dates between January 1, 2003, and December 31, 2004 (two years), resulting in a sample of 928 announcements. We eliminate 480 announcements that are either multiple announcements by the same firm or followed by additional announcements from the same firm within two years of the end of the announcement quarter.²² We eliminate 31 additional firms for which we do not have sufficient return data on CRSP for the study period and 41 firms without sufficient actual repurchase data on Compustat. Two more firms were eliminated during the analysis because of erroneous actual repurchase data. The final sample consists of 374 firms: 170 firms with announcements in 2003, and 204 with announcements in 2004.

¹⁹In the model, the level of uncertainty determines the degree of information asymmetry. In practice, however, uncertainty is a necessary but not a sufficient condition for information asymmetry.

²⁰When x/a is high and δ is low, there is a region where a partial repurchase equilibrium could exist if *c* is high enough. In this region, however, a full repurchase equilibrium always exists regardless of *c* (see Section 3.2.3).

²¹In its current setup, the model cannot predict narrowing of the spread. This is because our benchmark at t = 0 is a firm with no asymmetric information and no program, in which case the bid–ask spread is zero. It might be possible to generate narrowing of the spread in a setup in which information is already asymmetric at t = 0 and in which the announcement reveals some sort of information. The tension between information revealed and adverse selection created will determine whether, following an announcement, the outcome would be narrowing, widening, or no change (of the spread).

²²Firms that announce a repeated program, often do so before finishing the earlier program. Once the repeated program is announced, it is impossible to separate the actual repurchases to the different programs.

The mean and median program size in dollars was \$213.8 million and \$29.9 million, respectively. The smallest program was \$0.1 million, and the largest was \$8 billion (Viacom Inc.). The mean and median percentage of the shares sought was 7.8% and 5.7% of the outstanding shares, respectively. The smallest program was for 0.5% of the outstanding shares and the largest was for 59.1% of the outstanding shares.²³

4.1.2. Measuring program completion rate

The dollar amount spent on actual repurchases is reported on the Statement of Cash Flow and is provided on Compustat files as Purchases of Common and Preferred Stock (Compustat quarterly data item #93).²⁴ In the U.S., firms do not report the expected ending day of the program in their announcements. Stephens and Weisbach (1998) and Kahle (2002), however, suggest that firms repurchase most of what they will repurchase within two years of the announcement date. Accordingly, we calculate actual repurchase as the reported dollar amount spent on repurchases beginning with the quarter of the announcement and ending after the eighth quarter after the plan announcement (total of nine quarters). The completion rate of the program is then calculated as the ratio between the accumulated amount spent on repurchase and the size of the program in dollars stated in the announcement.

4.1.3. Measuring bid-ask spread

For the bid–ask spread, we first use the CRSP daily closing bid and ask prices. We calculate the daily percentage spread as $(Ask - Bid)/((Ask + Bid) \times 0.5) \times 100$. We then use the equally weighted average of the nine quarters as our first measure for the bid–ask spread. The correlation between this CRSP-based measure and Hasbrouck's (2006) Gibbs measure is 87%, suggesting this CRSP-based spread is a reasonable measure of the spread.²⁵

Second, because our theory deals with illiquidity, of which the bid–ask spread is only one measure, we adapt the ILLIQ measure in Amihud (2002) as a proxy for price impact. ILLIQ is the daily ratio of absolute stock return to its dollar volume in millions, averaged over some period. It can be interpreted as the daily price response associated with \$1 million of trading volume, providing a rough measure of price impact. Although this measure is based on daily data, and is hence less accurate than intra-day based measures (e.g., Brennan and Subrahmanyam, 1996; Easley et al., 2002), it is readily available. Our analysis is performed using the logarithm of ILLIQ (henceforth ln *ILLIQ*) rather than ILLIQ.²⁶ If our theory is correct, both the bid–ask spread and ln *ILLIQ* should be negatively correlated with the repurchase program completion rate.

²³SDC reports program size announced in dollars for all firms. We calculate the fraction of shares sought by dividing the program size in dollars by the market value of the firm on the announcement day.

