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ORIGINAL PAPER

On the different styles of large shareholders' activism

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Abstract This paper extends Noe's (Rev Financial Studies 15:289–317, 2002) model of large shareholder activism in two directions. First, it considers a framework in which large shareholders can choose not only when to monitor, but also how intensively they want to monitor the firm. Second, it considers the impact of laws and regulations by introducing a governance quality parameter that makes monitoring more cost effective. The model yields a new and rich characterization of activism. We find that share wealth (ownership concentration) induces monitoring for higher firm value through more frequent monitoring with unchanged intensity. Cash wealth motivates activism for trading gains, not higher firm value, through less frequent monitoring coupled with higher intensity. We also find that better governance leads to higher firm value through more frequent but less intense activism. When asymmetries within the group of large shareholders exist, the model predicts that the larger/wealthier/more efficient shareholders are more active. These results are broadly consistent with the empirical evidence.

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1 Introduction

Extensive research has been conducted on large shareholder activism. However, several important questions on the topic remain unanswered. First, there is a lot of variation in large shareholder activism. Some large shareholders such as the institutional investor CalPERS spend significant resources on monitoring the firms they invest in (e.g. Smith 1996), whereas others do not engage in activism at all. There is also great variation in styles of activism. Some large shareholders prefer long-term activism such as analyst coverage and relationship investing (e.g. Warren Buffett's long term relationship with Coca Cola); others engage in short-term but forceful activism (e.g. Carl Icahn's showdown with Time Warner, or Kirk Kerkorian's showdown with General Motors). It is unclear why such variation exists.

Second, it is unclear how activism enhances firm value, if at all. Some studies find activism to be associated with higher firm value, whereas others find no relation.[1] Moreover, not only activism (monitoring, takeovers, relationship-investing) but also alternative protective mechanisms such as laws, regulations, and corporate culture can affect firm value (e.g. Bebchuk et al. 2009). In particular, it is well established that countries with relatively strong shareholder protection, such as the US and UK, feature higher valuation of corporate assets relative to regions with poor shareholder protection, such as Italy and East Asia (Shleifer and Vishny 1997; La Porta et al. 2002). It is less clear, however, how activism interacts with such alternative protective mechanisms to determine firm value.

Lastly, most of the existing theoretical literature about activism considers only a single large shareholder.[2] However, increasing evidence suggests that most firms have multiple large shareholders (see, for example, Dlugosz et al. 2006; Holderness 2009). Activism of multiple large shareholders may differ from that of a single large shareholder. For example, noncoordinated monitoring may result in duplications and inefficiencies. Further, a large shareholder's trading decision may be different when the firm has multiple large shareholders. Raiders generally benefit based on the information about their monitoring by buying a toehold before engaging in activism and selling their shares abruptly when they decide to stop monitoring.[3] It is unclear how a raider's decision about monitoring and trading may change when she has to consider the decisions of other large shareholders.

[1] Studies that indicate an increase of value with activism (ownership concentration) include Smith (1996), Claessens (1997) and Becht et al. (2009). Studies that indicate no relation between firm value and activism include Karpoff (2001), Holderness (2003) and Bhagat et al. (2004).

[2] Models of activism that consider a single large shareholder include Shleifer and Vishny (1986), Admati et al. (1994), Kahn and Winton (1998), Maug (1998) and Instefiord (2009).

[3] In fact, empirical evidence suggests this trading behavior is true not only for corporate raiders but rather for large shareholders in general. See for, example, Bradley et al. (2009).

In this paper, we develop a model of large shareholders activism to answer these questions. The model has four important features. First, it has multiple (two) noncooperating large shareholders that can engage in activism (monitoring). Second, these large shareholders can choose both the frequency and the intensity with which they monitor. Third, financial markets in which large shareholders can trade to benefit from the private information that their activism generates are available. Lastly, firm value depends not only on activism but also on the quality of protective mechanisms that are beyond shareholders' control, namely exogenous governance mechanisms.[4] Under these assumptions we investigate the large shareholders' monitoring and trading decisions, and the resulting firm value.

Our paper uses Noe (2002) as a starting point. Noe considers multiple large shareholders who can both monitor and trade. He explicitly models the market, as well as the information asymmetry generated by activism where activism is identified with monitoring. He demonstrates that trading does not necessarily lead to the absence of monitoring, because market prices may dictate that monitoring and buying, or even monitoring and holding, can be more beneficial than selling.[5] Another elegant result of Noe's model is that investors choose different frequencies (probability) with which they monitor. However, large shareholders cannot choose the intensity with which they monitor in Noe's model. As a result the relation between ownership and monitoring is nonmonotonic. Namely, among investors who choose activism, those with the smallest holdings are the most aggressive. While this result is very interesting, it may not reflect most real life situations. We introduce the ability of large shareholders to choose the intensity with which they monitor, i.e., the amount they spend on monitoring when they do monitor, into Noe's framework. We also introduce the quality of exogenous governance mechanisms. In this generalized framework we then investigate how the monitoring decision, the associated trading decision, and firm value depend on the large shareholders characteristics (cash and share wealth) and on the quality of exogenous governance mechanisms.

Our results suggest that the optimal monitoring strategy for large shareholders is characterized by both frequency and intensity. In particular, we find that share wealth (ownership concentration) induces more frequent monitoring with unchanged intensity, which, in turn, results in higher firm value; Intuitively, a share-wealthy (vested) investor cares about the firm value. The more shares he owns, the higher his incentive to monitor and the higher the firm value. However, because of diminishing returns to monitoring he is better off monitoring more through increased frequency rather than through higher intensity. In contrast, we find that cash wealth induces less frequent monitoring coupled with higher intensity and does not enhance firm value; Intuitively, unlike a share-wealthy investor, a cash-wealthy investor cares about the firm value only when he buys shares. The greater his cash hoard, the more he wants to monitor, when

[4] Here, we categorize mechanisms as endogenous (activism) or exogenous according to whether investors currently have the ability to affect the mechanism. Thus, takeovers, although generally performed by outside investors, are considered endogenous. In contrast, corporate charter, although initially was set by the investors, is considered an exogenous mechanism.

[5] Kahn and Winton (1998) also show that trading does not necessarily lead to absence of monitoring. Their model considers a single large shareholder who can engage in activism.

he monitors. However, because his monitoring is motivated by trading gains rather than by value appreciation of shares held, he is better off "hiding" the information that he (buys shares and) monitors by reducing the frequency with which he monitors and increasing the intensity with which he monitors, when he monitors. Firm value is not enhanced because of the lower frequency of monitoring. With respect to the quality of exogenous governance mechanisms (laws, regulations), the model predicts that higher quality of these mechanisms leads to more frequent monitoring, but less spending on monitoring when monitoring occurs, which, in turn, results in higher firm value. Intuitively this is because of the substitutability between the quality of the exogenous mechanisms and monitoring and because of higher efficiency of the monitoring.

We also investigate how activism depends on asymmetries within the group of large shareholders. We find, that among large shareholders, larger ones monitor more because their benefit from enhancing the firm value is higher. Thus, unlike in Noe (2002), the relationship between share wealth and monitoring is monotonic. Similarly, cash-wealthier shareholders monitor more because their trading gains from monitoring are higher, and shareholders whose monitoring is more efficient monitor more because of higher marginal benefit from monitoring, both through value appreciation and trade. Lastly, our analysis suggests that monitoring of multiple large shareholders is similar in terms of frequency and intensity to that of a single large shareholder and that large shareholders use similar monitoring strategies in terms of intensity and frequency of monitoring whether or not they cooperate their monitoring with one another.

The model predictions are broadly consistent with the empirical evidence. Consistent with the model, vested investors such as pension funds rely heavily on high frequency–high intensity activism such as analyst coverage and investor relations rather than on special shareholder meetings or proxy fights, and their activism leads to value enhancement (see, for example, Smith 1996; Carlton et al. 1998; Becht et al. 2009). Consistent with the model, cash rich investors such as corporate raiders, monitor for trading gains (through value appreciation of shares purchased) rather than firm value, and tend to use low frequency–high intensity activism in the form of proxy fights, special shareholder meetings, referendums and scrutiny of management and their activism often does not result in value enhancement (see, for example, Bradley et al. 2009). An important implication of the model is that the ambiguity in empirical studies about the relation between activism and value, discussed above, exists because, as we show in the paper, activism may or may not lead to higher firm value. Specifically, activism motivated by ownership concentration enhances firm value, whereas activism motivated for trading gains may not.

Consistent with the model, corporate value is positively correlated with the quality of exogenous governance mechanisms (see, for example, La Porta et al. 1998; Bebchuk et al. 2009). We are unaware, however, of empirical evidence on the relation between the quality of governance and styles of activism. The model predicts, for example, that in countries with better investor protection through laws and regulations, such as the US and the UK, shareholder activism will be characterized with higher frequency and lower intensity in comparison to activism in countries with poor investor protection. Similarly, the model predicts that following legislation that improves the quality of laws and regulations such as the 2002 Sarbanes–Oxley Act, large shareholder activism will become more frequent and less intense.

Our main contributions are three: first, we characterize large shareholders activism on two dimensions: frequency and the intensity. Second, we investigate how the quality of exogenous governance mechanisms affects large shareholders activism and firm value. To our knowledge this is the first model to perform such an investigation. Lastly, our generalization of Noe (2002) allows us to explain and support a broader set of monitoring styles. For example, Noe's model predicts that among the shareholders who monitor, the larger ones monitor less, whereas our model supports the intuitive result that larger shareholders monitor more. A detailed comparison to Noe (2002) is given in Sect. 4.3.

In addition to Noe (2002), a number of other models consider multiple activists and the important tensions associated with the interaction among them. Winton (1993) investigates how monitoring of multiple large shareholders depends on capital structure. He shows that monitoring increases in the liability of their shares. Winton does not allow for trade and suggests that if large shareholders could trade freely they would sell off and there will be no monitoring. Cornelli and Li (2002), Attari et al. (2006) and Goldstein and Guembel (2008) study the trading strategies of multiple large shareholders. They investigate how these large shareholders trade to benefit from private information and how market prices are affected. They do not consider the agency problem, namely, they do not consider the value enhancement through monitoring.

The rest of this paper is organized as follows. Section 2 presents a simple model for large shareholders activism (monitoring) that abstracts from exogenous governance mechanisms and characterizes equilibrium monitoring strategies. Section 3 introduces a quality of governance parameter (i.e. the quality of laws, regulations) into the model. Section 4 solves the model and presents results on how ownership concentration, non-share wealth, and governance quality determine shareholders activism and firm value. Section 5 discusses empirical implications, and Sect. 6 concludes.

2 A simple model of shareholder activism

In this section we present a simple model of large shareholder activism with diminishing returns to activism, and provide a general characterization of equilibrium monitoring strategies and market prices. We defer the introduction of exogenous governance mechanisms to Sect. 3. Our notation is similar to that of Noe (2002).

2.1 Assumptions and notation

Assume a one period, risk-neutral economy with no taxes, no transaction costs, and an interest rate of zero. We consider an all-equity-financed firm with one share outstanding owned by $n-2$ nonstrategic small investors and two strategic large investors (henceforth "small investors" and "investors," respectively). The two strategic investors are indexed by subscript $i \in \{1, 2\}$. For these two investors, let (ϕ_i^0, b_i^0) denote investor i's endowment, where $\phi_i^0 > 0$ denotes share endowment, $b_i^0 > 0$ denotes riskless bond (cash) endowment, and the superscript 0 indicates beginning of period.

The firm is run by a manager. All investors (small or large) are outsiders in the sense that they have access to public information and have conflicts of interest with the manager. The terminal value of the firm depends solely on shareholder activism: higher level of activism reduces the probability that the manager will shirk or steal from the firm. The value of the firm is normalized to Q_0 when the manager shirks or steals and to Q_0+1 when he does not do so. For compactness of the strategy space Q_0 is assumed to be an arbitrary small but positive constant. We identify shareholder activism with monitoring and assume that monitoring is the only way for shareholders to affect the firm value.[6] Only the two large investors can monitor the manager. Let $0 \leq c_i$ denote the amount of money that investor i can spend on monitoring which is bounded by some (very) high limit, and let $c = (c_1, c_2)$. We follow Noe (2002) in assuming that the cost of monitoring c_i does not come out of the cash available for trade b_i^0.[7] Let $Q(c_1, c_2) : R_{0+}^2 \longmapsto [Q_0, Q_0 + 1]$ denote the firm value as a function of monitoring and let $Q(0, 0) = Q_0$. Equivalently, we can assume that $Q - Q_0$ represents the probability that monitoring will prevent the manager from shirking/stealing when investors 1 and 2 spend c_1 and c_2 on monitoring, respectively. Let v denote the expected firm value. Then $v = E[Q]$. Assume also that Q has continuous second-order derivatives. We make the following assumptions on the functional form for Q. First, $\partial Q/\partial c_i > 0$, i.e. firm value increases in monitoring of either of the large shareholders. Second, $\partial^2 Q/\partial c_i^2 < 0$, i.e. returns to monitoring are diminishing.[8] Last, $\partial Q\,(0, 0)\,/\partial c_i > 1$, i.e. the benefit from monitoring must be high enough to justify monitoring.

There is a market in which investors can trade their shares. At the beginning of the period, a zero-expected-profit market maker posts bid–ask prices $p = (p_A, p_B)$ at which he is willing to buy and sell any quantity of shares, respectively. The riskless bond price is 1 (since the interest rate is zero). Borrowing and shortselling are not allowed.[9] Let ϕ_i and b_i denote investor i's post-trade shares and post-trade bonds, respectively. For given (ϕ_i, b_i), the total expenditure by investor i on purchasing shares is $(\phi_i - \phi_i^0)^+ p_A$ where $x^+ \equiv \max[x, 0]$. The proceeds from the sale of endowment stock is $(\phi_i^0 - \phi_i)^+ p_B$. The budget constraint implies that bond holdings after trade are given by

$$b_i(\phi_i) = b_i^0 + (\phi_i^0 - \phi_i)^+ p_B - (\phi_i - \phi_i^0)^+ p_A. \tag{1}$$

Denote the largest possible post-trade holding for investor i with

$$\bar{\phi}_i = \frac{b_i^0}{p_A} + \phi_i^0. \tag{2}$$

6 We think of shareholder activism as including all actions that investors can take (naturally at a cost) to increase the value of the firm. In Sect. 3, the model is generalized to allow factors that are currently beyond the control of shareholders to affect firm value (i.e., laws, regulations, the corporate charters, etc.)

7 Relaxing this assumption should not change the qualitative results of the paper but will significantly complicate the analysis. Imposing a limit on c_i is to assure compactness of the monitoring strategy and is consistent with investors having limited wealth.

8 Noe (2002), for example, assumes a fixed cost of monitoring, c^*, and that monitoring by one investor is sufficient to prevent shirking. This is equivalent to assuming $Q(c_1, c_2) = 1$ if $\max(c_1, c_2) \geq c^*$, and $Q(c_1, c_2) = 0$ otherwise. See further discussion of the functional form of Q in Sect. 3.1.

