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The role of accounting disaggregation in detecting and mitigating earnings management

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Abstract Though ample empirical evidence alludes to the importance of disaggregated accounting data in the context of earnings management, extant theory considers biases in reporting mostly at the aggregated level of the accounting report. By introducing accounting disaggregation into the conventional theoretical framework of earnings management, this study highlights the essential role that disaggregated accounting data play in detecting and mitigating reporting manipulations. Disaggregated reports are shown to be especially effective when they consist of accounting items that are tightly interrelated by their fundamental economic nature, differ considerably in their sensitivity to reporting manipulations, and vary in their signs.

Keywords Information asymmetry · Accounting · Financial reporting · Reporting bias · Earnings management · Disaggregated accounting data · Financial ratios · Ratio analysis

JEL Classification D82 · G14 · M41 · M43

1 Introduction

Accounting measures are typically the aggregation of many components. Some of them have to be disclosed within the financial statements, while others may be disclosed in notes to the financial statements. Though ample empirical evidence alludes to the importance of disaggregated accounting data in the context of earnings management

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(e.g., Healy 1985; De Angelo 1986; Jones 1991; Dechow and Sloan 1991; Dechow et al. 1995; Balsam et al. 2002; Hirst et al. 2007), extant theory pertains to biases in reporting mostly at the aggregated level of the accounting report (e.g., Dye 1988; Stein 1989; Fischer and Verrecchia 2000; Kirschenheiter and Melumad 2002; Fischer and Stocken 2004; Ewert and Wagenhofer 2005; Guttman et al. 2006; Einhorn and Ziv 2012). By introducing accounting disaggregation into the conventional theoretical framework of earnings management, this study illuminates the essential role it plays in detecting and mitigating manipulations in reporting.

Our analysis is based on the observation that different line items of the financial statements tend to be stochastically interrelated (e.g., Nissim and Penman 2001). Examples include the ratios that usually exist between earnings components (such as net profit margin, operating profit margin, gross profit margin, and effective tax rate), ratios based on balance sheet line items (such as current ratio or financial leverage), and ratios that involve an earnings item and a balance sheet item (such as asset turnover or return on equity).While such accounting relations are usually noisy, because they are largely influenced by shocks in the business environment, they nevertheless typically follow a systematic stochastic behavior pattern that stems from the fundamental economic nature of the underlying accounting measures. The degree of noisiness to which relations between various accounting items are subject depends on the particular accounting items involved, as well as on industry and firm-specific characteristics.

We argue that reporting manipulations are likely to distort the fundamental stochastic behavior of the interrelations between the accounting components. Distortions in accounting ratios that involve two components with opposite signs (such as ratios between incomes and expenses or ratios between assets and liabilities) might arise due to the managerial wish to bias the two components in opposite directions. Accounting ratios that involve two components with the same sign are also likely to be distorted as a result of reporting manipulations but for a different reason. Here, the managerial incentives favor the same direction of bias, but differences in the managerial ability to manipulate each of the two components are likely to generate a report that distorts their expected fundamental relationship. It is normally the case that the degree of reporting leeway accorded to managers within accounting conventions varies across different components of the financial statements. In consequence, a reporting bias in a certain item is not likely to be accompanied by a perfectly proportional bias in another item, even if both items are biased in the same direction. We demonstrate the distorting impact of reporting manipulations on the fundamental expected relations between the different accounting items and explore its implications for the ability of external users of financial statements to detect manipulations in reporting and for the propensity of managers to engage in reporting manipulations. To do so, we extend the earnings management setup of Fischer and Verrecchia (2000) by introducing a disaggregated accounting report for which the components can be manipulated at different costs.

An investigation of the equilibrium in our reporting game highlights the power of disaggregated accounting data in detecting and mitigating reporting manipulations. In equilibrium, due to exogenous noise, the capital market investors are incapable of perfectly identifying and backing out the biases in reporting. Nevertheless, knowing that managers have different incentives and different degrees of discretion in manipulating various line items of the accounting report, the investors rationally infer that reporting manipulations are likely to distort the expected fundamental relationships between the components of the report. So, they can utilize the disaggregated accounting data to identify apparent irregularities in the accounting report. Such irregularities may either result from reporting manipulations or stem from economic noise and can thus serve as noisy indicators of the reporting bias. Using these indicators, investors can better adjust for biases in reporting and thereby more accurately evaluate the firm based on the accounting report. It should be noted that, even though such an analysis of disaggregated accounting data relies on the products of the reporting process, which are all tainted by the managerial manipulations, it nevertheless enables the market to imperfectly discern and back out manipulations in reporting.