²⁴This measure may overstate program completion because it aggregates all security repurchases and retirements during the quarter. Alternative measures are available but are less accurate (Banyi et al., 2005).

²⁵CRSP's reporting of bid and ask closing prices are inside quotations for the NASDAQ; for NYSE and AMEX securities, they are the prices from the last representative quote before the market close for each trading date. Bid and ask prices are set by CRSP to 0 when CRSP determines that the available quote was unrepresentative of trading activity. A small number of the spreads we calculated were negative and removed from the data. Because the Gibbs measure is yearly based, we could not use it for the analysis.

²⁶We use ln *ILLIQ* because several firms in the sample have extreme values of ILLIQ. The qualitative results are similar when ILLIQ is used.

4.2. Empirical results

4.2.1. Program completion rates

Table 2 provides statistics on the timing of actual repurchases by quarter following the announcement of the programs and by the percentage of firms repurchasing more or less than various benchmarks. The results suggest that (1) average actual repurchase rates are relatively high; (2) many firms do not repurchase any shares; and (3) many firms repurchase much more than what they announce. By the end of the eighth quarter after the announcement quarter, the unconditioned mean completion rate is 92%. About 15% of the firms spent more than 200% of what they originally announced, and about 35% of the firms spent less than 1% of what they originally announced. These findings are consistent with those reported in Stephens and Weisbach (1998).

4.2.2. Cross-sectional sample statistics and correlations

While the mean program completion rate (*CR*) is 92%, the median is 50%. The big difference between the mean and median is because many firms repurchase several times more than what they originally announced. The mean CRSP-based percentage spread (*SP*) is 0.60%, and the median is 0.20%. The mean $\ln ILLIQ$ measure is -4.64, and the median is -5.17.

The Pearson correlation between CR and SP is -0.16, with a *p*-value < 0.0001. The correlation between CR and $\ln ILLIQ$ is -0.31, with a *p*-value < 0.0001. These negative correlations strongly support the model prediction that the bid–ask spread and the program completion rate are negatively correlated. The correlation between SP and $\ln ILLIQ$ is 0.73, with a *p*-value < 0.0001.

We perform several difference of means tests. First, we sort the sample by CR and divide it into two equally sized groups: one with below-median CR and one with above-median CR. We then test for the difference of the mean of the spread (SP) between these groups. We repeat this test sorting the sample by SP and testing the difference of means of CRbetween these groups. In both tests, higher mean CR is associated with lower mean SP, consistent with the model prediction, and the difference of the means is statistically significant (p-value < 0.0001).

We then repeat these difference of means tests substituting $\ln ILLIQ$ for SP. Again, in both tests, higher mean CR is associated with lower mean SP, and the difference of the means is statistically significant (p-value <0.0001).

4.2.3. Cross-sectional regressions

Stoll (1978) shows that percentage spreads are positively correlated with return variance and negatively correlated with price and trade volume. Following Franz et al. (1995) and Miller and McConnell (1995), we perform a multivariate analysis that controls for these factors. Specifically, in addition to running a univariate regression in which the dependent variable is the bid–ask spread and the independent variable is the completion rate, we run a regression in which the dependent variable is the bid–ask spread and the independent variables are (1) price, (2) trade volume, (3) return volatility, and (4) completion rate.²⁷

²⁷We choose the bid–ask spread as the dependent variable and the completion rate as the independent variable rather than the other way around for consistency with the earlier literature on repurchases and bid–ask spreads. Regressions substituting firm size for trade volume as a control variable yield similar results.