9 This assumption is needed for compactness of the strategy space because agents are risk neutral. Limited borrowing or limited shortselling will not change the qualitative results.

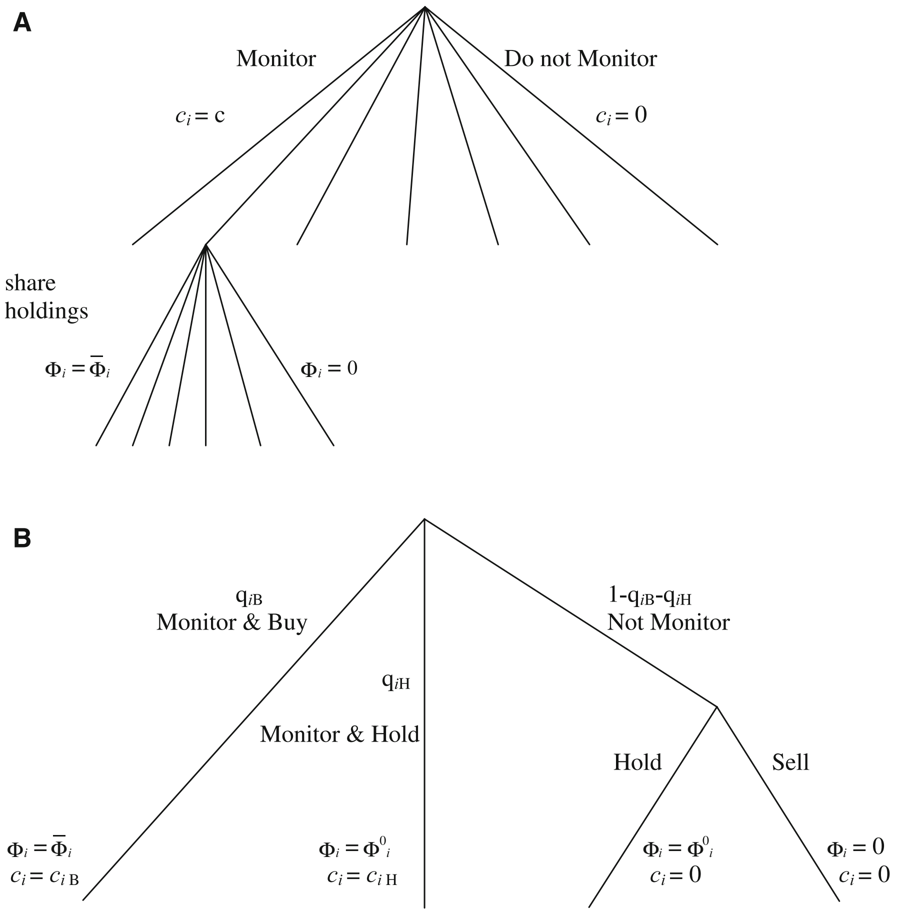

Fig. 1 **a** *Game tree*. Each investor $i\,(i = 1, 2)$ chooses monitoring level, c_i, in the range [0,c], and shareholdings, Φ_i, in the range $[0, \bar{\Phi}_i]$, simultaneously. **b** *Reduced game tree*. The tree from **a** can be simplified to the tree below. Investor i's after-trade shareholding is either $\bar{\Phi}_i$ (Buy), or Φ_i^0 (Hold), or 0 (Sell). Given shareholding choice each investor chooses a unique monitoring level $c_{i\mathrm{B}}$ when they buy and $c_{i\mathrm{H}}$ when they hold. Investors never sell when they monitor and never buy when they do not monitor. The probability that investor i assigns to {Monitor and Buy} is $q_{i\mathrm{B}}$ and the probability that investor i assigns to {Monitor and Hold} is $q_{i\mathrm{H}}$. If he does not monitor then the investor either holds or sells (he never mixes)

Also define $\phi^0 = (\phi_1^0, \phi_2^0)$, $b^0 = (b_1^0, b_2^0)$, $\phi = (\phi_1, \phi_2)$, $b = (b_1, b_2)$, and $\bar{\phi} = (\bar{\phi}_1, \bar{\phi}_2)$.

2.2 The game between investors

The game between the two investors is one with simultaneous moves with perfect information on endowments. Figure 1a describes the game tree. In this game, investor i's strategy specifies the number of shares to trade, $\phi_i - \phi_i^0$ (i.e., his choice of ϕ_i for a given ϕ_i^0) and the amount of cash to spend on monitoring, c_i. To allow for the possibility of mixed strategies, let $C_i \geq 0$ denote investor i's mixed monitoring strategy. Similarly let $\Phi_i \in [0, \bar{\phi}]$ denote investor i's mixed shareholding strategy. Note that c_i and ϕ_i, defined earlier, are elements of C_i and Φ_i, respectively. Also define $\Phi = (\Phi_1, \Phi_2)$ and $C = (C_1, C_2)$. Noncooperative monitoring implies $\text{corr}(C_1, C_2) = 0$.

Investors are expected utility maximizers. Let u_i denote investor i's utility function. Investor i chooses a strategy to maximize his expected utility given the strategy of the other investor. Specifically, the objective function for investor i is

$$\max_{C_i \geq 0 \;\; \Phi_i \in [0, \bar{\phi}_i]} \{E[u_i(\Phi, C, p)]\}. \tag{3}$$

Definition 1 An equilibrium is a triple, $(\Phi^*, C^*, p^*(\Phi^*, C^*))$, consisting of trading strategies Φ^*, monitoring strategies C^*, and bid and ask prices p^*. This triple satisfies the following conditions: (i) the strategies (Φ_1^*, C_1^*) and (Φ_2^*, C_2^*) are independent; (ii) the market maker's expected profit is zero, and (iii) for investor i

$$E[u_i(\Phi^*, C^*, p^*(\Phi^*, C^*))] \geq E\left[u_i\left(\left(\Phi_i, \Phi_{-i}^*\right), \left(C_i, C_{-i}^*\right), p(\Phi_i, \Phi_{-i}^*, C_i, C_{-i}^*)\right)\right]$$

for all Φ_i, C_i and $i = 1, 2$.

Proposition 1 *An equilibrium always exists.*

All proofs are relegated to the Appendix.

2.3 Investors' optimization problem

We now define the optimization problems for the large investors. Note that because both investors are risk-neutral, maximizing their expected utility is equivalent to maximizing their expected post-trade wealth. Investor 1 chooses C_1 and Φ_1 to maximize his expected post-trade wealth given C_2 and Φ_2. That is, investor 1 solves

$$\begin{aligned} \max_{C_1 \geq 0,\;\; \Phi_1 \in [0, \bar{\phi}_1]} \Bigg\{ E_{C_1, \Phi_1} \Bigg[&\left(b_1^0 + \left(\phi_1^0 - \Phi_1\right)^+ p_B - \left(\Phi_1 - \phi_1^0\right)^+ p_A \right) \\ &+ E_{C_2}\left[\Phi_1 Q\left(C_1, C_2\right) - C_1\right] \Bigg] \Bigg\} \end{aligned} \tag{4}$$

for given C_2 and Φ_2 where $E_{C_1, \Phi_1}[\cdot]$ is the expectation with respect to the joint distribution of C_1 and Φ_1. The first term inside $E_{C_1, \Phi_1}[\cdot]$ is the value of investor 1's after-trade bond holdings (a direct result of (1)). The second term is the value of his after-trade share holdings. The second term suggests that for given ϕ_1 and C_2 investor 1 will spend $c_1 > 0$ on monitoring only if

$$\phi_1 E_{C_2}[Q(c_1, C_2)] - c_1 > \phi_1 E_{C_2}[Q(0, C_2)], \tag{5}$$

where $E_{C_2}[\cdot]$ is the expectation with respect to the distribution of C_2. Investor 2 solves a symmetric problem.

2.4 Properties of equilibrium

In this subsection, we characterize the equilibrium of the game. We first describe the market mechanism and derive the equilibrium prices set by the market maker. We then identify properties of the investors' equilibrium strategies which would allow us to simplify the game tree and characterize equilibrium strategies and profits. By construction, if monitoring level is fixed our model boils down to Noe (2002). We compare our results to Noe's in Sect. 4.3.

2.4.1 Equilibrium market prices

Recall that $v = E[Q(C_1, C_2)]$ and let $v_{-i} = E[Q|C_i = 0]$. Namely, v_{-i} is the expected firm value given that investor i does not monitor. Our earlier assumption $Q'_{c_i} > 0$ implies that $v \geq v_{-i}$. Let $d_B(\Phi)$ and $d_S(\Phi)$ denote the aggregate *number* of shares that strategic investors choose to buy and sell, respectively. Then $d_B(\Phi) = \sum_{i=1}^{2}(\Phi_i - \phi_i^0)^+$ and $d_S(\Phi) = \sum_{i=1}^{2}(\phi_i^0 - \Phi_i)^+$. The *dollar* value of buy-side demand and sell-side supply from strategic investors is given by $D_B(\cdot) = p_A d_B(\cdot)$ and $D_S(\cdot) = p_B d_S(\cdot)$, respectively. Unlike the two strategic (large) investors, nonstrategic (small) investors trade for liquidity reasons. We assume that the dollar value of buy-side demand from nonstrategic investors is given by an exogenous random variable L_B and that the sell-side supply *in shares* from nonstrategic investors is given by an exogenous random number l_S.[10] The buy-side demand in shares from nonstrategic investors is given by $l_B(p) = L_B/p_A$. Also, denote $L = E[L_B]$ and $s = E[l_S]$. The zero expected profit condition for the market maker is:

$$E[(d_B(\Phi) + l_B(p))(p_A - Q(C_1, C_2))] = 0, \tag{6}$$
$$E[(d_S(\Phi) + l_S)(Q(C_1, C_2) - p_B)] = 0. \tag{7}$$

Solving Eqs. (6) and (7) for p_A^* and p_B^* yields the following Lemma.

Lemma 1 *The equilibrium ask price satisfies*

$$p_A^* = v^* + \frac{\operatorname{cov}(D_B^*, Q^*)}{E[D_B^*] + L}, \tag{8}$$

and the equilibrium bid price satisfies

$$p_B^* = v^* + \frac{\operatorname{cov}(d_s^*, Q^*)}{E[d_s^*] + s} \tag{9}$$

Note that $\operatorname{cov}(D_B^*, Q^*) > 0$ implies $p_A^* > v^*$. This is because when strategic purchases are positively correlated with monitoring, the market maker sets the ask price

10 We define buy-side liquidity on dollar amount and sell-side liquidity on the number of shares for tractability of the bid and ask prices, because strategic sell-side supply is limited by the number of shares the investor owns, whereas strategic buy-side demand is limited by the (cash) budget constraint.

to be higher than the unconditional expected value to compensate for this adverse selection. Similarly, $\mathrm{cov}(d_s^*, Q^*) < 0$ implies $p_B^* < v^*$ because when strategic sells are negatively correlated with monitoring, the market maker has to set the bid price to be lower than the expected value. Note also that since the stock value can never fall below Q_0 then p_B^* can never fall below Q_0 (Otherwise, the zero expected profit condition to the market maker in the sell market would not hold.) As a result, we have $Q_0 < p_B^* < v^* < p_A^*$.

2.4.2 Properties of equilibrium strategies

Next we characterize the equilibrium strategies of the large investors. To do so, we first establish (in Lemmas 2–4) properties that allow us to eliminate dominated strategies.

Lemma 2 *Given a monitoring level $c_i \geq 0$, optimally, investor i either sells all shares, or holds, or spends all available cash to buy shares.*

Intuitively, the utility function for investor i (the maximand in (4) for either investor) is convex in ϕ_i on each of the segments $[0, \phi_i^0]$, $[\phi_i^0, \bar{\phi}_i]$. Hence, the optimal shareholding strategy for investor i is one of the three end points 0, ϕ_i^0, and $\bar{\phi}_i$. Thus he has a bang–bang solution on portfolio weights. Denote the corresponding shareholding strategies for investor i with $\{\text{Sell}\}$, $\{\text{Hold}\}$, and $\{\text{Buy}\}$. The following Lemma further restricts equilibrium shareholdings.

Lemma 3 (i) *When investor i monitors, he never sells.* (ii) *When investor i does not monitor, he never buys.*

The intuition for (i) is that an investor will never spend resources in order to increase the value of a stock if he plans to sell. The intuition for (ii) is as follows: since the ask price reflects the probability of adverse selection through monitoring and buying, it is higher than the expected value, given that the investor does not monitor (i.e. $v_{-i}^* < p_A^*$ since $v_{-i}^* < v^*$ and $v^* < p_A^*$). Thus the strategy $\{\text{Not Monitor and Buy}\}$ is always strictly dominated by the strategy $\{\text{Not Monitor and Hold}\}$. In fact, it can be observed from the second term inside $E_{C_1,\Phi_1}[\cdot]$ in (4) that when investor i does not monitor, he sells if $p_B > v_{-i}^*$ and holds otherwise. This choice is less important than the decision to hold or buy when he does monitor (since no spending on monitoring is involved). Hence, with a little abuse of notation, we will generally use $\{\text{Not Monitor}\}$ for both $\{\text{Not Monitor and Hold}\}$ and $\{\text{Not Monitor and Sell}\}$ without specifying.

Lemma 4 *When investor i's strategy involves monitoring, he does not mix on his monitoring level c_i given shareholding choice ϕ_i. Specifically, investor i monitors with a unique monitoring level c_{iB} if he buys and a unique monitoring level c_{iH} if he holds.*

Intuitively, by assumption, the firm value is concave in the amount that investor i spends on monitoring. Hence, any strategy that involves mixing on monitoring expenditure is dominated by the strategy of spending the average amount. Based on the above Lemmas, we have the following Proposition:

Proposition 2 *Any equilibrium can involve only the strategies* $\{\{$*Monitor and Buy*$\}$, $\{$*Monitor and Hold*$\}$, $\{$*Not Monitor*$\}\}$, *where the strategy* $\{$*Monitor and Buy*$\}$ *involves monitoring with a unique monitoring level* c_{iB}, *and the strategy* $\{$*Monitor and Hold*$\}$ *involves monitoring with a unique monitoring level* c_{iH}. *With the strategy* $\{$*Not Monitor*$\}$ *investor* i *either sells or holds depending on whether or not* $p_B > v^*_{-i}$, *where* v^*_{-i} *is the expected share value given that he does not monitor.*

Proposition 2 allows us to simplify the decision tree for investor i from Fig. 1a to the three-branch decision tree in Fig. 1b. When investor i chooses not to monitor, he either holds or sells all his shares. When he chooses to monitor, he either holds and monitors with one level denoted by c_{iH}, or buys shares with all available cash and monitors with a possibly different monitoring level c_{iB}. Investors' strategies may still involve mixing on the pure strategies in Proposition 2. To allow for such mixing, we denote the probability that investor i assigns to {Monitor and Buy} with $q_{iB} \in [0, 1]$ and the probability that investor i assigns to {Monitor and Hold} with $q_{iH} \in [0, 1]$. Thus, c_{iB}, c_{iH} , represent the intensity with which the investors monitor whereas q_{iB}, q_{iH}, represent the frequency with which they monitor.