The power of accounting disaggregation in assisting the market to detect manipulations in reporting and adjust for them has two countervailing effects on managerial misreporting incentives. On one hand, recognizing the investors' ability to identify irregularities in the accounting report and thereby imperfectly detect biases in reporting, managers become more reluctant to manipulate the report in the first place. On the other hand, this ability of the investors results in a pricing rule that places more weight on the aggregate accounting report, enhancing the managerial incentives to bias the report. We show that the former force always dominates. Our analysis, therefore, demonstrates that ratio analysis of disaggregated accounting information, via its role in indicating reporting manipulations, might serve as an effective mechanism for mitigating managerial misreporting propensity.

Despite its role in revealing and restricting earnings management, accounting disaggregation is not necessarily undesirable from the viewpoint of managers. It has been well established in the literature that managers can be worse off with the option of biasing their accounting reports (Stein 1989). Knowing that investors will suspect their report in any case, managers might end up taking costly actions to bias their reporting even when they cannot fool the market. In the presence of exogenous noise that does not allow the investors to perfectly back out the reporting bias, some types of managers actually benefit from the reporting bias but at the expense of other types of managers (Fischer and Verrecchia 2000). We show that accounting disaggregation alleviates this problem by reducing the sensitivity of all types of managers to the bias option. By imperfectly revealing the managerial type, accounting disaggregation impedes the misreporting incentives of managers who are otherwise capable of fooling the market, while relaxing the pressure on all other managers to engage in inefficient actions of earnings management.

Our study suggests that expanded disaggregated disclosure may be advantageous in cases where accounting standards allow a relatively high degree of managerial discretion in reporting. Utilizing a comparative statics analysis, we also offer guidance to accounting policymakers in selecting among alternative accounting disaggregating procedures. The analysis highlights the merits of decomposing the accounting report into items that are fundamentally tightly proportional to each other (for example, sales and cost of sales). Disclosure of such accounting items is powerful in revealing reporting manipulations because their relationships are relatively weakly sensitive to economic noise, which implies that a substantial part of the irregularities in their relationships is caused by reporting biases. The analysis also sheds light on the advantages of decomposing the accounting report into items that differ considerably in the extent to which their reporting can be managed (for example, cash and accrual components of earnings) or items with opposite signs (for example, incomes and expenses in the income statement or assets and liabilities in the balance sheet). Disclosure of such accounting items is especially effective in exposing earnings management because their relationships are highly sensitive to reporting manipulations, so a large portion of the irregularities in their relationships can be ascribed to biases in reporting.

The paper proceeds as follows. In the next section, we model the reporting game on which our analysis is based. The equilibrium in this game is derived and analyzed in Sect. 3, demonstrating the role that disaggregated accounting data play in mitigating reporting manipulations. The final section summarizes and offers concluding remarks. Proofs appear in the "Appendix".

2 Model

Our model depicts a single-period reporting game between a privately informed manager of a publicly traded firm and capital market investors. In designing the model, we built on the earnings management setup of Fischer and Verrecchia (2000). We deviate from their model by considering a disaggregated accounting report rather than an aggregate accounting report. This allows us to highlight the role of accounting disaggregation in the context of earnings management. The remainder of this section details the parameters and assumptions underlying the model, which are all assumed to be common knowledge unless otherwise indicated.

We consider a firm that is traded in a capital market for one period. We model the firm's uncertain accounting earnings during the given period as a random variable $\tilde{a}$, which is normally distributed with mean μ and variance σ^2. We further assume that the earnings $\tilde{a}$ that the firm yields during the given period constitute its equity book value at the end of the period, as well as its economic equity value at the end of the period.[1] To capture the essence of almost any accounting measure as an aggregator, we assume that the accounting variable $\tilde{a}$ is the sum of two normally distributed components $\tilde{a}_1$ and $\tilde{a}_2$. That is, $\tilde{a} = \tilde{a}_1 + \tilde{a}_2$.

The covariance ratio $\mathrm{cov}(\tilde{a}, \tilde{a}_2)/\mathrm{cov}(\tilde{a}, \tilde{a}_1)$ deserves special attention in our analysis.[2]Throughout the analysis, we denote it by λ and refer to it as the fundamental proportion between the components $\tilde{a}_1$ and $\tilde{a}_2$ of the aggregate accounting measure $\tilde{a}$. This interpretation is rooted in the observation that the

[1] The assumption that the accounting variable $\tilde{a}$ equals the economic equity value of the firm is a simplifying assumption (which fits the single-period nature of the model). However, the analysis can be generalized to the case where the accounting variable is a noisy estimator of the firm's value without qualitatively affecting the results. Also, as we can interchangeably refer to the random variable $\tilde{a}$ as representing earnings or equity book value, our analysis applies to both earnings data and balance sheet data.