Repurchase measure	Quarter relative to announcement								
		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
Panel A: Cumulative quarterly repurchase									
Unconditioned accumulation (%)	15.9	28.2	36.9	47.1	56.1	64.4	74.8	85.5	92.3
Accumulation of >0 (%)	23.6	42.2	54.9	70.4	84.2	96.9	112.7	129.1	139.5
Accumulation of truncation at 2 (%)	15.6	27.3	35.2	42.8	50.2	56.5	63.0	69.0	72.5
Accumulation of >0 and truncation at 2 (%)	23.3	40.7	52.2	63.8	75.1	84.7	94.7	103.8	109.2
Panel B: Percentage of firms repurchasing more	e or less	than a	nnoun	ced am	ount				
More than 200% of announced shares (%)	0.27	1.3	2.7	4.8	5.9	8.3	10.4	13.1	15.0
More than 100% of announced shares (%)	2.67	6.7	9.4	13.6	18.4	23.0	26.5	30.5	32.9
More than 50% of announced shares (%)	10.16	20.3	29.7	35.0	39.6	43.9	47.3	49.2	49.7
Less than 20% of announced shares (%)	74.60	61.0	54.3	50.5	47.3	44.4	43.3	42.2	41.7
Less than 5% of announced shares (%)	57.22	46.5	43.6	42.5	40.1	38.2	38.2	37.2	37.2
Less than 1% of announced shares (%)	50.00	41.2	39.0	38.2	36.9	35.3	35.3	34.8	34.8

Table 2Program completion rate through time relative to announcement.

Panel A reports the average quarterly completion rate (*CR*) of repurchases relative to the announcement, beginning with the announcement quarter and ending with the eighth quarter after the announcement quarter. Completion rates are expressed as the average ratio between the accumulated dollar value spent on repurchase (Compustat data item #93) relative to the dollar size of the program declared in the announcement. The first line in Panel A reports unconditional completion rates. "Accumulation of >0" is *CR* of the sub-sample that excludes firms that did not repurchase any shares following the announcement of their program. "Accumulation of truncation at 2" is average *CR* where *CR* is set to 2 once it reaches 200% of the announcement. Panel B presents the number of firms in the sample that have purchased at least a specified percentage of shares, or that have not repurchased a specified percentage of shares by the end of the eighth quarter after the announcement quarter.

The regression model is

$$SP_i = \alpha + \beta_1 CR_i + \beta_2 PRC_i + \beta_3 VOL_i + \beta_4 STD_i + \varepsilon_i,$$
(24)

where *SP* is the average percentage daily spread during the repurchase period (nine quarters), *PRC* is the average stock price during the repurchase period, *VOL* is the average daily trading volume in the stock during the repurchase period in millions of dollars, *STD* is the average of the quarterly variance where the quarterly variance is calculated from the daily return, and *CR* is the percentage completion rate of the program. We then repeat the regression analysis using ln *ILLIQ* as the dependent variable instead of the spread:

$$\ln ILLIQi = \alpha + \beta_1 CR_i + \beta_2 PRC_i + \beta_3 VOL_i + \beta_4 STD_i + \varepsilon_i.$$
⁽²⁵⁾

The results are reported in Table 3. The coefficient β_1 captures the correlation between the spread and actual repurchase. This coefficient is negative and statistically significant in all regressions (Columns 1–4). Our analysis thus suggests that the correlation between the bid–ask spread and the program completion rate is negative and significant, whether or not we control for price, trade volume, and variance of return.

In the multivariate regressions (Columns 2 and 4 in Table 3), the coefficients of *PRC* and *VOL* (β_2 and β_3 , respectively) are negative and significant, and the coefficient of *STD* (β_4) is positive and significant. The signs of these coefficients are consistent with the findings in Stoll (1978), Franz et al. (1995), and Miller and McConnell (1995).

	SP		Ln ILLIQ	
	(1)	(2)	(3)	(4)
Intercept	0.72	0.61	-3.87	-2.06
	(11.73)	(3.56)	(19.10)	(4.87)
CR	-0.13	-0.09	-0.83	-0.54
	(3.20)	(2.37)	(6.24)	(5.79)
PRC		-0.01		-0.08
		(2.98)		(9.95)
VOL		-0.14		-1.02
		(4.25)		(12.76)
STD		0.20		0.32
		(3.88)		(2.50)
R^2	0.027	0.189	0.095	0.567
Number of observations	374	374	374	374

Table 3 Cross-sectional regressions.