Definition 2 A mixed equilibrium is an equilibrium in which, at least for some investor i, $q_{iB} \in (0, 1)$ or $q_{iH} \in (0, 1)$. That is, at least one investor mixes between two or more of the pure strategies {Monitor and Buy},{Monitor and Hold}, {Not Monitor}. A fully mixed equilibrium is an equilibrium in which $q_{iB} \in (0, 1)$, $q_{iH} \in (0, 1)$, and $q_{iB} + q_{iH} < 1$, for $i = 1, 2$.

Theorem 1 *Every equilibrium is a mixed equilibrium.*

By Theorem 1, a pure strategy equilibrium is not possible because of the free riding problem. Note, however, that we do not rule out an equilibrium in which one investor plays a mixed strategy and the other plays a pure strategy. In such an equilibrium, the investor that holds would free ride (gain) when the other investor monitors but loses when the other investor does not monitor. Indeed, Sect. 4.2 demonstrates, for example, that with enough asymmetry between the large shareholders one investor never monitors and the other investor monitors with probability between 0 and 1.

2.4.3 Equilibrium profits

Next, we characterize equilibrium profits. Recall that c_{iB} is the optimal monitoring level for investor i when he monitors and buys, and let

$$v_{+iB} \equiv E\big[Q|C_i = c_{iB}, \Phi_{-i}, C_{-i}\big]. \tag{10}$$

Namely, v_{+iB} is the expected firm value when investor i spends c_{iB} on monitoring with probability 1, given the other investor's strategy, (Φ_{-i}, C_{-i}), which might involve mixing. Similarly let

$$v_{+iH} \equiv E\big[Q|C_i = c_{iH}, \Phi_{-i}, C_{-i}\big]. \tag{11}$$

In any equilibrium $v_{+iH} \leq p_A \leq v_{+iB}$. (Suppose not, then, if $v_{+iH} > p_A$, investor i will buy rather than hold, alternatively, if $v_{+iB} < p_A$, investor i will not buy.) Hence, $c_{iH} \leq c_{iB}$.

Define MP_{iB} to be the *excess* expected profit for investor i from the strategy {Monitor and Buy} *as opposed to* the strategy {Not Monitor} given the other investor's strategy (henceforth "monitoring profit"). Then

$$MP_{iB} = b_i^0 \left(\frac{v_{+iB}}{p_A} - 1 \right) + \phi_i^0 [v_{+iB} - \max(p_B, v_{-i})]. \tag{12}$$

The first term on the right-hand side of Eq. (12) is the trading gain on the cash endowment. With the strategy {Monitor and Buy} each cash-endowment dollar invested in shares yields $\frac{v_{+iB}}{p_A}$ whereas with the strategy {Not Monitor} the cash-endowment is retained. The second term is the gain from monitoring on the share endowment. With the strategy {Monitor and Buy} the value of each share becomes v_{+iB}. With the strategy {Not Monitor} the shares are sold if $p_B > v_{-i}$ and held otherwise, so that the proceeds on each share are $\max(p_B, v_{-i})$. Similarly, define MP_{iH} to be the excess expected monitoring profit for investor i from the strategy {Monitor and Hold} as opposed to the strategy {Not Monitor}. Then

$$MP_{iH} = \phi_i^0 [v_{+iH} - \max(p_B, v_{-i})]. \tag{13}$$

This profit reflects only the gain on the share endowment, as in both strategies, {Monitor and Hold} and {Not Monitor}, the cash endowment is retained (there are no trading profits). Note that MP_{iB} and MP_{iH} are nonnegative. The market maker compensates himself for the expected losses to informed (large) investors with profits that he makes from trading with the liquidity (small) investors.

Given (Φ_{-i}, C_{-i}) and p, investor i's *net* profit (i.e., monitoring profit less monitoring cost) from {Monitor and Buy} as opposed to {Not Monitor} is $MP_{iB} - c_{iB}$; and the net profit from {Monitor and Hold} as opposed to {Not Monitor} is $MP_{iH} - c_{iH}$. His best response is the strategy that yields the highest profit. For example, if

$$0 < MP_{iH} - c_{iH} \quad \text{and} \quad MP_{iB} - c_{iB} < MP_{iH} - c_{iH},$$

then investor i's best response is {Monitor and Hold} with probability 1. The following Lemma characterizes the relation between the probabilities with which each of these three strategies are played in equilibrium and the corresponding equilibrium profits.

Lemma 5 *In any equilibrium, for each investor i,*

$$q_{iB}^* \left(1 - q_{iB}^* - q_{iH}^*\right) \left(MP_{iB}^* - c_{iB}^*\right) = 0 \tag{14}$$
$$q_{iH}^* \left(1 - q_{iB}^* - q_{iH}^*\right) \left(MP_{iH}^* - c_{iH}^*\right) = 0 \tag{15}$$

Lemma 5 summarizes restrictions on equilibrium strategies and profits based on the requirement that all strategies played in equilibrium must provide the investor with the same net gain. For example, if investor i's equilibrium strategy is to mix between

{Monitor and Buy} and {Not Monitor}, then it must be the case that both strategies provide the same gain. That is, investor i's net equilibrium profit from monitoring is zero. Indeed, since in this case $q_{iB}^* > 0$ and $\left(1 - q_{iB}^* - q_{iH}^*\right) > 0$, Eq. (14) requires that $MP_{iB}^* = c_{iB}^*$. Similarly, in a fully mixed equilibrium (i.e., an equilibrium in which all three strategies are played with positive probabilities by both investors), it must be the case that for both investors the excess expected monitoring profit equals the associated monitoring costs. Indeed, for each investor i, $q_{iB}^* > 0$ and $q_{iH}^* > 0$ and $\left(1 - q_{iB}^* - q_{iH}^*\right) > 0$ imply that $MP_{iB} = c_{iB}$ and $MP_{iH} = c_{iH}$.

3 Governance quality

In this section, we investigate how varying corporate governance quality, both in terms of external factors such as national laws, regulations, corporate culture, and corporate charter, and in terms of investor specific factors, affects shareholder activism. Investors may rely on varying sources of information, for instance some investors use advice from own analysts whereas other investors rely on internal contacts, and these investor specific factors may make them more or less effective as active shareholders. We first outline how we can define parameters that introduce governance quality into the framework, and then show how equilibrium is defined given the new parameters.

3.1 The specialized governance function

In this subsection we describe how we can augment the activism function $Q(c_1, c_2)$ with an augmented governance function $Q(\alpha, c) \equiv Q(\alpha_1, \alpha_2, c_1, c_2)$ that incorporates governance quality parameters α_1 and α_2 to represent the quality of the governance provision by shareholder 1 and 2, respectively. We assume the following specific form of $Q(\alpha, c)$:

$$Q(\alpha, c) = Q_0 + 1 - e^{-(\alpha_1 c_1 + \alpha_2 c_2)} \tag{16}$$

where $\alpha_i > 1$. This functional form captures substitutability between the investors' activism as well as increasing but diminishing returns to monitoring across investors (i.e., $\partial^2 Q/\partial c_1 \partial c_2 < 0$). It captures substitutability between quality of exogenous mechanisms α and monitoring c.[11] It also captures increasing but diminishing returns to α. The requirement $\alpha_i > 1$ assures that $\partial Q(0, 0)/\partial c_i > 1$ holds for this specific functional form. Other functional forms can be used, but may not be as good in capturing the above features. The functional form (16) preserves all the properties of Q in relation to c that we assumed in Sect. 2; thus, all results from Sect. 2 still hold. Under (16) we can simplify

[11] We acknowledge that in practice exogenous mechanisms and monitoring may at times be complements rather than substitutes (e.g. higher disclosure requirements may lead to a drop in monitoring costs). Our priors, however, are that substitutability generally dominates complementarity. Setting $\alpha_1 \neq \alpha_2$ allows us to deviate from the case of perfect substitution of the monitoring.

$$v^* = E[Q(\alpha, C)] = Q_0 + 1 - E[e^{-\alpha_1 C_1}]E[e^{-\alpha_2 C_2}], \tag{17}$$
$$v^*_{-1} = E[Q(\alpha, C)|_{C_1=0}] = Q_0 + 1 - E[e^{-\alpha_2 C_2}], \tag{18}$$
$$v^*_{-2} = E[Q(\alpha, C)|_{C_2=0}] = Q_0 + 1 - E[e^{-\alpha_1 C_1}], \tag{19}$$

where

$$E[e^{-\alpha_1 C_1}] = q_{1B} e^{-\alpha_1 c_{1B}} + q_{1H} e^{-\alpha_1 c_{1H}} + (1 - q_{1B} - q_{1H}), \tag{20}$$

and

$$E[e^{-\alpha_2 C_2}] = q_{2B} e^{-\alpha_2 c_{2B}} + q_{2H} e^{-\alpha_2 c_{2H}} + (1 - q_{2B} - q_{2H}). \tag{21}$$

As indicated in Proposition 2, under the strategy {Not Monitor}, investor i may either hold or sell depending on whether or not $v^*_{-1} \geq p^*_B$. In order to solve for the equilibrium it is helpful to define a variable for shareholding level under this strategy. Let ϕ_{iNM} denote the number of shares investor i holds when he does not monitor. Accordingly,

$$\begin{aligned} \phi_{1NM} &= \phi_1^0 \qquad \text{when } v^*_{-1} \geq p^*_B \\ &= 0 \qquad \text{otherwise,} \end{aligned} \tag{22}$$

and

$$\begin{aligned} \phi_{2NM} &= \phi_2^0 \qquad \text{when } v^*_{-2} \geq p^*_B \\ &= 0 \qquad \text{otherwise} \end{aligned} \tag{23}$$

and where v^*_{-1} and v^*_{-2} are given in (18) and (19), respectively.

3.2 Equilibrium

Based on the analysis in Sect. 2 and given the governance function (16), we can derive a system of equations that determines the equilibrium. To do so, we first establish the equilibrium prices and the optimal monitoring in Lemmas 6 and 7, respectively. Upon substitution of the governance function (16) into the bid and ask prices (8) and (9) given in Lemma 1, we have the following results on equilibrium prices:

Lemma 6 *Under the governance function* (16) *the equilibrium ask price is*

$$p^*_A = v^* + \frac{q_{1B} b_1^0 E[e^{-\alpha_2 C_2}] \left(E[e^{-\alpha_1 C_1}] - e^{-\alpha_1 c_{1B}}\right) + q_{2B} b_2^0 E[e^{-\alpha_1 C_1}] \left(E[e^{-\alpha_2 C_2}] - e^{-\alpha_2 c_{2B}}\right)}{q_{1B} b_1^0 + q_{2B} b_2^0 + L} \tag{24}$$

and the equilibrium bid price is

$$p_B^* = v^* + \frac{(1-q_{1B}-q_{1H})(\phi_1^0-\phi_{1NM})\left(1-E[e^{-\alpha_2 C_2}]\right)+(1-q_{2B}-q_{2H})(\phi_2^0-\phi_{2NM})\left(1-E[e^{-\alpha_1 C_1}]\right)}{(1-q_{1B}-q_{1H})(\phi_1^0-\phi_{1NM})+(1-q_{2B}-q_{2H})(\phi_2^0-\phi_{2NM})+s} \tag{25}$$

where v^*, $E[e^{-\alpha_1 C_1}]$, $E[e^{-\alpha_2 C_2}]$, ϕ_{1NM}, *and* ϕ_{2NM} *are given in* Eqs. (17), (20)–(23), *respectively.*

Upon substitution of the governance function (16) into the optimization problem (4) we have the following results on optimal monitoring levels:

Lemma 7 *Under the governance function* (16) *the optimal monitoring levels are as follows:*

$$c_{1B}^* = \frac{1}{\alpha_1}\left(\ln\bar{\phi}_1^* + \ln\alpha_1 + \ln E\left[e^{-\alpha_2 C_2^*}\right]\right), \tag{26}$$

$$c_{1H}^* = \frac{1}{\alpha_1}\left(\ln\phi_1^0 + \ln\alpha_1 + \ln E\left[e^{-\alpha_2 C_2^*}\right]\right), \tag{27}$$

$$c_{2B}^* = \frac{1}{\alpha_2}\left(\ln\bar{\phi}_2^* + \ln\alpha_2 + \ln E\left[e^{-\alpha_1 C_1^*}\right]\right), \tag{28}$$

$$c_{2H}^* = \frac{1}{\alpha_2}\left(\ln\phi_2^0 + \ln\alpha_2 + \ln E\left[e^{-\alpha_1 C_1^*}\right]\right), \tag{29}$$

where $\bar{\phi}_i^* = \phi_i^0 + (b_i^0/p_A^*)$

In equilibrium, the market maker sets prices p_A^*, p_B^*, and each large investor i chooses a mixed strategy that includes the following:

(1) Probability of playing the pure strategy {Monitoring and Buy}, q_{iB}^*, and the corresponding monitoring intensity, c_{iB}^* , where under this strategy he spends all his cash endowment on buying shares and eventually holds $\bar{\phi}_i^* = \phi_i^0 + (b_i^0/p_A^*)$ shares.
(2) Probability of playing the pure strategy {Monitor and Hold}, q_{iH}^*, and the corresponding monitoring intensity, c_{iH}^* , where under this strategy he does not buy or sell shares, i.e., he holds ϕ_i^0 shares.
(3) With probability $1 - q_{iB}^* - q_{iH}^*$ he chooses the pure strategy {Not Monitor}, in which case he holds ϕ_{iNM}^* shares. Namely, he holds on to his share endowment ϕ_i^0 if $v_{-1}^* \geq p_B^*$ and sells all his shares otherwise. (This is formally indicated in Eqs. (22) and (23) above.)

Proposition 3 *Given the governance function* (16) *and the following*:

(1) *governance quality parameters* α_1, α_2,
(2) *the large investors' share endowments* ϕ_1^0, ϕ_2^0,
(3) *the large investors' cash endowments* b_1^0, b_2^0,

(4) *and expected liquidity in the buy and sell markets* L, s, *respectively, equilibrium prices and strategies can be derived by solving the system of* 12 *Eqs.* (35)–(46) *given in the proof of Proposition* 3 *for*:
   (1) *prices* p_A^*, p_B^*,
   (2) *probabilities for* {*Monitor and Buy*} q_{1B}^*, q_{2B}^*, *and corresponding monitoring intensities* c_{1B}^*, c_{2B}^*,
   (3) *probabilities for* {*Monitor and Hold*} q_{1H}^*, q_{2H}^*, *and corresponding monitoring intensities* c_{1H}^*, c_{2H}^*,
   (4) *shareholdings under* {*Not Monitor*} ϕ_{1NM}^*, ϕ_{2NM}^*.