[2] Since $\sigma^2 = \mathrm{cov}(\tilde{a}, \tilde{a}_1) + \mathrm{cov}(\tilde{a}, \tilde{a}_2)$ is strictly positive, either the covariance $\mathrm{cov}(\tilde{a}, \tilde{a}_1)$ or the covariance $\mathrm{cov}(\tilde{a}, \tilde{a}_2)$ must differ from zero. So, without loss of generality, we assume that $\mathrm{cov}(\tilde{a}, \tilde{a}_1) \neq 0$, ensuring that the covariance ratio $\mathrm{cov}(\tilde{a}, \tilde{a}_2)/\mathrm{cov}(\tilde{a}, \tilde{a}_1)$ is always welldefined. The covariance ratio can be either positive or negative, but it cannot be -1 because the variance $\sigma^2 = \mathrm{cov}(\tilde{a}, \tilde{a}_2) + \mathrm{cov}(\tilde{a}, \tilde{a}_1)$ of the aggregate accounting measure $\tilde{a}$ is strictly positive.

normally distributed random variable $(\tilde{a}_2 - E(\tilde{a}_2)) - \lambda(\tilde{a}_1 - E(\tilde{a}_1))$, denoted $\tilde{\varepsilon}$, is not correlated with the aggregate accounting measure $\tilde{a}$ and its mean equals zero.[3] Hence, the covariance ratio λ stochastically links between the two accounting variables $\tilde{a}_1$ and $\tilde{a}_2$ in the following linear fashion: $\tilde{a}_2 = m + \lambda\tilde{a}_1 + \tilde{\varepsilon}$, where $m = E(\tilde{a}_2) - \lambda E(\tilde{a}_1)$ is a scalar, and $\tilde{\varepsilon}$ is a normally distributed random variable with a mean of zero that is independent of the aggregate accounting measure $\tilde{a}$. For example, in the special case where $\tilde{a}_1$ represents the sales and $\tilde{a}_2$ represents the cost of goods sold, the scalar m measures the average level of the fixed costs, whereas the scalar λ measures the fundamental proportion between the variable costs $\tilde{a}_2 - m$ and the sales $\tilde{a}_1$. It should be noted, however, that λ represents a stochastic proportion between the measures $\tilde{a}_1$ and $\tilde{a}_2$, and thus deviations from the relationship $\tilde{a}_2 = m + \lambda\tilde{a}_1$ could occur due to the underlying economic noise $\tilde{\varepsilon}$. The variance of the noise term $\tilde{\varepsilon}$, denoted s^2, represents the extent to which the relationship $\tilde{a}_2 = m + \lambda\tilde{a}_1$ between the two accounting components $\tilde{a}_1$ and $\tilde{a}_2$ is noisy.

The absence of any correlation between the noise term $\tilde{\varepsilon}$ and the aggregate accounting measure $\tilde{a}$ serves to control for size, implying that deviations from the linear relation $\tilde{a}_2 = m + \lambda\tilde{a}_1$ between the two accounting components $\tilde{a}_1$ and $\tilde{a}_2$ are described independently of their total sum $\tilde{a}$.[4] It also implies that the stochastic relation $\tilde{a}_2 = m + \lambda\tilde{a}_1 + \tilde{\varepsilon}$ is symmetrical, as long as $\lambda \neq 0$, in the sense that it can be represented as $\tilde{a}_1 = -\lambda^{-1}m + \lambda^{-1}\tilde{a}_2 - \lambda^{-1}\tilde{\varepsilon}$ using the scalars $-\lambda^{-1}m$ and λ^{-1} with the normally distributed random variable $-\lambda^{-1}\tilde{\varepsilon}$, which is independent of $\tilde{a}$, and its mean equals zero. The independent variables $\tilde{a} = \tilde{a}_1 + \tilde{a}_2, \tilde{\varepsilon} = \tilde{a}_2 - m - \lambda\tilde{a}_1 \sim (\mu, 0, \sigma^2, s^2, 0)$ can substitute for the variables $\tilde{a}_1$ and $\tilde{a}_2$ in constituting the primitives of the model. We accordingly refer to the parameters μ, σ^2, m, λ, and s^2 throughout the analysis as primitive parameters (instead of using the parameters underlying the joint distribution of $\tilde{a}_1$ and $\tilde{a}_2$). The parameters m, λ, and s^2 are of special interest in our analysis because they determine the stochastic relationship $\tilde{a}_2 = m + \lambda\tilde{a}_1 + \tilde{\varepsilon}$ between the accounting components $\tilde{a}_1$ and $\tilde{a}_2$.