The table presents the coefficients from the cross-sectional regression of the firm's liquidity variables on the completion rate (CR) variable and the control variables. The sample contains 374 firms. Each firm has one observation based on nine quarters starting with the quarter in which the program is announced. SP is the CRSPbased bid-ask spread (in %), calculated as the equally weighted average of the nine quarter averages, where each quarter average is the daily equally weighted spread average based on closing bid and ask quotes on CRSP. Ln ILLIQ is the logarithm of the Amihud (2002) ILLIQ measure. ILLIQ, in turn, is calculated as the equally weighted average of the nine quarter averages, where each quarter average is the daily ILLIQ equally weighted average. CR is the aggregate of the nine quarter repurchases relative to the announcement. PRC is the equally weighted average of the nine quarter averages, where each quarter average is the daily split-adjusted close price equally weighted average. VOL is the equally weighted average of the nine quarter averages where each quarter average is the daily split adjusted trade volume equally weighted average (in millions). STD is the equally weighted average of the nine quarters standard deviation of the firm's returns, where each quarter standard deviation is the standard deviation of the daily returns in the quarter (in %). Columns 1 and 2 give the results of univariate and multivariate regressions, respectively, where SP is the dependent variable. Columns 3 and 4 give the results of the univariate and the multivariate regressions, respectively, where ln ILLIO is the dependent variable. t-statistics are in parentheses below the coefficients.

Because many firms repurchase much more than what they originally announce, we also run the cross-sectional regressions with Win-censoring at 100% and 200% (CR = 1 and 2). The results obtained were similar.

Overall, the results of our empirical analysis strongly support the prediction that bid–ask spreads and program completion rates are negatively correlated. More generally, they suggest that illiquidity and program completion rate are negatively correlated, as the model predicts.

5. Conclusion

In this paper we perform a theoretical investigation of the execution of open-market stock repurchase programs. Our model suggests that the execution depends on the agency costs of free cash that are realized, and information asymmetry that develops only after the program is announced. The results highlight important properties of open-market stock repurchase programs that other payout methods (e.g., self-tender-offers, dividends) do not have: they give the firm the option to avoid payout when cash is needed for operations, yet they also disburse free cash as long as the stock is not severely overpriced. Because they are performed at management discretion, however, repurchase programs also redistribute wealth among shareholders. The model generates predictions about the completion rate of repurchase programs and about the bid–ask spread during the repurchase period that might explain inconsistencies in earlier empirical studies. One new and testable prediction of our theory is that program completion rates and bid–ask spreads are negatively correlated. Tests using a sample of U.S. firms yield evidence consistent with this prediction.

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Appendix A. Proofs of lemmas and propositions

Proof of Lemma 1. In the states v = a - x and v = a + x, the firm does not have cash. Thus, by assumption, it cannot buy. In the state v = a + x + c, the firm will repurchase whenever the value of a terminal share with repurchase is higher than its value without it, i.e., whenever

$$a + x + \delta c < \frac{A + X}{N - \frac{C}{p}} = \frac{a + x}{1 - \frac{c}{p}},$$

which can be rearranged to

$$p < \frac{a+x}{\delta} + c,$$

which always holds, otherwise, the firm will never repurchase, implying that the market-maker is making a positive profit. \Box

Proof of Lemma 2. In any full repurchase equilibrium, condition (3) must hold, and, in addition, condition (7) must *not* hold. Upon substitution of A = Na, C = Nc, and X = Nx, and with rearrangement, we can write (3) as

$$p^{2} - p\left(a + c - \frac{Nc}{2Q}\right) + \left[\frac{ca}{2} - (a + c)\frac{Nc}{2Q}\right] = 0$$

or,

$$p = \frac{a+c-\frac{Nc}{2Q} \pm \sqrt{\left((a+c)-\frac{Nc}{2Q}\right)^2 + 2c\left[(a+c)\frac{N}{Q}-a\right]}}{2}$$

We can further reduce to the "+" solution (the only feasible solution), to get (5).