Solving for equilibrium requires finding equilibrium values of the prices p_A, p_B, monitoring probabilities $q_{1B}, q_{2B}, q_{1H}, q_{2H}$, monitoring intensity levels c_{1B}, c_{1H}, c_{2B}, c_{2H}, and shareholdings under the strategy {Not Monitor} ϕ_{1NM}, ϕ_{2NM}. The system (35)–(46), given in the proof of Proposition 3, provides a framework with which to solve for the values of these parameters.

4 Results on corporate governance

This section presents results on the impacts of share wealth (ownership concentration), cash wealth, and quality of the protective mechanisms that are beyond the shareholders control (exogenous governance mechanisms) on the large shareholder monitoring strategy (frequency and intensity), the associated trading strategy (buy, sell, or hold), and the resulting firm value. We derive these results by solving the corporate governance model from Sect. 3 (essentially the system of Eqs. 35–46) and performing comparative statics. Because the system is highly nonlinear, an analytical solution is not available to our knowledge. Therefore, numerical techniques (Newton–Raphson method) were used.[12] Section 4.1 reports the results for the symmetric case, i.e., when both investors own the same number of shares, have the same amount of cash, and have access to identical monitoring technology. Section 4.2 describes how the results change when these symmetries are relaxed. Section 4.3 compares our results to those of Noe (2002). Before moving to the results, the next paragraph offers introductory intuition for the analysis to follow.

Our analysis suggests that five major forces shape the equilibrium outcome. These forces are the substitutability between the quality of exogenous mechanisms and monitoring, diminishing return to monitoring, lack of coordination of monitoring, adverse selection, and free riding. The first force, the substitutability between the quality of exogenous mechanisms and monitoring, implies that higher quality of

[12] We solved the system of 12 Eqs. (35)–(46) using an algorithm designed to implement the Newton-Raphson method (Press et al. 1988, pp. 270–288). First, initial values of all the variables are chosen. Then, new values of these variables are computed through iterations until a convergence criterion is satisfied. Because the Newton–Raphson method requires that the equations to which the method is applied have continuous first-order derivatives whereas the left hand sides of Eqs. (37) and (38) are discontinuous in ϕ_1 and ϕ_2, respectively, each iteration is carried out in two steps. In the first step of the iteration, new values of all variables except for ϕ_1 and ϕ_2 are computed. In the second step, new values of ϕ_1 and ϕ_2 are computed from Eqs. (37) and (38) using the new values of all other variables (that were computed in the first step of the iteration).

exogenous mechanisms leads to lower spending on monitoring when monitor occurs (lower intensity). The second force, diminishing returns to activism (monitoring), motivates the large shareholders to monitor more frequently and spend less on monitoring when they monitor. The third force, lack of coordination of monitoring, causes duplication between the monitoring of the large shareholders and results in inefficient monitoring because of diminishing returns to monitoring (i.e., excessive monitoring). This force motivates less frequent monitoring, with more spending on monitoring when monitoring occurs, in order to avoid duplication. The fourth force, adverse selection, results from the information asymmetry generated by monitoring (only shareholders who monitor are informed about their monitoring). Like the third force, adverse selection encourages less frequent monitoring, with more spending on monitoring when monitoring occurs as its purpose is to generate trading gains, rather than to increase firm value. The last force, free riding, exists because only investors who monitor bear the costs of monitoring, whereas gains are enjoyed by all shareholders. The tension among the above forces determines the equilibrium outcome.

4.1 Symmetric equilibrium

In this subsection we assume that investors are symmetric, i.e. $\phi_1^0 = \phi_2^0 = \phi^0$, $b_1^0 = b_2^0 = b^0$, $\alpha_1 = \alpha_2 = \alpha$. We consider the symmetric equilibrium in which equilibrium strategies are symmetric and for which we can write

$$(q_{iB}, c_{iB}, q_{iH}, c_{iH}) = (q_B, c_B, q_H, c_H) \quad \text{for } i = 1, 2.$$

We investigate how the equilibrium (the solution to the system of Eqs. 35–46) changes in response to changes in initial stake in the firm, ϕ^0, cash wealth, b^0, and quality of the exogenous mechanisms, α. We change *one* parameter at a time while holding all other parameters fixed. All symmetric equilibria obtained were nonfully mixed. The strategy {Monitor and Hold} was never played in symmetric equilibrium. That is, in all symmetric equilibria obtained, $q_H = 0$, $c_H = 0$. Both investors mix between {Monitor and Buy} and {Not Monitor}, and they always sell when they do not monitor (because $p_B > v_{-1} = v_{-2}$). The comparative statics results presented below are obtained using the parameter values $\phi^0 = 0.1$, $b^0 = 0.1$, $\alpha = 5$, $L = 0.05$, $s = 0.1$ as the benchmark, and $Q_0 = 0.00001$. The qualitative results are not sensitive to the specific parameter values with which they were obtained. (We have tried numerous different sets of parameter values for which the same results were obtained).

The main results from the analysis of the symmetric case are as follows: We find that share wealth (ownership concentration) induces monitoring for firm value. This is done through monitoring with higher frequency but unchanged intensity because of diminishing returns to monitoring. In contrast, nonshare wealth induces monitoring for trading gains rather than value. This is done through monitoring with higher intensity but lower frequency, so that the large shareholders can better hide the information that they monitor and increase their trading gains. With respect to the quality of exogenous governance mechanisms, we find that higher quality of the exogenous mechanisms is associated with monitoring with higher frequency but lower intensity because of the substitutability between the quality of the exogenous mechanism and monitoring.

Figures 2, 3 and 4 use graphs to illustrate comparative statics results for the symmetric case. Figure 2 demonstrates that higher ownership concentration induces higher frequency of monitoring, with unchanged intensity. As a result, expected expenditure on monitoring and expected firm value both increase with ownership concentration. Intuitively, a share-wealthy (vested) investor cares about the firm value. The more shares he owns, the higher his incentive to monitor becomes. However, because of diminishing returns to monitoring, he is better off monitoring more by increasing the frequency of monitoring than by increasing the intensity. The bid–ask spread increases with ownership concentration when ownership concentration is low, but decreases with ownership concentration when ownership concentration is high, reflecting how adverse selection changes with stock endowment. Adverse selection, in turn, depends on the variability in the firm value introduced through monitoring, and on the investors' ability to explore this variability through trade. Thus, small ownership means both little ability for strategic selling (because the ownership is small) and low benefits from strategic buying because buying is rare. Large ownership means both little ability for strategic buying (because as the ask price increases the number of shares each large shareholder can buy with fixed cash is small) and low benefits from strategic selling because selling is rare. (See Fig. 2 for further discussion.)

Figure 3 shows that higher cash wealth induces more intensive but less frequent monitoring while expected expenditure on monitoring, and hence expected monitoring and firm value is unchanged. Intuitively, unlike a share-wealthy investor, a cash-wealthy investor cares about the firm value only when he buys shares. The higher his cash hoard is, the more he wants to monitor, if he does monitor (his monitoring level must be optimal given the size of his holdings). However, in order to have mixing between monitoring and not monitoring in equilibrium, the probability of monitoring must fall. Reducing the frequency helps large investors "hide" the information that they monitor, whereas increasing the intensity increases the variability in the firm value, and therefore increases their trading gains. The figure also reveals that the ask price increases with cash wealth, because large investors have more cash for purchasing stock when they monitor, and hence buy-side adverse selection is higher. In contrast, the bid price is hardly affected, because stock ownership is fixed so that sell-side adverse selection does not change (short selling is not allowed). As a result, the bid–ask spread increases with cash wealth.

Figure 4 depicts that higher quality of the exogenous governance mechanisms induces more frequent but less intensive monitoring. Expected firm value increases with the quality of the exogenous mechanisms, although expected expenditure on monitoring hardly increases. This outcome reflects the substitutability between the quality of the exogenous mechanisms and monitoring. Intuitively, as the quality of the governance mechanisms increases, returns to monitoring diminish more quickly; hence, the better the exogenous mechanisms, the lower the optimal amount each investor spends on monitoring when he monitors. Because higher quality of exogenous mechanisms implies that monitoring is more effective, investors monitor more frequently (with higher probability). However, the frequency of monitoring cannot be too large. If so, inefficiencies from lack of coordination of monitoring (duplication between the

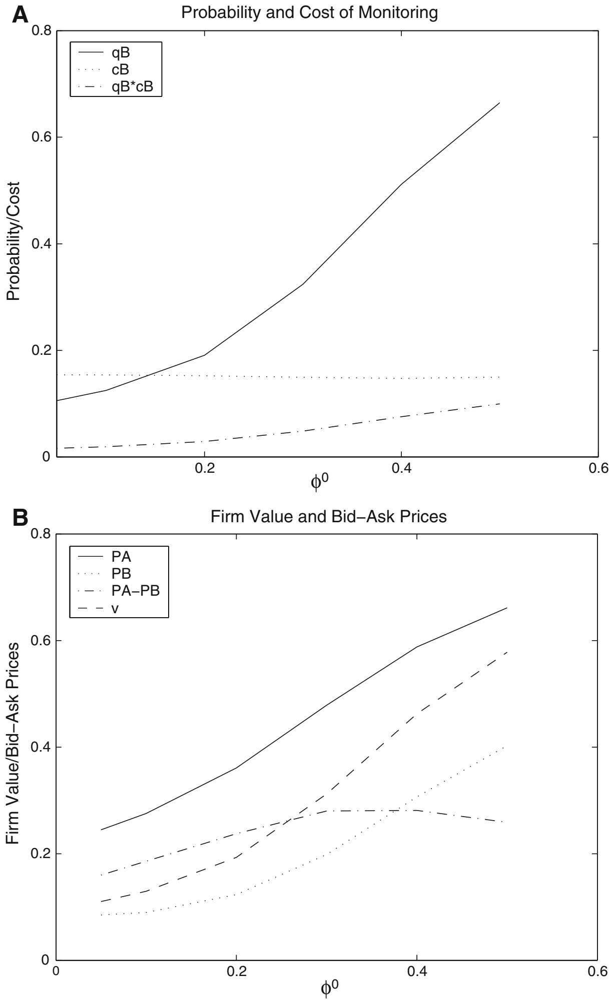

Fig. 2 *Monitoring, firm value, and prices as a function of share wealth—symmetric case.* This figure describes how the equilibrium results change with the stock endowment size, ϕ^0 (i.e. with ownership concentration). **a** Demonstrates that the frequency of monitoring increases with stock endowment, while intensity of monitoring when it occurs, maintains a steady level. As a result, expected expenditure on monitoring, $q_B * c_B$, increases moderately. **b** Shows that the firm value increases with ϕ^0 because expected spending on monitoring increases with ϕ^0. Consequently, both the bid and ask prices increase with ϕ^0. The bid–ask spread increases for small values of ϕ^0 but decreases for large values of ϕ^0, reflecting how adverse selection changes with stock endowment. Adverse selection, in turn, depends on the variability in the firm value introduced through monitoring (i.e., how different are v_{-1}, v, and v_{+1}), and the investors' ability to explore this variability through trade. Low values of ϕ^0 mean both little ability for strategic selling because the ownership is small and low benefits from strategic buying because q_B is low (monitoring and hence buying is rare). High values of ϕ^0 mean both little ability from strategic buying because the number of shares each large shareholder can buy is small and low benefits from strategic selling because q_B is high (monitoring is high and hence selling is rare)

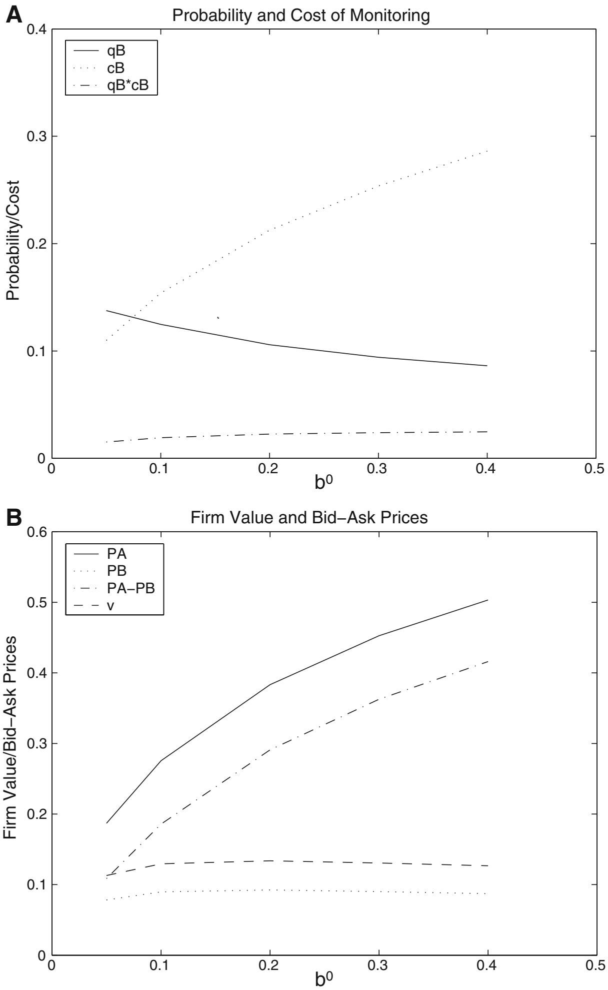

Fig. 3 *Monitoring, firm value, and prices as a function of cash wealth—symmetric case.* This figure describes how equilibrium results change with the bond endowment, b^0. **a** Demonstrates that as b^0 increases, the expected spending on monitoring remains relatively flat. However unlike with increases in ϕ^0, the spending on monitoring when it occurs increases, while the frequency (probability) of monitoring actually decreases. **b** Shows that the expected firm value hardly increases with cash endowment since expected monitoring is relatively unchanged. The figure also depicts that, the ask price increases with b^0 but the bid price is unchanged. As a result the bid–ask spread increases with cash wealth. The ask price increases with b^0 because adverse selection in the buy market increases as large investors have more cash to buy stock with when they monitor. In contrast, the bid price is hardly affected by changes in b^0 because ϕ^0 is fixed; thus sell-side adverse selection does not change

monitoring of the investors) and the incentive to free ride can become very large. The bid–ask spread increases in the quality of exogenous governance mechanisms and reflects how adverse selection changes with it. Adverse selection, in turn, increases because better exogenous governance mechanisms induce higher variability in the