The manager privately observes the realization a of the aggregate accounting measure $\tilde{a}$, as well as the realizations a_1 and a_2 of its two components $\tilde{a}_1$ and $\tilde{a}_2$, respectively. Based on her private information, she issues an accounting report $r = r_1 + r_2$ of the aggregate measure a, which is decomposed into two reports r_1 and r_2 of the accounting components a_1 and a_2, respectively.[5] Our simplifying

[3] That is, $\text{cov}(\tilde{a}, \tilde{\varepsilon}) = \text{cov}(\tilde{a}, \tilde{a}_2) - \lambda\text{cov}(\tilde{a}, \tilde{a}_1) = \text{cov}(\tilde{a}, \tilde{a}_2) - \frac{\text{cov}(\tilde{a}, \tilde{a}_2)}{\text{cov}(\tilde{a}, \tilde{a}_1)}\text{cov}(\tilde{a}, \tilde{a}_1) = 0$ and $E(\tilde{\varepsilon}) = E(\tilde{a}_2) - E(\tilde{a}_2) - \lambda(E(\tilde{a}_1) - E(\tilde{a}_1)) = 0$.

[4] While it follows from $\text{cov}(\tilde{a}, \tilde{\varepsilon}) = 0$ that deviations from the relation $\tilde{a}_2 = m + \lambda\tilde{a}_1$ are independent of the aggregate accounting measure $\tilde{a}$, they might be correlated with each of its two components $\tilde{a}_1$ and $\tilde{a}_2$. Hence, $\text{cov}(\tilde{a}_1, \tilde{\varepsilon})$ and $\text{cov}(\tilde{a}_2, \tilde{\varepsilon})$ are not necessarily zero. The absence of any correlation between $\tilde{\varepsilon}$ and $\tilde{a}$ only implies that $\text{cov}(\tilde{a}_1, \tilde{\varepsilon}) = -\text{cov}(\tilde{a}_2, \tilde{\varepsilon})$, so that the implication of any deviation from the relation $\tilde{a}_2 = m + \lambda\tilde{a}_1$ for one of the two accounting components is expected to offset its countervailing implication for the other component.

[5] Since the accounting report does not usually entirely detail all its components, each of the two accounting components a_1 and a_2 can be viewed as an aggregate measure of more elementary items, which are observable only to the manager. Such decomposition of the accounting components a_1 and a_2 into more elementary items does not have any influence on our analysis.

assumption that the accounting variable $\tilde{a}$ equals the economic equity value of the firm implies that disaggregation of the accounting report into its two components is irrelevant to investors for valuation purposes when the realization of the aggregate accounting measure $\tilde{a}$ is truthfully reported to them. Hence any valuation implications of accounting disaggregation in our model must be due to reporting manipulations. We indeed assume that the manager exercises discretion in reporting, which allows her to bias both components of the accounting report. The difference $r_1 - a_1$ is the reporting bias embedded in the first component of the report. The difference $r_2 - a_2$ is the reporting bias in the second component. The overall reporting bias is $r - a$.

We further assume that biasing the accounting report is costly for the manager.[6] In common with the literature (e.g., Fischer and Verrecchia 2000; Dye and Sridhar 2004; Guttman et al. 2006; Einhorn and Ziv 2012), we assume a quadratic biasing cost function. Unlike prior models, however, which assume a biasing cost function that is quadratic in the overall reporting bias, our biasing cost is quadratic in the reporting bias in each of the two components of the report. That is, when the manager observes the accounting measures (a_1, a_2) and reports (r_1, r_2), she bears a biasing cost of $c_1(r_1 - a_1)^2 + c_2(r_2 - a_2)^2$, where $c_1, c_2 > 0$.[7] Hence we allow the manager to bias both accounting components but at potentially different costs. This structure of the biasing cost function captures the differences in the reporting discretion that accounting conventions accord to managers with respect to various components of the financial statements. By their nature, some accounting items can be more easily managed than others. It is generally accepted, for example, that accrual-based earnings items (such as bad debts and loss reserves, depreciations and amortizations, asset impairments) are more easily managed than cash-based earnings items. Segment reporting, where the consolidated accounting report of a firm is decomposed into reports of its operations in various business or geographical segments, also typically exhibits a considerable variety of managerial discretion in reporting different accounting items.

The cost ratio c_1/c_2 measures the extent to which the second accounting component is manageable relative to the first accounting component. The higher the ratio c_1/c_2, the more manageable is the second component relative to the first.[8] This interpretation of the ratio c_1/c_2 becomes apparent when the cost function

[6] The assumption of a costly reporting bias is typical of earnings management models (e.g., Stein 1989; Fischer and Verrecchia 2000), distinguishing them from cheap talk models, where misreporting is costless (e.g., Crawford and Sobel 1982) and precluding "babbling" equilibria in which no information is conveyed. Biases in reporting can be associated with a variety of costs, such as litigation costs, reputation erosion costs, costs that emerge from conflicts with auditors and audit committees, and the costs of reducing the reporting flexibility in future reports.