Next, in any partial repurchase equilibrium, condition (4) must hold, and, in addition, condition (7) must hold. Upon substitution of A = Na, C = Nc, and X = Nx, and with rearrangement, we can write (4) as

$$p^{2} - \left[a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c\right]p + \frac{c}{4}\left[(3a - x + \delta c) - (a + x + c)\frac{N}{Q}\right] = 0$$

or,

$$p = \frac{\left(a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c\right) \pm \sqrt{\left(a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c\right)^2 + c\left((a + x + c)\frac{N}{Q} - (3a - x + \delta c)\right)}}{2}.$$

We can further reduce to the "+" solution (the only feasible solution), to get (6). \Box

Proof of Lemma 3. Upon substitution of (5) into the complement of (7) (i.e., into $p \le (a - x)/\delta + c$), a necessary and sufficient condition for a full repurchase equilibrium is

$$\frac{a+c-\frac{Nc}{2Q}+\sqrt{\left((a+c)-\frac{Nc}{2Q}\right)^2+2c\left[(a+c)\frac{N}{Q}-a\right]}}{2} \le \frac{a-x}{\delta}+c,$$
(26)

which can be rearranged to

$$\frac{2a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right] \le c\left[\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(2+\frac{N}{Q}\right)-\left(1+\frac{N}{Q}\right)\right],$$

and further rearranged to get (8).

Upon substitution of (6) into (7), a necessary and sufficient condition for a partial repurchase equilibrium is

$$p = \frac{a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c + \sqrt{\left(a + \left(1 + \frac{\delta}{4} - \frac{N}{4Q}\right)c\right)^2 + c\left((a + x + c)\frac{N}{Q} - (3a - x + \delta c)\right)}}{2}$$
$$> \frac{a - x}{\delta} + c, \tag{27}$$

which can be rearranged to

$$\frac{4a}{\delta}\left(1-\frac{x}{a}\right)\left[1-\frac{1}{\delta}\left(1-\frac{x}{a}\right)\right] > c\left(\frac{1}{\delta}\left(1-\frac{x}{a}\right)\left(\frac{N}{Q}+4\right)-2-\left(1+\frac{x}{a}\right)\frac{N}{Q}\right),$$

and further rearranged to get (9). \Box

Proof of Proposition 1. When $\delta = 1$, conditions (8) and (9) become

$$2x\left(1-\frac{x}{a}\right) \le c\left[1-\frac{x}{a}\left(\frac{N}{Q}+2\right)\right]$$
(28)

and

$$2x\left(1-\frac{x}{a}\right) > c\left(1-\frac{x}{a}\left(\frac{N}{Q}+2\right)\right),$$

respectively, and are mutually exclusive. Thus, full repurchase is the outcome whenever (28) holds. Otherwise, the outcome is partial repurchase. Now if

$$1 \le \frac{x}{a} \left(\frac{N}{Q} + 2\right),$$

which is

$$\frac{1}{\frac{N}{Q}+2} \le \frac{x}{a},$$

condition (28) never holds (because x < a and x, a, and c are positive), so that the firm will never fully repurchase (always partially repurchase). Otherwise, if

$$\frac{x}{a} < \frac{1}{\frac{N}{Q} + 2},$$

condition (28) can be written as (11). Thus, in this case, the firm will fully repurchase if (11) holds and partially repurchase otherwise. \Box

Proof of Proposition 2. If $\delta = 1$, it is immediate from Proposition 1 that the firm will always fully repurchase. So consider the case with $0 < \delta < 1$. Conditions (8) and (9) become

$$\frac{4a}{\delta}\left(1-\frac{1}{\delta}\right) \le c\left(\frac{2}{\delta}\left(\frac{N}{Q}+2\right) - 2\left(\frac{N}{Q}+1\right)\right)$$
(29)

and

$$\frac{4a}{\delta}\left(1-\frac{1}{\delta}\right) > c\left(\frac{1}{\delta}\left(\frac{N}{Q}+4\right) - \left(\frac{N}{Q}+2\right)\right),\tag{30}$$

respectively. Condition (29) always holds because the LHS is negative and the multiplier of c in the RHS is always positive. Condition (30) never holds because the LHS is negative and the multiplier of c in the RHS is always positive. \Box

Proof of Proposition 3. Consider the condition for full repurchase equilibrium (8). The LHS of (8) is negative whenever

$$1 < \frac{1}{\delta} \left(1 - \frac{x}{a} \right),$$

that is, whenever

$$\delta < 1 - \frac{x}{a}.\tag{31}$$

The RHS of (8) is negative whenever

$$\frac{1}{\delta} \left(1 - \frac{x}{a} \right) \left(\frac{2N}{Q} + 4 \right) < 2 \left(\frac{N}{Q} + 1 \right),$$

that is, whenever

$$\left(1-\frac{x}{a}\right)\frac{\left(\frac{N}{Q}+2\right)}{\left(\frac{N}{Q}+1\right)} < \delta.$$