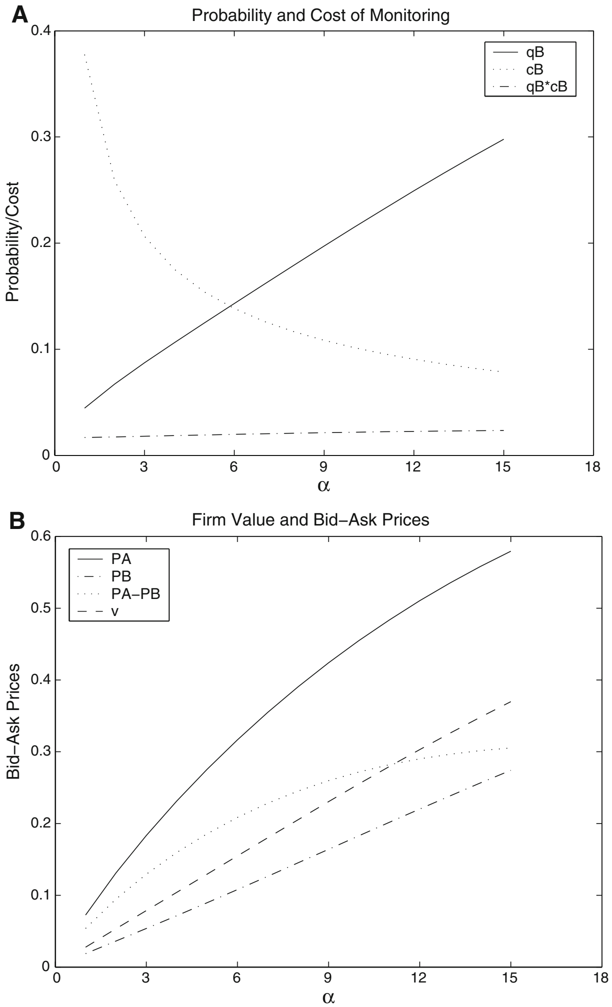

Fig. 4 *Monitoring, firm value, and prices as a function of exogenous governance quality—symmetric case.* This figure describes how the equilibrium results change with the quality of the exogenous mechanisms, α. **a** Demonstrates that as α increases, the amount each investor spends in a monitoring event, c_B, decreases, but the frequency each investor monitors with, q_B, increases. As a result, expected spending on monitoring, $q_B * c_B$, is hardly changed. **b** Depicts an increase in firm value with α (even though expected spending on monitoring hardly changes, as shown in **a**). This is because higher quality of exogenous mechanisms makes monitoring more effective. **b** Also shows that both the bid and the ask prices increase with α since the firm value increases with α. The bid–ask spread increases in α because adverse selection introduced through monitoring increases with α. This is, in turn, because variability in the firm value introduced through the equilibrium value of αc_B and q_B (the frequency of monitoring) increases in α. Although c_B decreases in α, as shown in Fig. 2a (because of diminishing returns), αc_B actually increases in α, thus increasing adverse selection. Adverse selection depends also on purchasing power captured by b_0/P_A. Since P_A increases with α this acts to reduce adverse selection. However, this effect is not dominant

firm value through monitoring (despite the resulting decrease in the expenditure on monitoring when monitoring occurs).

Naturally, the main influence of an increase in liquidity, either on the sell side or on the buy side (L or s), is to reduce the bid–ask spread because the impact of the informed trade on the prices is moderated by the volume (The market maker always makes zero profit). However, lower bid–ask spread, in turn, increases the motivation to monitor because it increases the trading gains and hence results in higher firm value (figures omitted).

4.2 Asymmetric equilibrium

In this subsection, we report results upon the introduction of asymmetries between the large shareholders, i.e., when investors have different initial stakes in the firm ($\phi_1^0 \neq \phi_2^0$), differ in their initial cash wealth ($b_1^0 \neq b_2^0$), and differ in their monitoring efficiency ($\alpha_1 \neq \alpha_2$). We first introduce asymmetry in only one parameter at a time, holding the other parameters fixed. Asymmetry in two parameters is considered in Sect. 4.2.1. We repeated this analysis for different parameter values. With asymmetry in one parameter only, all equilibria obtained were nonfully mixed. The strategy {Monitor and Hold} was never played.[13] As will be described in details below, in these equilibria, either both investors mix between {Monitor and Buy} and {Not Monitor}, or one investor mixes between {Monitor and Buy} and {Not Monitor} and the other investor plays {Not Monitor} with probability one. The comparative statics results presented below are obtained using the symmetric case with parameter values $\phi_1^0 = \phi_2^0 = 0.1$, $b_1^0 = b_2^0 = 0.1$, $\alpha_1 = \alpha_2 = 5$, $L = 0.05$, $s = 0.1$ as the benchmark, and $Q_0 = 0.00001$. The qualitative results are not sensitive to the specific parameter values with which they were obtained.

The main results from the analysis of the asymmetric case are that the investor that is larger/cash-wealthier/more efficient in monitoring, monitors more (both frequency and intensity), and that when asymmetry is sufficiently high, the other investor stops monitoring. Figures 5, 6, and 7 illustrate comparative statics results for the asymmetric case; in each figure asymmetry in a different parameter is introduced. Figure 5 demonstrates that the larger investor monitors more. Intuitively, the larger investor has more shares and thus benefits more from monitoring through value appreciation, whereas the smaller investor has an incentive to free ride. As the asymmetry in ownership becomes high, the smaller investor stops monitoring because he does not have enough shares to benefit on from monitoring, while the larger investor's monitoring becomes similar to the symmetric case, i.e., the larger investor's monitoring increases with ownership through higher frequency but unchanged intensity (see Fig. 2).

Figure 6 shows that the investor with higher cash wealth monitors more (both frequency and intensity). The wealthier investor has higher incentive to benefit from trade, while the less wealthy investor has an incentive to free ride, as potential gains from informed trade for him are low. As the asymmetry in cash wealth becomes

[13] In Sect. 4.2.1 we demonstrate a situation in which the strategy {Monitor and Hold} is played. This requires introducing asymmetry in more than one parameter.

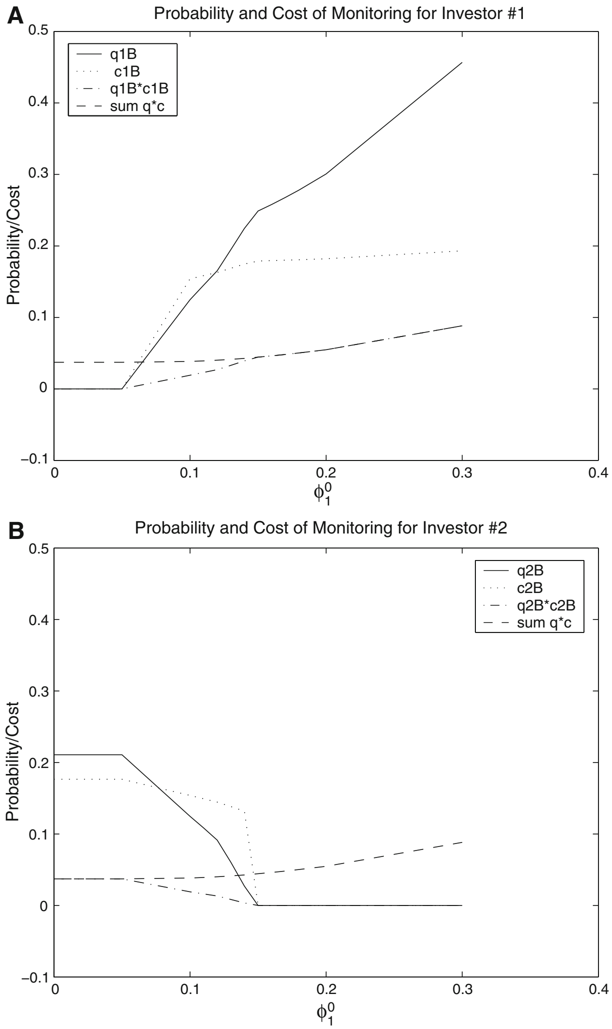

Fig. 5 *Monitoring under asymmetric share endowments.* This figure describes how equilibrium monitoring strategies of both investors change with the share endowment to investor 1, ϕ_1^0, when the share endowment to investor 2, ϕ_2^0, is held fixed. **a**, **b** Characterize the equilibrium monitoring of investor 1 and investor 2, respectively. **a** Shows that for relatively low values of ϕ_1^0, investor 1 does not monitor because he does not have enough shares to benefit on from monitoring. As ϕ_1^0 increases, investor 1's expected monitoring increases. **b** Shows that investor 2's expected monitoring decreases and as ϕ_1^0 becomes, relatively high, investor 2 stops monitoring. **a** Shows that within the range in which both investors monitor, investor 1's monitoring increases both through spending on monitoring when he monitors and through the frequency of monitoring. However, as investor 1 becomes the lead/sole monitor, his monitoring responses to changes in ϕ_1^0 becomes similar to those in the symmetric case Fig. 2a. Namely, his monitoring increases in frequency, but spending on monitoring when monitoring occurs hardly changes, primarily because of diminishing returns to monitoring. Unlike in the symmetric case, as investor 2 reduces his monitoring and investor 1 becomes the lead monitor, the inefficiencies associated with non-coordinated monitoring disappear. Yet, as in the symmetric case, the aggregate expected monitoring (sum $q * c = q_{1B}c_{1B} + q_{2B}c_{2B}$) moderately increases with ϕ_1^0 (shown in both **a** and **b**)

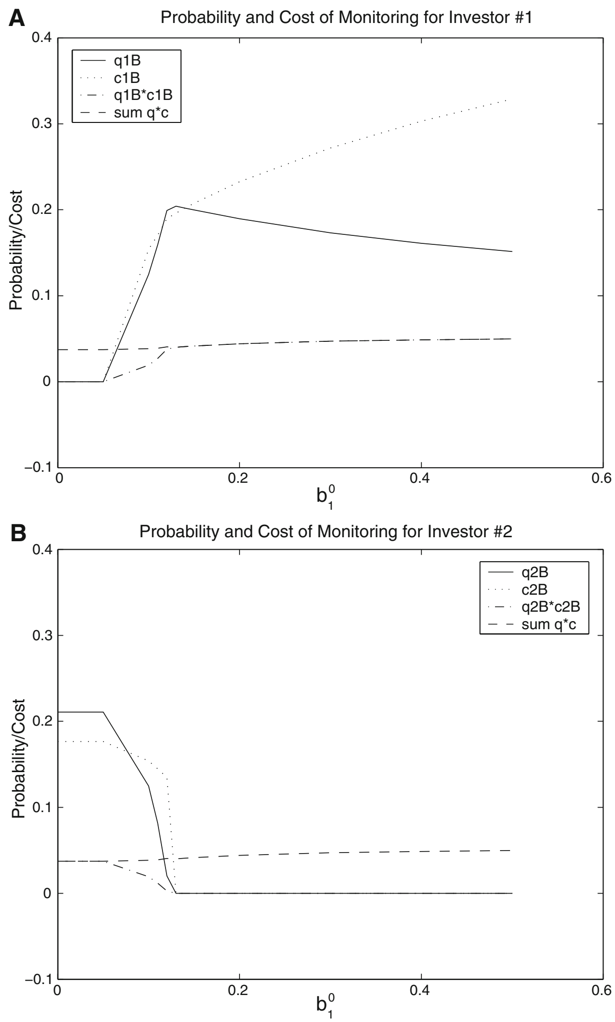

Fig. 6 *Monitoring under asymmetric cash endowments.* This figure describes how equilibrium monitoring strategies of both investors change with the cash endowment to investor 1, b_1^0, when the cash endowment to investor 2, b_2^0, is held fixed. **a**, **b** Characterize the equilibrium monitoring of investor 1 and investor 2, respectively. **a** Shows that for relatively low values of b_1^0 investor 1 does not monitor because he does not have enough cash in order to benefit from adverse selection associated with monitoring. As b_1^0 increases, investor 1's expected monitoring increases. **b** Shows that investor 2's expected monitoring decreases with b_1^0; moreover, as b_1^0 becomes relatively high, investor 2 stops monitoring. **a** Shows that within the range in which both investors monitor, investor 1's monitoring increases both through expenditure on monitoring when he monitors and through the frequency of monitoring. However, as investor 1 becomes the lead monitor, his monitoring responses to changes in b_1^0 become similar to those in the symmetric case (Fig. 3a). Namely, he spends more on monitoring when monitoring occurs, but the frequency with which he monitors actually decreases. As in the symmetric case, the aggregate expected monitoring (sum $q * c$), hardly increases with b_1^0 (shown in both **a**, **b**)

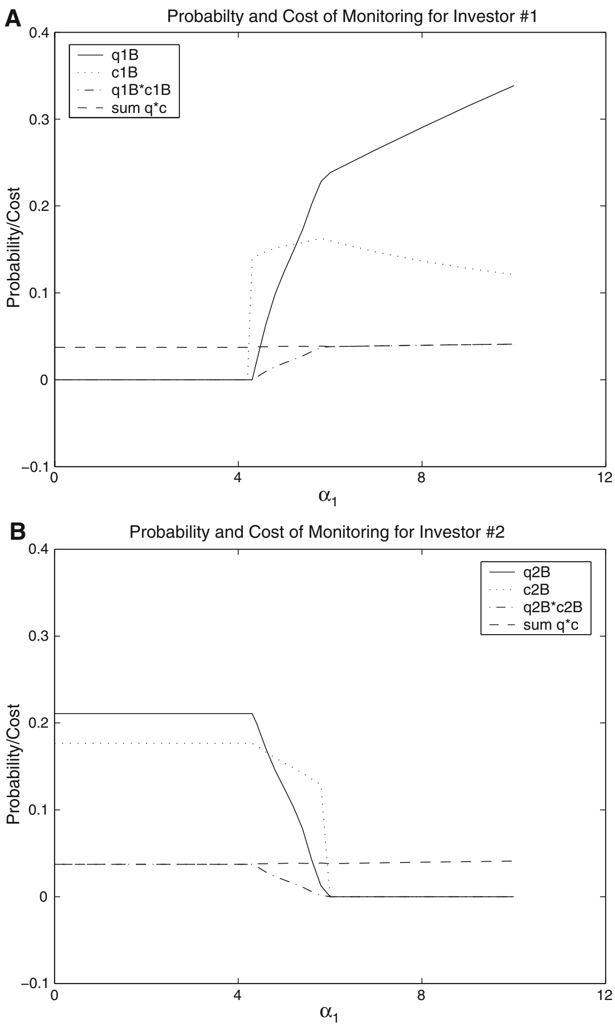

Fig. 7 *Monitoring under asymmetric monitoring efficiency.* This figure describes how equilibrium monitoring strategies of both investors change with the monitoring efficiency of investor 1, α_1 when the monitoring efficiency of investor 2, α_2, is held fixed. **a**, **b** Characterize the equilibrium monitoring of investor 1 and investor 2, respectively. **a** Shows that, given fixed α_2, for relatively low values of α_1 investor 1 does not monitor because his monitoring is not effective. As α_1 increases, investor 1's expected monitoring increases. Meanwhile **b** shows that investor 2's expected monitoring decreases, and, as α_1 becomes relatively high, investor 2 stops monitoring. **a** Shows that within the range in which both investors monitor, investor 1's monitoring increases both through higher expenditure on monitoring when he monitors and through higher frequency of monitoring. However, as investor 1 becomes the lead monitor, the way his monitoring responds to changes in α_1 become similar to that in the symmetric case Fig. 4a. Namely, his monitoring increases through the frequency of monitoring but spending on monitoring when monitoring occurs decreases, because, as investor 2 stops monitoring, returns to monitoring diminish more quickly with α_1. Unlike in the symmetric case, here an additional factor comes into play: as investor 1 becomes the sole monitor, the inefficiencies associated with noncoordinated monitoring disappear. As a result, like in the symmetric case, the aggregate expected monitoring (sum $q * c$), hardly increases with α_1 (shown in both **a**, **b**)

high, the investor with less cash stops monitoring because he does not have enough cash to generate the trading gains required to cover the costs of monitoring, whereas the more cash-wealthy investor's monitoring becomes similar to the symmetric case (see Fig. 3). That is, the (cash) wealthier investor's monitoring increases through higher intensity rather than the frequency of monitoring.