[7] The biasing cost function can be generalized to be $c_1(r_1 - a_1)^2 + c_2(r_2 - a_2)^2 + c_3(r - a)^2$, where $c_1, c_2 > 0$ and $c_3 \geq 0$, without qualitatively changing the results. The additive structure of the cost function is assumed to enable a tractable analysis.

[8] The extreme case where c_1 converges to infinity (as does the ratio c_1/c_2) describes situations where only the second component of the accounting report is manageable. The opposite extreme case where c_2 converges to infinity (and the ratio c_1/c_2 thus converges to zero) describes situations where only the first component of the accounting report is manageable. In both these extreme cases, the reporting of one of the accounting components must be truthful, and this component can thus be viewed as an exogenous public signal.

$c_1(r_1 - a_1)^2 + c_2(r_2 - a_2)^2$ is equivalently described (after algebraic rearrangements) as $\frac{c_1 c_2}{c_1+c_2}(r-a)^2 + \frac{1}{c_1+c_2}(c_2(r_2-a_2) - c_1(r_1-a_1))^2$. Here, $\frac{c_1 c_2}{c_1+c_2}(r-a)^2$ can be viewed as the cost associated with the overall reporting bias $r - a$, whereas $\frac{1}{c_1+c_2}(c_2(r_2-a_2) - c_1(r_1-a_1))^2$ is the cost associated with the inner allocation of the bias between the two reported components. Keeping the overall reporting bias $r - a$ constant, the cost associated with the inner allocation of the reporting bias decreases as the ratio $(r_2 - a_2)/(r_1 - a_1)$ between the biases in the two components of the accounting report approaches the cost ratio c_1/c_2, and it equals zero when the two ratios coincide and $(r_2 - a_2)/(r_1 - a_1) = c_1/c_2$.

The manager's compensation is linked to the firm's stock price, and thus the manager makes her reporting decision in conjunction with her expectations about the impact of the accounting report on the market price of the firm. For any given disaggregated accounting report (r_1, r_2), we define $P(r_1, r_2)$ as the market price of the firm. We denote by x the benefit to the manager from shifting the market price of the firm upward by one additional unit. Accordingly, given that the manager reports (r_1, r_2) after observing (a_1, a_2), her utility is $xP(r_1, r_2) - c_1(r_1 - a_1)^2 - c_2(r_2 - a_2)^2$. Similar to Fischer and Verrecchia (2000) and subsequent papers, it is assumed that x is privately known only to the manager.[9] The investors do not observe x, and they consider it as the realization of some independent normally distributed random event $\tilde{x}$ with mean μ_x and variance σ_x^2.[10] The uncertainty of the investors about the manager's reporting objective prevents them from perfectly backing out the reporting bias, precluding fully revealing equilibria.

Investors are assumed to be risk neutral. Accordingly, they set the firm's market price equal to the firm's expected value, conditional on all the available information. In particular, they use their expectations about the manager's reporting strategy in an effort to detect opportunistic biases in reporting and thereby most effectively use the information conveyed in the accounting report in pricing the firm. Given that a_1 and a_2 are the realizations of the accounting measures $\tilde{a}_1$ and $\tilde{a}_2$, respectively, and x is the realization of $\tilde{x}$, we define $B_i(a_1, a_2, x)$ as the reporting bias in the i'th $(i = 1, 2)$ component of the accounting report. Although the investors do not observe a_1, a_2, and x, they rationally anticipate the manager's reporting strategy and thus elicit from the manager's report (r_1, r_2) the information $\tilde{a}_1 + B_1(\tilde{a}_1, \tilde{a}_2, \tilde{x}) = r_1$ and $\tilde{a}_2 + B_2(\tilde{a}_1, \tilde{a}_2, \tilde{x}) = r_2$.

The model is, therefore, a single-period game with two interrelated decisions made by two players—the reporting decision of the firm's manager and the pricing decision

[9] Following Fischer and Verrecchia (2000), uncertainty on the part of investors about the reporting objective of managers is widely assumed in the disclosure literature (e.g., Fischer and Stocken 2004; Dye and Sridhar 2004; Ewert and Wagenhofer 2005; Einhorn 2007).

[10] Since $\tilde{x}$ is normally distributed, it might be either positive or negative, implying that the manager might have the incentive to either inflate or deflate the market price of the firm. While managerial incentives to inflate the stock price are prevalent, there are points in time at which managers have the incentive to drive the stock price of their firm downward (Einhorn 2007). We can, however, approximately preclude such scenarios from the model by assuming that the mean μ_x of $\tilde{x}$ is a positive number that exceeds its variance σ_x^2 by a large amount, so that the probability that the manager wishes to deflate the price is very close to zero.