Note that

$$1 - \frac{x}{a} < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 2\right)}{\left(\frac{N}{Q} + 1\right)},$$

and hence, whenever (31) holds, (32) never holds, and whenever (32) holds, (31) never holds. Now suppose that condition (31) holds. Then condition (32) does *not* hold, and we can write (8) as $c \ge c^F$, where c^F is defined in (15). In this case, in c^F the numerator is negative, while the denominator is positive and, because *c* is positive, in this situation the condition $c \ge c^F$ always holds, i.e., (8) always holds. *Conclusion*: If condition (31) holds (i.e., if the inequality in (12) holds strictly), full repurchase equilibrium always exists.

Next, if condition (32) holds, we can write (8) as $c \le c^F$, where c^F is defined in (15). Since condition (32) holds, the denominator in c^F is negative. But if condition (32) holds, condition (31) never holds, so that c^F is negative. Since c is positive, in this situation, the condition $c \le c^F$ never holds, i.e., (8) never holds. *Conclusion*: Whenever (32) holds (i.e., if the inequality in (13) holds strictly), full repurchase equilibrium never exists.

In the intermediate region for δ indicated in (14), both conditions (31) and (32) do not hold. Because (32) does not hold, (8) can be written as $c \ge c^F$, where c^F is defined in (15). Because condition (31) also does not hold, c^F is positive (because in this region both the numerator and the denominator of c^F are positive). Thus, in this region, existence of full repurchase equilibrium depends on the value of c, i.e., a full repurchase equilibrium exists if c is sufficiently high, such that $c \ge c^F$.

Last, because it is always the case that

$$1 - \frac{x}{a} < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 2\right)}{\left(\frac{N}{Q} + 1\right)},$$

then, when (12) holds with equality, condition (8) always holds (i.e., a full repurchase equilibrium always exists), and when (13) holds with equality, condition (8) never holds (i.e., a full repurchase equilibrium never exists). \Box

Proof of Proposition 4. Consider the condition for partial repurchase equilibrium (9). The LHS of (9) is negative whenever

$$1 < \frac{1}{\delta} \left(1 - \frac{x}{a} \right),$$

(32)

which can also be written as

$$\delta < 1 - \frac{x}{a}.\tag{33}$$

The RHS of (9) is negative whenever

$$\frac{1}{\delta} \left(1 - \frac{x}{a} \right) \left(\frac{N}{Q} + 4 \right) < 2 + \left(1 + \frac{x}{a} \right) \frac{N}{Q},$$

which can also be written as

$$\left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}} < \delta.$$
(34)

The relation between conditions (33) and (34) depends on whether or not

$$1 - \frac{x}{a} \le \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}}$$

which after rearrangement is

$$\frac{x}{a} \le \frac{2Q}{N}.$$
(35)

We need to consider three cases: the case where (35) holds with strict inequality (Case 1), the case when it does not hold (Case 2), and the case when it holds with equality (Case 3).

Case 1: Suppose first that in condition (35) the inequality holds strictly. Then

$$1 - \frac{x}{a} < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}}$$

and, hence, whenever (33) holds, (34) never holds, and whenever (34) holds, (33) never holds. Now suppose that (33) holds. We can then write (9) as $c < c^P$, where c^P is defined in (19). Because in Case 1, a hold in (33) implies that (34) does not hold, then in c^P the numerator is negative whereas the denominator is positive so that c^P is negative. Because c is always positive, then in this situation of Case 1, the condition $c < c^P$ can never hold, i.e., condition (9) can never hold. *Conclusion*: In Case 1, if condition (33) holds (i.e., if the inequality in (16) holds strictly), partial repurchase equilibrium never exists.