Figure 7 demonstrates that the shareholder whose monitoring is more efficient monitors more (both frequency and intensity). Intuitively, this is because his marginal benefit from monitoring is higher. As the asymmetry in monitoring efficiency becomes high, the shareholder who is less efficient in monitoring stops monitoring and free rides because his monitoring is not efficient enough to induce a sufficient increase in firm value. Meanwhile, the more efficient investor's monitoring becomes similar to the symmetric case (see Fig. 4). That is, the more efficient investor's monitoring increases through more frequent monitoring, but with less expenditure on monitoring when he monitors because of diminishing returns to monitoring.

The similarity of the monitoring characteristics (frequency and intensity) of the single large shareholder in Fig. 5 relative to Fig. 2, in Fig. 6 relative to Fig. 3, and in Fig. 7 relative to Fig. 4, suggests that the monitoring of multiple large shareholders is similar to the monitoring of a single large shareholder and the monitoring of multiple large shareholders when they can collude, with respect to share holdings, cash holdings, and quality of governance.

4.2.1 Special cases

As noted in Sect. 4.1, the strategy {Monitor and Hold} is never played when large investors are symmetric or when asymmetry involves only one parameter. For a large investor to adopt the strategy {Monitor and Hold}, gains from monitoring should be high in comparison to the potential gains from trade. This requires asymmetry not only in shareholdings but also in cash wealth. Figure 8 describes such a case. The figure demonstrates that when one large investor has more cash while the other owns more shares, the former buys when he monitors and the latter holds when he monitors. The investor that has more cash monitors for trading gains, whereas the investor that has more shares monitors only for value. The share-rich but cash-poor investor has too little cash to benefit on, from trade, in order to justify the extra expenditure on monitoring that would increase the expected firm value above the ask price (which is high because of the adverse selection introduced by the other investor).

4.3 Comparison to Noe (2002)

Because our paper is essentially an extension of Noe (2002) a comparison is warranted. Noe assumes a fixed level of monitoring. We generalize Noe's model and allow for varying intensity of monitoring. We show that this generalization yields a rich characterization of monitoring in terms of intensity and frequency. Specifically, for a share-rich (holding) investor it may be profitable to lessen intensity and increase frequency but for a cash-rich (trading) investor it may be profitable to increase intensity

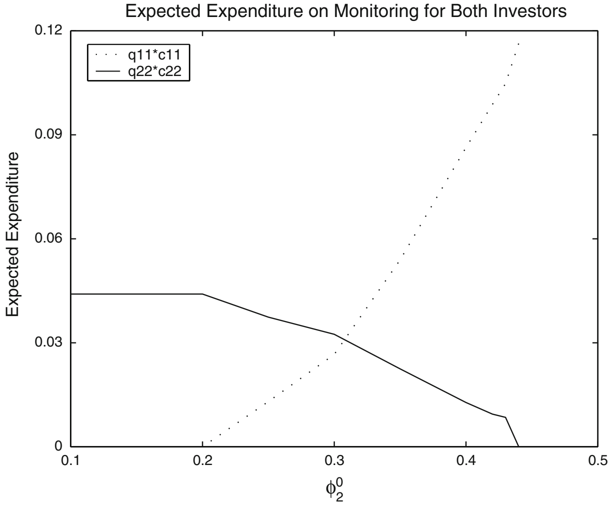

Fig. 8 *Monitoring expenditure—asymmetry in both share and cash endowments.* This figure describes a situation with asymmetries in both share and cash wealth. In both panels we fix $\alpha_1 = \alpha_2 = 5$, $L = 0.1$, $s = 0.05$. We introduce asymmetry in cash wealth by assigning $b_1^0 = 0.2 > b_1^0 = 0.1$. That is, investor 1 is (cash) wealthier than investor 2. We change investor 2's ownership concentration ϕ_2^0 while investor 1's ownership is fixed to $\phi_1^0 = 0.1$. The figure describes how both investor's monitor. When investor 1 is wealthier than investor 2 and owns more shares he is the sole monitor and mixes between {Monitor and Buy} and {Not Monitor} (q_{11} is positive while q_{12} is zero). As the ownership concentration of investor 2 increases, he also starts monitoring (with positive probability) because his benefits from monitoring through value enhancement becomes significant. However, he holds rather than buys shares when he monitors (q_{21} is positive while q_{22} is zero). This is because he does not have enough cash to benefit on, from trade, in order to justify the extra expenditure on monitoring that would increase the expected firm value above P_A. Investor 1 still mixes between {Monitor and Buy} and {Not Monitor}, but as the asymmetry in ownership concentration changes in favor of investor 2, investor 1 monitors less while investor 2 monitors more

and decrease frequency as this makes trading gains greater. This feature just does not emerge in Noe (2002).

Furthermore, the varying monitoring intensity allows an equilibrium in which among shareholders who monitor, the larger shareholder monitors more, whereas in Noe's model a fixed monitoring intensity yields the counterfactual result that smaller shareholder monitors more. The result that larger shareholders monitor more can only be obtained by allowing investors to vary both the frequency and the intensity of monitoring. Intuitively, in Noe's model, if both shareholders monitor, they monitor with the same intensity and both must be randomizing. The only way the larger shareholder can be deterred from switching to playing a pure strategy of always monitoring, given his larger stake in the firm, is if he thinks that his monitoring will have a smaller effect than monitoring by the small shareholder. Given rational expectations, this is only possible if his monitoring does have a smaller effect than monitoring by the smaller shareholder. This smaller effect is possible only if the small shareholder monitors more. Now, by allowing a variation in monitoring intensity there is another way to lower the marginal gain to the large shareholder sufficiently to prevent switching to the pure strategy of always monitoring: lowering the marginal return from monitoring by raising monitoring intensity.

Another effect of allowing for varying intensity is that monitoring without buying additional shares is sometimes used by the large shareholders. In Noe's model investors always buy shares when they monitor.

5 Empirical implications

In this section we discuss empirical implications of the results from Sect. 4. The model predicts two general types of activism: high frequency–low intensity and low frequency–high intensity. We argue that activism is indeed characterized in this manner. For example, analysts' coverage and investor relations are high frequency–low intensity forms of activism. They require exerting effort on a regular basis with actions such as telephone calls, memos, presentations and client reports. These actions are relatively inexpensive. In contrast, special meetings, proxy fights and referendums are low frequency–high intensity forms of activism. They tend to be rare events but are time consuming and are relatively costly.

The model predicts that vested (share rich) investors, will engage high frequency–low intensity activism and that their monitoring will be motivated by value enhancement rather than by trading gains. Consider institutional investors. They are generally vested investors and their activism generally relies on continuous analyst coverage and investor relations. They rarely engage in proxy fights or initiate special shareholder meetings. Indeed, Carlton et al. (1998) investigate activism of institutional investors such as TIAA-CREF and SWIB (State of Wisconsin Investment Board), and document that these vested investors focus on "quiet" activism such as investor relations and do not use intensive and public activism such as proxy fights. Their monitoring is long term and enhances shareholders value. Similar findings are documented in Smith (1996) for CalPERS, another large institutional investor. Smith finds that CalPERS is active in the firms it invests in for the long run. CalPERS does not buy or sell abruptly for trading gains, but rather monitors in order to increase value. More recently, Becht et al. (2009) investigate the activism of Hermes, a UK pension fund. They too show that activism by Hermes is ongoing. Hermes monitors the firms it invests in on a regular basis and does not engage in intensive activism such as proxy fights. Consistent with our model, Becht et al. (2009) also show that the activism of Hermes increases firms' value.

On the other hand, the model predicts that cash rich investors will engage low intensity–high frequency activism, and that their monitoring will be motivated by trading gains rather than value enhancement. Consider corporate raiders. They are generally cash rich and short term investors. Raiders tend to monitor rarely but intensively and are motivated to monitor by trading gains (through value appreciation of shares purchased) rather than by firm value enhancement. Typically corporate raiders accumulate shares quietly and then monitor forcefully with proxy fights, special shareholders meetings, and intensive scrutiny of management. Examples include Karl Icahn's activism in Time Warner, and Kirk Kerkorian's activism in General Motors. Another example of low frequency–high intensity activism is the pressure on Deutsche Bourse to abandon its bid for the London Stock Exchange by a London based hedge fund manager who had taken a large stake in the firm, and as was rumored at the time, had also taken a large short position in the London Stock Exchange. More recently, William Ackman, a cash rich investor quickly and quietly accumulated a 9.8% position in Target and then started aggressive activism. Ackman is known for low frequency–high intensity activism and intensive trading, such as his investments in Wendy's and McDonald's. Consistent with the model, Becht et al. (2009) control for collaborative (high frequency–low intensity) vs. confrontational interventions (high intensity–low frequency) activism, and find that collaborative activism enhances value more than confrontational activism. Bradley et al. (2009) investigate activism in closed-end funds. They find that arbitrageurs have become very active in initiating proxy contests and referendums targeted at open-ending or liquidating deeply discounted closed end funds. These findings too, suggest that activism by cash rich investors is associated with intensity and with significant trade. Bradley et al. (2009) find that the terminal firm value is either enhanced or reduced. Our results thus offer an explanation for the ambiguity in earlier empirical studies about the relation between activism and firm value discussed in the introduction. Namely, our model suggests that ownership concentration motivates activism for firm value, whereas cash wealth motivates activism for trading gains and may not enhance firm value.

Although the model is not dynamic, it has implications about how activism strategies change with investment in equity. When a potentially large investor starts to invest in a firm, his ownership is small and his activism is thus focused on trading gains more than on value enhancement (low frequency with high intensity). The new investor might initially purchase shares in the market quietly, and, once becoming a significant shareholder, engage in a one time intensive activism (proxy fight, takeover threat, and special shareholder meeting to remove the incumbent manager). This generally pushes the market price up with the anticipation for improvement. This now-large investor can then exercise trading gains either by selling in the market or by settling on Greenmail. Alternatively, the investor might continue to accumulate shares, in which case the stake in the firm will increase and activism will become more focused on value enhancement than on trading gains (i.e., higher frequency with lower intensity). This prediction is supported by the findings of Becht et al. (2009). They find that when the institutional investor Hermes makes its first investment in a firm, activism is intense. However, over time, as Hermes becomes vested, its activism becomes less intense but more frequent.

The prediction that better exogenous governance mechanisms lead to higher firm value is also consistent with the empirical evidence. Firms in countries with better exogenous governance mechanisms have higher market valuation, despite greater dispersion of ownership. La Porta et al. (1998) find that across countries, concentration of ownership is negatively related to investor protection. Because both the quality of governance and ownership concentration are exogenous in our model, we cannot investigate the interaction between them. However, the model does predict that higher ownership concentration will result in more spending on monitoring (see Fig. 2a) while lower α does not (see Fig. 4a). Because, in the model, firm value is concave in the quality of governance, one implication that can be drawn outside our model is that ownership concentration will be used to incentivize higher spending on monitoring when quality of governance is low, consistent with La Porta et al. (1998). Bebchuk et al. (2009) concludes that when governance quality is high, firm value is enhanced with less exercise of power, and Bebchuk et al. (2009) show that higher quality of governance results in higher firm value. We are unaware of empirical investigations that study the relation between the quality of governance mechanism and the style of activism. The model predicts, for example, that in countries with better investor protection through laws and regulations, such as the US and the UK, large shareholder activism will be characterized with analysts coverage and investor relations (high frequency–low intensity) more than with proxy fights and management scrutiny (low frequency–high intensity) in comparison to countries with poor investor protection. Similarly, the model predicts that following legislation that improves the quality of laws and regulations such as the 2002 Sarbanes–Oxley Act, large shareholder activism will become more frequent and less intense.

Our analysis of asymmetric equilibria predicts that the investors who are larger, cash-wealthier, or more efficient in monitoring are more active. We are not aware of any systematic empirical investigation into activism within the group of large shareholders. The prediction that, among the large shareholders, the larger ones monitor more is different from the prediction in Noe (2002) that, among the large shareholders, the smaller ones monitor more. As in Noe (2002), the prediction that larger shareholders monitor even when smaller shareholders do not implies that free riding does not necessarily discourage activism, because investors who monitor can benefit not only from an increase in firm value, but also from trade.

The result that the monitoring of a single large shareholder is similar to the monitoring of multiple symmetric large shareholders has important implications. It predicts, for example, that if two large shareholders were initially monitoring the management separately and then begin to cooperate, the qualitative characteristics of their activism will not change. We are not aware of any empirical investigation of this prediction and suggest this as an interesting direction for further research.

6 Conclusion

In this paper we have investigated how large investors execute their activism (monitoring) under diminishing returns to monitoring when financial markets are available, recognizing two dimensions of activism: frequency and intensity. We characterize how ownership concentration, cash wealth, and the quality of exogenous governance mechanisms (laws, regulations) affect activism and firm value. Consistent with the empirical evidence, we find that activism does not always lead to appreciation in firm value. Specifically, ownership concentration motivates activism to enhance firm value, whereas cash-wealth motivates activism for trading gains. We also suggest that better exogenous governance mechanisms lead to higher valuation through more frequent but less intense activism. When asymmetries within the group of large shareholders exist, the model predicts that the investor that is larger/cash-wealthier/more efficient in monitoring is more active. An interesting implication of the model is that monitoring styles of multiple large shareholders are similar whether or not they cooperate and that these styles are similar to those of a single large shareholder.