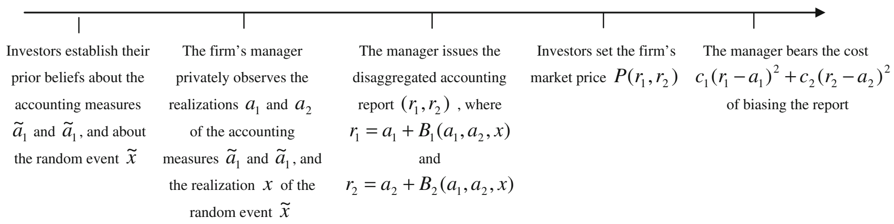

Fig. 1 A time line depicting the sequence of events in the model

of the investors. Figure 1 provides a timeline depicting the sequence of events in the model. Equilibrium in the model consists of three functions: $B_1,\ B_2 : \Re^3 \rightarrow \Re$, representing the manager's strategy in biasing the two components of the accounting report, and $P : \Re^2 \rightarrow \Re$, representing the pricing rule applied by the investors. In equilibrium, the manager chooses the reporting strategy (B_1, B_2) based on her rational expectations about the market pricing rule P, which, in turn, is based on the investors' rational expectations regarding the manager's reporting strategy $(B_1,\ B_2)$. We look for Bayesian equilibrium, formally defined as a vector of functions $(B_1,\ B_2 : \Re^3 \rightarrow \Re,\quad P : \Re^2 \rightarrow \Re)$, which satisfies two conditions. The first equilibrium condition pertains to the manager's reporting strategy (B_1, B_2), requiring for any realizations $a_1, a_2, x \in \Re$ of $\tilde{a}_1$, $\tilde{a}_2$ and $\tilde{x}$, respectively, that $(B_1(a_1, a_2, x), B_2(a_1, a_2, x)) \in \arg\max_{(b_1,b_2)\in\Re^2} xP(\,a_1 + b_1,\quad a_2 + b_2\,) - \ c_1 b_1^2 - c_2 b_2^2$. The second equilibrium condition describes the pricing rule P applied by the capital market investors, requiring for any accounting report $(r_1,\ r_2) \in \Re^2$ that $P(r_1, r_2) = E(\,\tilde{a}\,|\,\tilde{a}_1 + B_1(\tilde{a}_1, \tilde{a}_2, \tilde{x}) = r_1,\quad \tilde{a}_2 + B_2(\tilde{a}_1, \tilde{a}_2, \tilde{x}) = r_2\,)$.

We restrict the analysis to equilibria with a linear pricing rule, imposing $P(r_1, r_2)$ to be a linear function of the two components r_1 and r_2 of the accounting report. Linear equilibria are commonly assumed in the earnings management literature.[11] When combined with a quadratic biasing cost function and a normal distribution of the firm's value and accounting measures, a linear pricing rule enables a tractable analysis and yields equilibrium outcomes that can be analytically characterized and intuitively explained.[12] As linearity restrictions are commonly made in empirical research, the assumption of a linear pricing rule also allows us to link our analytical results to empirical findings and make predictions that map into linear empirical frameworks.

3 Equilibrium analysis

As a benchmark, we start the analysis by considering the case of an aggregated accounting regime, where the accounting report provides only the aggregated datum

[11] An exception is the analysis of Guttman et al. (2006), which explains kinks and discontinuities in the distribution of the reported earnings by focusing on nonlinear equilibria.

[12] Einhorn and Ziv (2012) show that restricting the pricing rule to be linear in a conventional earnings management setting is equivalent to restricting the out-of-equilibrium beliefs to be reasonable in the sense of the D1 criterion of Cho and Kreps (1987).

r without disclosing its decomposition into the components r_1 and r_2. This case provides a natural point of reference because it represents the traditional framework of earnings management. It thus yields equilibrium outcomes that coincide with those of Fischer and Verrecchia (2000) and do not depend at all upon the determinants m, λ, and s^2 of the stochastic relationship $\tilde{a}_2 = m + \lambda\tilde{a}_1 + \tilde{\varepsilon}$ between the accounting components $\tilde{a}_1$ and $\tilde{a}_2$. Observation 1 establishes the existence and uniqueness of equilibrium in the benchmark case, denoted by the superscript b, and characterizes its form.