Next, if condition (34) holds, we can write condition (9) as $c > c^P$. Since condition (34) holds, the denominator is negative. But we have seen that if condition (34) holds then condition (33) never holds, so that c^P is negative. Since c is positive, in this situation of Case 1, condition $c > c^P$ always holds, i.e., (9) always holds. *Conclusion*: In Case 1, if condition (34) holds (i.e., if the inequality in (17) holds strictly), a partial repurchase equilibrium always exists.

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In the intermediate region for δ

$$1 - \frac{x}{a} < \delta < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}}$$

$$(36)$$

both conditions (33) and (34) do not hold. Because condition (34) does not hold, (9) can be written as $c < c^{P}$. Because condition (33) also does not hold, in this region, c^{P} is positive (because in this region both the numerator and the denominator of c^{P} are positive). Thus, in this region, the existence of a partial repurchase equilibrium depends on the value of c, i.e., a partial repurchase equilibrium exists if c is sufficiently low, such that $c < c^{P}$.

Last, because in Case 1 it is always the case that

$$1 - \frac{x}{a} < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}},$$

then when (16) holds with equality, condition (9) never holds (i.e., a partial repurchase equilibrium never exists), and when (17) holds with equality, condition (9) always holds (i.e., a partial repurchase equilibrium always exists).

Case 2: Suppose now, instead, that condition (35) does not hold. Then

$$\left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}} < 1 - \frac{x}{a}$$

Now if

$$\delta < \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}},\tag{37}$$

we can write (9) as $c < c^P$. But in Case 2, if (37) holds, condition (33) always holds, and hence, in c^P the numerator is negative and the denominator is positive, so that c^P is always negative. Because c is always positive, in this situation of Case 2, the condition $c < c^P$ never holds, i.e., condition (9) never holds. *Conclusion*: In Case 2, if condition (37) holds (i.e., if the inequality in (20) holds strictly), partial repurchase equilibrium never exists.

Next, if

$$1 - \frac{x}{a} < \delta, \tag{38}$$

condition (37) never holds so that we can write (9) as $c > c^P$. But in Case 2, if (38) holds, condition (34) always holds and (33) never holds, and hence, in c^P the denominator is negative and the numerator is positive so that c^P is negative. Because c is positive, the condition $c > c^P$ always holds, i.e., condition (9) always holds. *Conclusion*: In Case 2, if condition (37) holds (i.e., if the inequality in (21) holds strictly), equilibrium with partial repurchase always exists.

In the intermediate region for δ

$$\left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}} < \delta < 1 - \frac{x}{a}$$

both conditions (33) and (34) hold. Because condition (34) holds, condition (9) can be written as $c > c^P$. Also, condition (33) holds. Thus, in this region of δ , c^P is positive (because in this region, both the numerator and the denominator in c^P are negative). Thus, in this region, the existence of partial repurchase equilibrium depends on the value of c, i.e., a partial repurchase equilibrium exists if c is sufficiently high, such that $c > c^P$.

Last, because in Case 2 it is always the case that

$$\left(1-\frac{x}{a}\right)\frac{\left(\frac{N}{Q}+4\right)}{2+\left(1+\frac{x}{a}\right)\frac{N}{Q}} < 1-\frac{x}{a},$$

then when (20) holds with equality, condition (9) never holds (i.e., a partial repurchase equilibrium never exists), and when (21) holds with equality, condition (9) always holds (i.e., a partial repurchase equilibrium always exists).

Case 3: Suppose now that condition (35) holds with equality. Then conditions (33) and condition (34) are mutually exclusive and there is no intermediate area to consider. There are only two regions to consider, and the analysis and the results in these regions are as in Cases 1 and 2. That is, if

$$\delta \le 1 - \frac{x}{a} = \left(1 - \frac{x}{a}\right) \frac{\left(\frac{N}{Q} + 4\right)}{2 + \left(1 + \frac{x}{a}\right)\frac{N}{Q}},$$

a partial repurchase equilibrium never exists, and it always exists otherwise. \Box

Proof of Proposition 5. The existence of a full repurchase equilibrium depends on c only in the region (14) of δ . Consider the definition of c^F in (15). The numerator in c^F is increasing in x/a and the denominator is decreasing in x/a, hence c^F is increasing in x/a. In the region (14), the numerator in c^F is increasing in δ because, by derivation, it is increasing in δ if $\delta < 2(1 - x/a)$, which contains the region (14) (since by assumption N/Q > 3). The denominator of c^F is decreasing in δ and hence c^F is increasing in δ , establishing the first part of Proposition 5.