Appendix

Proof of Proposition 1 Because it is always true that $p_A > Q_0$ (otherwise the market maker is making a positive profit) then shareholding strategy ϕ_i is bounded and closed by (2). By an earlier assumption, the monitoring strategy c_i is also bounded and closed. Hence, because the strategy space $\{(\Phi_i, C_i), i = 1, 2\}$ is a nonempty compact subset of a metric space, and the payoff functions u_i are continuous, by theorem (Glicksberg 1952) there exists a Nash Equilibrium in mixed strategies.[14] □

Proof of Lemma 1 Multiply the market maker zero-profit condition (6) by p_A to get

$$E[(p_A d_B^*(\Phi) + p_A^* l_B(p))(p_A^* - Q^*(C_1, C_2))] = 0.$$

Substitute $p_A d_B^*(\Phi) = D_B^*$ and $p_A l_B(p) = L_B$ to get

$$E[(D_B^* + L_B)(p_A^* - Q^*(C_1, C_2))] = 0.$$

By definition, $L = E[L_B]$, so that we can use $E[YZ] = E[Y]E[Z] + \text{cov}[Z, Y]$ to write

$$(E[D_B^*] + L)(p_A^* - E[Q^*(C_1, C_2)]) - \text{cov}(D_B^*, Q^*) = 0$$

and rearrange to get (8). Use $E[YZ] = E[Y]E[Z] + \text{cov}[Z, Y]$ to write (7) as

$$(E[d_S^*] + l_S)(E[Q^*(C_1, C_2)] - p_B^*) + \text{cov}(d_S^*, Q^*) = 0$$

and rearrange to get (9). □

Proof of Lemma 2 Consider investor 1's optimization problem in (4). The argument of $E_{C_1,\Phi_1}[\cdot]$ is convex in ϕ_1 over $[0, \phi_1^0]$ and over $[\phi_1^0, \bar{\phi}_1]$. To see this, note that the first part of the argument is linear and decreasing in ϕ_1 on each of the segments $[0, \phi_1^0]$, $[\phi_1^0, \bar{\phi}_1]$, with slopes $-p_B$ and $-p_A$, respectively. The second part is linear and increasing with slope v_{-1}^* on $\left[0, \frac{c_1}{E_{C_2}[Q(c_1,C_2)]-v_{-1}^*}\right]$ and linear and increasing with slope $E_{C_2}[Q(c_1, C_2)]$ for $\phi_1 > \frac{c_1}{E_{C_2}[Q(c_1,C_2)]-v_{-1}^*}$ and therefore is convex over $[0, \bar{\phi}_1]$ (because $E_{C_2}[Q(c_1, C_2)] > v_{-1}^*$ for all $c_i > 0$, and $\frac{c_1}{E_{C_2}[Q(c_1,C_2)]-v_{-1}^*}$ can be either larger or smaller than $\bar{\phi}_1$). Hence the summation must be convex on each of the segments $[0, \phi_1^0]$, $[\phi_1^0, \bar{\phi}_1]$. Convexity implies that the optimum is attained on extreme points, so, for investor 1, optimal shareholding is either 0, ϕ_1^0, or $\bar{\phi}_1$, for all (c_1, C_2).

[14] The strategy space is compact since we have assumed that the amount of money available for monitoring is fixed and because $\bar{\phi}$ is bounded since $p_A \geqslant Q_0$. For a more detailed discussion of existence of Nash equilibrium see Fudenberg and Tirole (1991), pp. 35–36.

(In the special case of linearity with zero slope, any choice of ϕ_1 is optimal, hence $0, \phi_1^0$, and $\bar{\phi}_1$ are also optimal. By making the reasonable assumption that whenever indifferent, investor i holds, this pathological case can be ignored.) With a symmetric argument, optimal shareholding choice for investor 2 is either $0, \phi_2^0$, or $\bar{\phi}_2$, for all (C_1, c_2). □

Proof of Lemma 3 For (i), observe from the second term inside $E_{C_1,\Phi_1}[\cdot]$ in (4) that it is never optimal to monitor when $\phi_1 = 0$. For (ii), observe that {Not Monitor and Hold} yields $b_1^0 + \phi_1^0 v_{-1}^*$, whereas {Not Monitor and Buy} yields $\bar{\phi}_1 v_{-1}^* = (\frac{b_1^0}{p_A} + \phi_1^0) v_{-1}^*$. To establish that the latter wealth is strictly lower, we only need to show that $p_A > v_{-1}^*$. Suppose not, then $p_A = v^* = v_{-1}^*$. The second equality here implies that investor 1 never monitors. This, in turn, implies that $v_{-2}^* = Q_0$ and that investor 2 must be monitoring with positive probability $1 > q_2 > 0$ (otherwise, if $q_2 = 0$ then $p_A = Q_0$ and some investor will deviate by monitoring and buying, and if $q_2 = 1$ then $p_A = v^* = v_{+2}^*$ and investor 2 will deviate by not monitoring and selling). But then, since $v^* = q_2 v_{+2}^* + (1 - q_2) v_{-2}^*$, and $p_A = v^*$, it must be the case that $v_{+2}^* > p_A$ (because $v_{-2}^* = 0$ and $1 > q_2 > 0$). Thus $v_{+2}^* > p_A > v_{-2}^* = Q_0$ implying that investor 2 must be buying when he monitors. Thus, $\mathrm{cov}(D_B, Q) > 0$, so that by Lemma 1 it must be the case that $p_A > v^*$, which constitutes a contradiction. The above arguments work symmetrically for investor 2; hence, {Not Monitor and Hold} always dominates {Not Monitor and Buy} for either investor. □

Proof of Lemma 4 We show that, when they monitor, each investor chooses a specific monitoring level. This level depends on whether the investor holds or buys. Without loss of generality, consider investor 1's strategy (on monitoring) $C_1 = \{c_{1j}\}_{j=1}^n$ with respective probabilities $\{P_{1j}\}_{j=1}^n$, given investor 2's strategy $C_2 = \{c_{2k}\}_{k=1}^m$ with respective probabilities $\{P_{2k}\}_{k=1}^m$. Then

$$\begin{aligned} E\left[u_1(C_1, C_2)\right] &= E\left[E\left[u_1(C_1, C_2) | C_1 = c_1\right]\right] \\ &= \sum_{j=1}^n \left(\sum_{k=1}^m \phi_1(c_{1j}) Q(c_{1j}, c_{2k}) P_{2k} + b_1(\phi_1(c_{1j})) - c_{1j} \right) P_{1j} \\ &= \sum_{j=1}^n \left(\phi_1(c_{1j}) \left(\sum_{k=1}^m Q(c_{1j}, c_{2k}) P_{2k} \right) + b_1(\phi_1(c_{1j})) - c_{1j} \right) P_{1j}. \end{aligned}$$

By strict concavity of the monitoring function, Q in C_2, we have that, for all j,

$$\sum_{k=1}^m Q(c_{1j}, c_{2k}) P_{2k} < Q(c_{1j}, \bar{c}_2)$$

where $\bar{c}_2 = \sum_{k=1}^m c_{2k} P_{2k}$ and hence

$$E\left[u_1(C_1, C_2)\right] < \sum_{j=1}^n [\phi_1(c_{1j}) Q(c_{1j}, \bar{c}_2) + b_1(\phi_1(c_{1j})) - c_{1j}] P_{1j}$$

By Lemma 3, when he monitors, optimally, investor 1 either holds ($\phi_1 = \phi_1^0$, $b_1 = b_1^0$) or uses all his wealth to buy stock ($\phi_1 = \bar{\phi}$, $b_1 = 0$). Hence

$$\begin{aligned}
&\sum_{j=1}^{n}\left[(\phi_1(c_{1j})Q(c_{1j},\bar{c}_2) + b_1(\phi_1(c_{1j})) - c_{1j})P_{1j}\right] \\
&\quad \le P_{1B}\sum_{j=1}^{n}\left[(\bar{\phi}_1 Q(c_{1j},\bar{c}_2) + 0 - c_{1j})I_j\{B\}\right] \\
&\qquad + P_{1H}\sum_{j=1}^{n}\left[(\phi_1^0 Q(c_{1j},\bar{c}_2) + b_1^0 - c_{1j})I_j\{H\}\right],
\end{aligned}$$

where $I_j\{B\}$ and $I_j\{H\}$ are the indicator functions on whether investor 1's strategy is {Monitor and Buy} and {Monitor and Hold} in state j respectively, and where P_{1B} and P_{1H} are the aggregate (unconditional) probabilities that investor 1's strategy is {Monitor and Buy} and {Monitor and Hold} in state j, respectively.

Let $\bar{c}_{1B} = \frac{1}{P_{1B}}\sum_{j=1}^{n} c_{1j}P_{1j}I_j\{B\}$ and similarly let $\bar{c}_{1H} = \frac{1}{P_{1H}}\sum_{j=1}^{n} c_{1j}P_{1j}I_j\{H\}$, then by strict concavity of Q in C_1

$$\sum_{j=1}^{n}[(\bar{\phi}_1 Q(c_{1j},\bar{c}_2) - c_{1j})I_j\{B\}] < P_{1B}\left(\bar{\phi}_1 Q(\bar{c}_{1B},\bar{c}_2) - \bar{c}_{1B}\right),$$

and

$$\sum_{j=1}^{n}[\left(Q(c_{1j},\bar{c}_2) + b_1^0 - c_{1j}\right)I_j\{H\}] < P_{1H}\left(\phi_1^0 Q(\bar{c}_1,\bar{c}_2) + b_1^0 - \bar{c}_{1H}\right).$$

Hence

$$\begin{aligned}
E\left[u_1(C_1,C_2)\right] &< P_{1B}\left(\bar{\phi}_1 Q(\bar{c}_{1B},\bar{c}_2) - \bar{c}_{1B}\right) + P_{1H}\left(\phi_1^0 Q(\bar{c}_1,\bar{c}_2) + b_1^0 - \bar{c}_{1H}\right) \\
&= P_{1B}u_1(c_{1B},\bar{c}_2) + P_{1H}u_1(c_{1H},\bar{c}_2). \qquad (30)
\end{aligned}$$

From (30) we can conclude that, given investor 1's choice of shareholding level (buy or hold), there is a specific monitoring level for this investor that strictly dominates any strategy that involves mixing on the monitoring level. However, the level of monitoring may be different depending on whether he buys or holds. The only situation where we may have mixing on positive monitoring level is when the investor is indifferent between {Monitor and Buy} and {Monitor and Hold}. □

Proof of Theorem 1 We need to show that we can rule out the pure strategies (C_1, C_2) for fixed values $c_i \ge 0$. There are four strategies to consider: $(0, 0)$, $(c_1, 0)$, $(0, c_2)$, and (c_1, c_2), for $c_1 > 0$, $c_2 > 0$. First, the strategy profile $(0, 0)$ (i.e., both investors never monitor with probability 1) is ruled out because in this case the market maker sets

$p_B = p_A = v = Q_0$. The stock is almost free (since Q_0 was earlier assumed to be very small however positive for compactness), so someone will deviate and buy and profit from monitoring since by earlier assumption $\partial Q(0,0)/\partial c_i > 1$. Also, we can rule out $(c_1, 0)$ (i.e., investor 1 monitors with probability 1 and investor 2 does not monitor). Suppose not, then the market maker sets $p_B = p_A = v = Q(c_1, 0) > Q_0$. Investor 1 is thus better off deviating to $C_1 = 0$ and selling, because $p_B > Q(0,0) = Q_0$. With a symmetric argument we can rule out $(0, c_2)$. Last, (c_1, c_2) with any fixed values $c_1 > 0, c_2 > 0$ is ruled out because in this case the market maker will set $p_B = p_A = v = Q(c_1, c_2) > Q_0$, and each investor i has the incentive to deviate to $C_i = 0$ and sell all shares. □

Proof of Lemma 5 If $\left(1 - q^*_{iB} - q^*_{iH}\right) = 0$, then both (14) and (15) hold. So suppose, alternatively, that $0 < \left(1 - q^*_{iB} - q^*_{iH}\right) < 1$, i.e., {Not Monitor} is part of the equilibrium strategy. Then, in turn, it must be the case that neither the strategy {Monitor and Buy} nor the strategy {Monitor and Hold} yield a higher profit than the strategy {Not Monitor}. That is, for investor i, both $MP^*_{iB} - C^*_{iB} \le 0$ and $MP^*_{iH} - C^*_{iH} \le 0$. If $MP^*_{iB} - C^*_{iB} < 0$, then {Monitor and Buy} cannot be part of the equilibrium strategy because it yields strictly lower profit than {Not Monitor}. Thus, it must be the case that $q^*_{iB} = 0$, which establishes (14). Similarly, if $MP^*_{iH} - C^*_{iH} < 0$, then {Monitor and Hold} cannot be part of the equilibrium strategy because it yields a strictly lower profit than {Not Monitor}; that is, it must be the case that $q^*_{iH} = 0$, establishing (15). □

Proof of Lemma 6 For p^*_A, from Lemma 1,

$$p^*_A = v^* + \frac{\operatorname{cov}(D^*_B, Q^*)}{E[D^*_B] + L}.$$

First, note that $\operatorname{cov}(D^*_B, Q^*) = E[D^*_B Q^*] - E[D^*_B]E[Q^*] = E\left[D^*_B\,(Q^* - Q_0)\right] - E[D^*_B]E\left[Q^* - Q_0\right]$ and recall that $E[Q^*] = v^*$. Since the investors bid independently, we can write

$$E[D^*_B] = E[D^*_{B1}] + E[D^*_{B2}] = q_{1B}b^0_1 + q_{2B}b^0_2.$$

Next we find $E[Q^* - Q_0] = v^* - Q_0$

$$\begin{aligned}
E[Q^* - Q_0] = v^* - Q_0 = E[Q(\alpha, C_1, C_2) - Q_0] = E[1 - e^{-(\alpha_1 C_1 + \alpha_2 C_2)}]\\
= 1 - [q_{1B}q_{2B}e^{-(\alpha_1 c_{1B} + \alpha_2 c_{2B})} + q_{1B}q_{2H}e^{-(\alpha_1 c_{1B} + \alpha_2 c_{2H})}\\
+ q_{1B}(1 - q_{2B} - q_{2H})e^{-\alpha_1 c_{1B}} + q_{1H}q_{2B}e^{-(\alpha_1 c_{1H} + \alpha_2 c_{2B})}\\
+ q_{1H}q_{2H}e^{-(\alpha_1 c_{1H} + \alpha_2 c_{2H})} + q_{1H}(1 - q_{2B} - q_{2H})e^{-\alpha_1 c_{1H}}\\
+ (1 - q_{1B} - q_{1H})q_{2B}e^{-\alpha_2 c_{2B}} + (1 - q_{1B} - q_{1H})q_{2H}(1 - e^{-\alpha_2 c_{2H}})\\
+ (1 - q_{1B} - q_{1H})(1 - q_{2B} - q_{2H})].
\end{aligned}$$

Last, we need to find $E[D_B^*(Q^* - Q_0)]$

$$\begin{aligned}E[D_B^*\left(Q^* - Q_0\right)] &= E[(D_{1B}^* + D_{2B}^*)\left(Q^* - Q_0\right)]\\ &= q_{1B}b_1^0[q_{2B} - q_{2B}e^{-(\alpha_1 c_{1B} + \alpha_2 c_{2B})} + q_{2H} - q_{2H}e^{-(\alpha_1 c_{1B} + \alpha_2 c_{2H})}\\ &\quad + (1 - q_{2B} - q_{2H}) + (1 - q_{2B} - q_{2H})(1 - e^{-\alpha_1 c_{1B}})]\\ &\quad + q_{2B}b_2^0[q_{1B} - q_{1B}e^{-(\alpha_1 c_{1B} + \alpha_2 c_{2B})} + q_{1H} - q_{1H}e^{-(\alpha_1 c_{1H} + \alpha_2 c_{2B})}\\ &\quad + (1 - q_{1B} - q_{1H}) - (1 - q_{1B} - q_{1H})(1 - e^{-\alpha_2 c_{2B}})].\end{aligned}$$

upon simplification

$$E[D_B^*\left(Q^* - Q_0\right)] = q_{1B}b_1^0(1 - e^{-\alpha_1 c_{1B}}E[e^{-\alpha_2 c_2}]) + q_{2B}b_2^0(1 - e^{-\alpha_2 c_{2B}}E[e^{-\alpha_1 c_1}])\,.$$

Now we can calculate

$$\begin{aligned}\operatorname{cov}(D_B^*, Q^*) &= E[D_B^*\left(Q^* - Q_0\right)] - E[D_B^*]E[Q^* - Q_0]\\ &= q_{1B}b_1^0 E[e^{-\alpha_2 C_2}]\left(E[e^{-\alpha_1 C_1}] - e^{-\alpha_1 c_{1B}}\right)\\ &\quad + q_{2B}b_2^0 E[e^{-\alpha_1 C_1}]\left(E[e^{-\alpha_2 C_2}] - e^{-\alpha_2 c_{2B}}\right).\end{aligned}$$

and note that $\operatorname{cov}(D_B^*, Q^*) > 0$.