Observation 1 *In the benchmark case of an aggregate reporting regime, where the accounting report includes only the aggregate datum r (while all other modeling assumptions are kept intact), there exists a unique linear equilibrium* $\left(B_1^b,\ B_2^b : \Re^3 \rightarrow \Re,\quad P^b : \Re^2 \rightarrow \Re\right)$. *The benchmark equilibrium is independent of* $m,\ \lambda,$ *and* s^2. *It takes the following form:* $P^b(r_1,\ r_2) = \alpha_0^b + \alpha_1^b(r_1 + r_2)$, $B_1^b(a_1,\ a_2,\ x) = \frac{\alpha_1^b}{2c_1}x$ *and* $B_2^b(a_1,\ a_2,\ x) = \frac{\alpha_1^b}{2c_2}x$ *for any* $a_1,\ a_2,\ x,\ r_1,\ r_2 \in \Re$, *where* $\alpha_0^b = \left(1 - \alpha_1^b\right)\mu - \alpha_1^b\left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)\mu_x$ *and* α_1^b *is the unique scalar that satisfies the equation* $\alpha_1^b = \frac{\sigma^2}{\sigma^2 + \sigma_x^2\left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)^2}$.

Observation 1 indicates that, in equilibrium, the benchmark overall reporting bias equals $B_1^b(a_1, a_2, x) + B_2^b(a_1, a_2, x) = \frac{1}{2}\left(\frac{c_1c_2}{c_1 + c_2}\right)^{-1}\alpha_1^b x$, whereas its allocation across the two components of the report satisfies $B_2^b(a_1, a_2, x)\big/B_1^b(a_1, a_2, x) = c_1/c_2$. Under an aggregate reporting regime, the inner allocation of the reporting bias is unobservable to the investors and thus does not affect the market price of the firm, so the optimal allocation of the reporting bias is such that $B_2^b(a_1, a_2, x)\big/B_1^b(a_1, a_2, x) = c_1/c_2$, nullifying the term $\frac{1}{c_1 + c_2}(c_2(r_2 - a_2) - c_1(r_1 - a_1))^2$ in the biasing cost function. In the benchmark case, therefore, the biasing cost function $\frac{c_1c_2}{c_1 + c_2}(r - a)^2 + \frac{1}{c_1 + c_2}(c_2(r_2 - a_2) - c_1(r_1 - a_1))^2$ is practically reduced to $\frac{c_1c_2}{c_1 + c_2}(r - a)^2$. The sign of the overall reporting bias $\frac{1}{2}\left(\frac{c_1c_2}{c_1 + c_2}\right)^{-1}\alpha_1^b x$ is the same as the sign of x. Its absolute value is increasing in the importance that the manager attaches to the market price of the firm, as captured by the absolute value of x, increasing in the weight α_1^b assigned by the pricing function $P^b(r_1,\ r_2) = \alpha_0^b + \alpha_1^b(r_1 + r_2)$ to the aggregate accounting report $r = r_1 + r_2$, but decreasing in the marginal biasing cost $\frac{c_1c_2}{c_1 + c_2}$ associated with the bias. Obviously, the benchmark equilibrium pricing rule $P^b(r_1, r_2) = \alpha_0^b + \alpha_1^b(r_1 + r_2)$ relies solely on the aggregate accounting report $r_1 + r_2$. Since investors do not observe the realization x of the random event $\tilde{x}$, they cannot precisely detect the reporting bias $\frac{1}{2}\left(\frac{c_1c_2}{c_1 + c_2}\right)^{-1}\alpha_1^b x$ and thus cannot decipher the manager's private information from her report. Being unable to unravel the reporting bias, the market uses the accounting report $r = r_1 + r_2$ only as a noisy signal of the underlying accounting measure $\tilde{a}$. Consequently, the weight α_1^b that the benchmark equilibrium pricing function $P^b(r_1,\ r_2) = \alpha_0^b + \alpha_1^b(r_1 + r_2)$ assigns to the accounting report $r = r_1 + r_2$ is lower than 1.

Having analyzed the benchmark of an aggregate reporting regime, we now turn to analyzing the case where reporting is disaggregated. By comparing the equilibrium under a disaggregated reporting regime against the benchmark equilibrium outcomes obtained under an aggregate reporting regime, we aspire to highlight the role that accounting disaggregation plays in mitigating the practice of earnings management. The manager's reporting decision in our model consists of two interrelated tiers: the choice of the aggregate reporting bias and choice of its allocation across the two components of the report. We first restrict our attention to the managerial choice of allocating the total bias across the two components of the report, keeping the level of the total bias fixed. For any given level of the aggregate reporting bias, the manager faces two countervailing forces in choosing its optimal allocation between the two components of the accounting report. On one hand, the manager aims at maximizing the market price of the firm and thus seeks to obscure any possible trace of evidence for the reporting bias by ensuring that the bias proportion $B_2(a_1, a_2, x)/B_1(a_1, a_2, x)$ remains close to the fundamental proportion λ. On the other hand, she wishes to minimize the term $\frac{1}{c_1+c_2}(c_2(r_2 - a_2) - c_1(r_1 - a_1))^2$ in the biasing cost function by keeping the bias proportion $B_2(a_1, a_2, x)/B_1(a_1, a_2, x)$ as close as possible to the cost ratio c_1/c_2. The only case where these two forces coincide is the case where λ exactly equals c_1/c_2. In the special case of $\lambda = c_1/c_2$, the manager optimally allocates the reporting bias between the components of the report such that $B_2(a_1, a_2, x)/B_1(a_1, a_2, x) = \lambda = c_1/c_2$ and thereby leaves no trace of evidence for the reporting bias in the disaggregated report and at the same time nullifies the term $\frac{1}{c_1+c_2}(c_2(r_2 - a_2) - c_1(r_1 - a_1))^2$ in the biasing cost function. In all other more prevalent cases where $\lambda \neq c_1/c_2$, the manager's optimal choice of the bias allocation is such that the bias proportion $B_2(a_1, a_2, x)/B_1(a_1, a_2, x)$ lies between the covariance ratio λ and the cost ratio c_1/c_2. This intuition lies at the base of Propositions 2 and 3, which establish the existence and uniqueness of a linear equilibrium under a disaggregated reporting regime, characterize its form, and compare it with the benchmark equilibrium.