For the existence of a partial repurchase equilibrium, first consider the case where x/a < 2Q/N. In this case, the existence of a partial repurchase equilibrium depends on c only in the region (18) of δ . In the region (18), both the numerator and the denominator of c^P (defined in (19)) are positive. The numerator is increasing in δ because, by derivation, it is increasing in δ if $\delta < 2(1 - x/a)$, which contains the region (18) (since by assumption N/Q > 3). The denominator of c^P is decreasing in δ and hence c^P is increasing in δ . Also, the numerator in c^P is increasing in x/a, whereas the denominator is decreasing in x/a, so that c^P is increasing in x/a. Next, consider the case where x/a > 2Q/N. In this case, the existence of a partial repurchase equilibrium depends on c only in the region (22) of δ . In

the region (22), both the numerator and the denominator of c^P are negative. The numerator is increasing in δ (becomes less negative) because, by derivation, it is increasing in δ if $\delta < 2(1 - x/a)$, which contains the region (22). The denominator is decreasing in δ (becomes more negative), so that c^P is decreasing in δ . The numerator in c^P is increasing in x/a (becomes less negative) and the denominator is increasing in x/a (becomes more negative) and hence in this region c^P is decreasing in x/a, establishing the second part of Proposition 5. \Box

Proof of Lemma 4. There is no adverse selection in the sell market. In order to make zero expected profit in the sell market, the market-maker must set $P_B = E[p_2]$. At time zero, all information is symmetric, and, since we take it as given that a program has been announced, the competitive price for which shares would be traded at t = 0 (bid or ask) must be equal to the expected sell price (at t = 1 or 2). \Box

Proof of Proposition 6. The result for the completion rate and social wealth is established in Section 3.3.1. Wealth expropriation is an increasing function of the bid–ask spread, so it is sufficient to show that the bid–ask spread is higher. Consider the market-maker conditions (3) and (4), and let p^F and p^P denote the prices for which they hold, respectively. Coexistence implies that $(A - X)/(N - C/p^F) \ge a - x + \delta c$ and $(A - X)/(N - C/p^P) < a - x + \delta c$, hence it must be the case that $p^P > p^F$. Next, since $p^P > p^F$, then $(A + X)/(N - C/p^F) > (A + X)/(N - C/p^P)$ and $(A - X)/(N - C/p^F) > (A - X)/(N - C/p^P)$, hence $E[v_2^F] > E[v_2^P]$; and since $v_2 = p_2$ (no adverse selection at t = 2), then $E[p_2^F] > E[p_2^P]$. \Box

Proof of Proposition 7. First, wealth transfers are an increasing function of the bid-ask spread, hence they increase whenever the bid-ask spread increases and decrease whenever the bid-ask spread decreases. Next, in a full repurchase equilibrium, the market-maker condition (3) is independent of x, hence given that a is fixed, p and $E[p_2]$ are independent of x/a. In a partial repurchase equilibrium, given that a is fixed, an increase in x/a implies that x is increasing. Consider the market-maker condition in a partial repurchase equilibrium (4). Observe that $E[v_2]$ increases in x because the changes in v_2 in the states a and a + x offset each other, while the decrease in v_2 in the state a - x + c is smaller than the increase in the state a + x + c because of the nonlinearity introduced though the firm's trade in the later state. But, if $E[v_2]$ increases in x, p must increase in x to provide the market-maker with zero expected profit. Furthermore, in the state a + x + c, the marketmaker is losing not only on a per-share basis but also because the quantity traded is higher than in all other states. Thus, for the market-maker to break even, it must be that the increase in p is larger than the increase in $E[v_2]$, establishing that $p - E[v_2]$ is increasing in x/a. Now since $E[v_2] = E[p_2] = p_B$, then the bid-ask spread is increasing in x/a (when a is fixed).

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