For p_B^*, from Lemma 1,

$$p_B^* = v^* + \frac{\operatorname{cov}(d_S^*, Q^*)}{E[d_S^*] + s}.$$

$\operatorname{cov}(d_S^*, Q^*) = E[d_S^* Q^*] - E[d_S^*]E[Q^*] = E\left[d_S^*(Q^* - Q_0)\right] - E[d_S^*]E\left[Q^* - Q_0\right]$ where $E[Q^*] = v^*$. Since the investors bid independently, we can write

$$\begin{aligned}E[d_S^*] &= E[d_{S1}^*] + E[d_{S2}^*] = E[(\phi_1^0 - \phi_{1NM}) + (\phi_2^0 - \phi_{2NM})]\\ &= (1 - q_{1B} - q_{1H})(\phi_1^0 - \phi_{1NM}) + (1 - q_{2B} - q_{2H})(\phi_2^0 - \phi_{2NM}).\end{aligned}$$

where

$$\begin{aligned}\phi_{iNM} &= \phi_i^0 \quad \text{if } v_{-i}^* \geq p_B\\ &= 0 \qquad \text{otherwise.}\end{aligned}$$

Next, we calculate $E[d_S^*(Q^* - Q_0)]$.

$$\begin{aligned}E[d_S^*\left(Q^* - Q_0\right)] &= E\left[\left((\phi_1^0 - \phi_{1NM}) + (\phi_2^0 - \phi_{2NM})\right)\left(1 - e^{-(\alpha_1 C_1 + \alpha_2 C_2)}\right)\right]\\ &= (1 - q_{1B} - q_{1H})(\phi_1^0 - \phi_{1NM})\left(1 - E[e^{-\alpha_2 C_2}]\right)\\ &\quad + (1 - q_{2B} - q_{2H})(\phi_2^0 - \phi_{2NM})\left(1 - E[e^{-\alpha_1 C_1}]\right).\end{aligned}$$

Now we can calculate:

$$
\begin{aligned}
\operatorname{cov}(d_S^*, Q^*) &= E[d_S^*(Q^* - Q_0)] - E[d_S^*]E[Q^* - Q_0] \\
&= (1 - q_{1B} - q_{1H})(\phi_1^0 - \phi_{1NM})E[e^{-\alpha_2 C_2}]\left(E[e^{-\alpha_1 C_1}] - 1\right) \\
&\quad + (1 - q_{2B} - q_{2H})(\phi_2^0 - \phi_{2NM})E[e^{-\alpha_1 C_1}]\left(E[e^{-\alpha_2 C_2}] - 1\right).
\end{aligned}
$$

and note that $\operatorname{cov}(d_S^*, Q^*) \leq 0$. □

Proof of Lemma 7 Consider the maximand in (4):

$$
\begin{aligned}
E\Bigg[&\left(b_1^0 + \left(\phi_1^0 - \Phi_1\right)^+ p_B - \left(\Phi_1 - \phi_1^0\right)^+ p_A\right) \\
&+ \max\left\{E_{C_2}\left[\Phi_1 Q\left(C_1, C_2\right) - C_1\right], \Phi_1 E_{C_2}\left[Q\left(0, C_2\right)\right]\right\}\Bigg]
\end{aligned}
\tag{31}
$$

To find c_{1B}^* (monitoring level for investor 1, given that he monitors and buys), recall our earlier result that, when investor 1 monitors and buys, he chooses $\phi_1 = \bar{\phi}_1$, hence $b_1 = 0$. Recall that b_1 is equal to the first term in (31). His expected utility is thus

$$
\begin{aligned}
E[u_1(\phi, c_{1B}, C_2, p)] &= E[\bar{\phi}_1 Q(\alpha, c_{1B}, C_2) - c_{1B}] \\
&= \bar{\phi}_1 \begin{bmatrix} q_{2B} Q\left(\alpha, c_{1B}, c_{2B}\right) + q_{2H} Q\left(\alpha, c_{1B}, c_{2H}\right) \\ + \left(1 - q_{2B} - q_{2H}\right) Q(\alpha, c_{1B}, 0) \end{bmatrix} - c_{1B},
\end{aligned}
\tag{32}
$$

where the first term on the right hand side of the last equality reflects investor 1's expectation over the three states $\{q_{2B}, C_2 = c_{2B} > 0\}$, $\{q_{2H}, C_2 = c_{2H} > 0\}$, and $\{(1 - q_{2B} - q_{2H}), C_2 = 0\}$. To maximize his expected utility, investor 1 solves

$$
\frac{\partial E[u_1(\phi, c_{1B}, C_2, p)]}{\partial c_{1B}} = 0. \tag{33}
$$

Use (16) to write

$$
\frac{\partial Q\left(\alpha, c_{1B}, C_2\right)}{\partial c_{1B}} = \alpha_1 e^{-(\alpha_1 c_{1B} + \alpha_2 C_2)}. \tag{34}
$$

Substitute (32) and (34) into (33) to get

$$
\frac{\partial E[u_1(\Phi, c_{1B}, C_2, p)]}{\partial c_{1B}} = \bar{\phi}_1 \alpha_1 e^{-\alpha_1 c_{1B}} E\left[e^{-\alpha_2 c_2}\right] - 1 = 0.
$$

Rearrange to get (26). The equilibrium monitoring levels c_{1H}^*, c_{2B}^*, and c_{2H}^* can be found to be (27)–(29), respectively, using a similar analysis. □

Proof of Proposition 3 Equilibrium prices and strategies can be derived by solving the system of 12 Eqs. (35)–(46).

$$p_A - 1 + \frac{q_{1B} b_1^0 (1-v_{-1}) e^{-\alpha_1 c_{1B}} + q_{2B} b_2^0 (1-v_{-2}) e^{-\alpha_2 c_{2B}} + L (1-v_{-1})(1-v_{-2})}{q_{1B} b_1^0 + q_{2B} b_2^0 + L} = 0 \tag{35}$$

$$p_B - (v_{-1} + v_{-2}) + \frac{(1-q_{1B}-q_{1H})(\phi_1^0-\phi_{1NM})v_{-2}+(1-q_{2B}-q_{2H})(\phi_2^0-\phi_{2NM})v_{-1}+s v_{-1} v_{-2}}{(1-q_{1B}-q_{1H})(\phi_1^0-\phi_{1NM})+(1-q_{2B}-q_{2H})(\phi_2^0-\phi_{2NM})+s} = 0 \tag{36}$$

$$\phi_{1NM} - \phi_1^0 \min\left(\frac{\max(v_{-1} - p_B, 0)}{v_{-1} - p_B}, 1\right) = 0 \tag{37}$$

$$\phi_{2NM} - \phi_2^0 \min\left(\frac{\max(v_{-2} - p_B, 0)}{v_{-2} - p_B}, 1\right) = 0 \tag{38}$$

$$c_{1B} - \frac{1}{\alpha_1}\left(\ln\left(\frac{b_1^0}{p_A} + \phi_1^0\right) + \ln \alpha_1 + \ln(1 - v_{-1})\right) = 0 \tag{39}$$

$$c_{1H} - \frac{1}{\alpha_1}\left(\ln \phi_1^0 + \ln \alpha_1 + \ln(1 - v_{-1})\right) = 0 \tag{40}$$

$$c_{2B} - \frac{1}{\alpha_2}\left(\ln\left(\frac{b_2^0}{p_A} + \phi_2^0\right) + \ln \alpha_2 + \ln(1 - v_{-2})\right) = 0 \tag{41}$$

$$c_{2H} - \frac{1}{\alpha_2}\left(\ln \phi_2^0 + \ln \alpha_2 + \ln(1 - v_{-2})\right) = 0 \tag{42}$$

$$\begin{aligned} &q_{1B}(1 - q_{1B} - q_{1H})\left(\left(\frac{b_1^0}{p_A} + \phi_1^0\right)\left(1 - (1 - v_{-1}) e^{-\alpha_1 c_{1B}}\right)\right. \\ &\left.-b_1^0 - \phi_1^0 \max(p_B, v_{-1}) - c_{1B}\right) = 0 \end{aligned} \tag{43}$$

$$\begin{aligned} &q_{1H}(1 - q_{1B} - q_{1H})\left(\phi_1^0\left(\left(1 - (1 - v_{-1}) e^{-\alpha_1 c_{1H}}\right)\right.\right. \\ &\left.\left.- \max(p_B, v_{-1})\right) - c_{1H}\right) = 0 \end{aligned} \tag{44}$$

$$\begin{aligned} &q_{2B}(1 - q_{2B} - q_{2H})\left(\left(\frac{b_2^0}{p_A} + \phi_2^0\right)\left(1 - (1 - v_{-2}) e^{-\alpha_2 c_{2B}}\right)\right. \\ &\left.-b_2^0 - \phi_2^0 \max(p_B, v_{-2}) - c_{2B}\right) = 0 \end{aligned} \tag{45}$$

$$\begin{aligned} &q_{2H}(1 - q_{2B} - q_{2H})\left(\phi_2^0\left(\left(1 - (1 - v_{-2}) e^{-\alpha_2 c_{2H}}\right)\right.\right. \\ &\left.\left.- \max(p_B, v_{-2})\right) - c_{2H}\right) = 0 \end{aligned} \tag{46}$$

The first two equations, Eqs. (35) and (36) are the price equations that result from substituting (18)–(21) into the price Eqs. (24) and (25) from Lemma 6, and rearranging. The next two equations, Eqs. (37) and (38) are compact presentations of the equations of shareholding under {Not Monitor} strategy (22) and (23), respectively. Equations (39)–(42) are the optimal monitoring Eqs. (26)–(29) upon substituting of Eqs. (18)–(21) from Lemma 7 and rearrangement. The last four equations, Eqs. (43)–(46) are derived from the restrictions on monitoring profits and monitoring probabilities that were developed in Sect. 2 in Eqs. (10)–(15). To find these last four equations, use (14) and (15) from Lemma 5 to write

$$q_{1B}^{*}\left(1-q_{1B}^{*}-q_{1H}^{*}\right)\left(MP_{1B}^{*}-c_{1B}^{*}\right)=0, \tag{47}$$
$$q_{1H}^{*}\left(1-q_{1B}^{*}-q_{1H}^{*}\right)\left(MP_{1H}^{*}-c_{1H}^{*}\right)=0, \tag{48}$$
$$q_{2B}^{*}\left(1-q_{2B}^{*}-q_{2H}^{*}\right)\left(MP_{2B}^{*}-c_{2B}^{*}\right)=0, \tag{49}$$
$$q_{2H}^{*}\left(1-q_{2B}^{*}-q_{2H}^{*}\right)\left(MP_{2H}^{*}-c_{2H}^{*}\right)=0, \tag{50}$$

and use (12) and (13) to write

$$MP_{1B}^{*}=\left(\frac{b_1^0 v_{+iB}^{*}}{p_A^{*}}-b_1^0\right)+\phi_1^0\left[v_{+1B}^{*}-\max\left(p_B^{*},v_{-1}^{*}\right)\right], \tag{51}$$
$$MP_{1H}=\phi_1^0\left[v_{+1H}^{*}-\max\left(p_B^{*},v_{-1}^{*}\right)\right], \tag{52}$$
$$MP_{2B}^{*}=\left(\frac{b_2^0 v_{+2B}^{*}}{p_A^{*}}-b_2^0\right)+\phi_2^0\left[v_{+2B}^{*}-\max\left(p_B^{*},v_{-2}^{*}\right)\right], \tag{53}$$
$$MP_{2H}=\phi_2^0\left[v_{+2H}^{*}-\max\left(p_B^{*},v_{-2}^{*}\right)\right], \tag{54}$$

where, based on Eqs. (10) and (11), we have

$$\begin{aligned}
v_{+1B}^{*}&=E\left[Q(\alpha,C_1,C_2)|_{C_1=c_{1B}^{*},C_2=C_2^{*}}\right]=Q_0+1-E\left[e^{-\alpha_2 C_2^{*}}\right]e^{-\alpha_1 c_{1B}^{*}}\\
&=Q_0+1-e^{-\alpha_1 c_{1B}^{*}}\left[q_{2B}^{*}e^{-\alpha_2 c_{2B}^{*}}+q_{2H}^{*}e^{-\alpha_2 c_{2H}^{*}}+\left(1-q_{2B}^{*}-q_{2H}^{*}\right)\right] \qquad (55)\\
v_{+1H}^{*}&=E\left[Q(\alpha,C_1,C_2)|_{C_1=c_{1H}^{*},C_2=C_2^{*}}\right]=Q_0+1-E\left[e^{-\alpha_2 C_2^{*}}\right]e^{-\alpha_1 c_{1H}^{*}}\\
&=Q_0+1-e^{-\alpha_1 c_{1H}^{*}}\left[q_{2B}^{*}e^{-\alpha_2 c_{2B}^{*}}+q_{2H}^{*}e^{-\alpha_2 c_{2H}^{*}}+\left(1-q_{2B}^{*}-q_{2H}^{*}\right)\right] \qquad (56)\\
v_{-1}^{*}&=E\left[Q(\alpha,C_1,C_2)|_{C_1=0,C_2=C_2^{*}}\right]=Q_0+1-E\left[e^{-\alpha_2 C_2^{*}}\right]\\
&=Q_0+1-\left[q_{2B}^{*}e^{-\alpha_2 c_{2B}^{*}}+q_{2H}^{*}e^{-\alpha_2 c_{2H}^{*}}+(1-q_{2B}^{*}-q_{2H}^{*})\right]\\
&=Q_0+q_{2B}^{*}(1-e^{-\alpha_2 c_{2B}^{*}})+q_{2H}^{*}\left(1-e^{-\alpha_2 c_{2H}^{*}}\right). \qquad (57)
\end{aligned}$$

$v_{+2B}^{*}, v_{+2H}^{*}, v_{-2}^{*}$ are defined symmetrically. Substitute into (47)–(50) and rearrange to get Eqs. (43)–(46), the last four equations of Proposition 3. □

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