Proposition 2 *The model yields a unique linear equilibrium* $(B_1,\ B_2 : \Re^3 \to \Re,\quad P : \Re^2 \to \Re)$. *The equilibrium is characterized by two scalars* α_1 *and* α_2, *such that* $P(r_1, r_2) = \alpha_0 + \alpha_1 r_1 + \alpha_2 r_2$, $B_1(a_1, a_2, x) = \frac{\alpha_1}{2c_1}x$ *and* $B_2(a_1, a_2, x) = \frac{\alpha_2}{2c_2}x$ *for any* $a_1,\ a_2,\ x,\ r_1,\ r_2 \in \Re$, *where* $\alpha_0 = \left(1 - \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}\right)\mu + \frac{\alpha_1 - \alpha_2}{1+\lambda}m - \left(\frac{\alpha_1^2}{2c_1} + \frac{\alpha_2^2}{2c_2}\right)\mu_x$.

Proposition 3 *If* $\lambda = c_1/c_2$, *then the equilibrium coincides with the benchmark equilibrium (i.e.,* $\alpha_0 = \alpha_0^b$, $\alpha_1 = \alpha_2 = \alpha_1^b$ *and* $B_2(a_1, a_2, x)/B_1(a_1, a_2, x) = \lambda = c_1/c_2$*). Otherwise,* $\alpha_1 \neq \alpha_2$ *and* $B_2(a_1, a_2, x)/B_1(a_1, a_2, x)$ *lies between* c_1/c_2 *and* λ *but differs from both of them.*

Proposition 2 suggests that the equilibrium overall reporting bias under a disaggregated reporting regime is $B_1(a_1, a_2, x) + B_2(a_1, a_2, x) = \left(\frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}\right)x$ and the equilibrium market pricing rule is $P(r_1, r_2) = \alpha_0 + \alpha_1 r_1 + \alpha_2 r_2$. It follows from Proposition 3 that these equilibrium outcomes coincide with those of the benchmark case only in the special case of $\lambda = c_1/c_2$. As λ captures the fundamental proportion between the two accounting components $\tilde{a}_1$ and $\tilde{a}_2$, $\lambda = c_1/c_2$ implies that the ability of the manager to manipulate the two components is proportional to their expected amounts, and in this sense the two components are proportionally exposed to managerial manipulations. In the case of $\lambda = c_1/c_2$, the equilibrium reporting bias preserves the fundamental proportion between the two accounting components, making the disaggregated data irrelevant to investors (who know that a deviation of the reported data r_1 and r_2 from the relation $r_2 = m + \lambda r_1$ is only due to economic noise and does not convey any clue about the reporting bias) and leading to an equilibrium price that depends solely upon the aggregate accounting report $r_1 + r_2$. However, due to the differences in the reporting discretion that accounting conventions accord to managers with respect to various components of the financial statements, the biasing cost ratio c_1/c_2 is not likely to exactly equal the fundamental proportion λ between the two accounting components $\tilde{a}_1$ and $\tilde{a}_2$.

In all the more prevalent cases where $\lambda \neq c_1/c_2$, the ability of the manager to manipulate the two accounting components $\tilde{a}_1$ and $\tilde{a}_2$ is not proportional to their expected amounts, implying that the two components vary in their relative exposure to managerial manipulations. The magnitude of the gap in their exposure to reporting manipulations is depicted by the distance between λ and c_1/c_2. When $\lambda \neq c_1/c_2$, misreporting works to distort the natural relationship between the two components of the report, leaving in the disaggregated accounting report a trace of evidence for the reporting bias that is captured by the estimator $r_2 - m - \lambda r_1$. The observable estimator $r_2 - m - \lambda r_1$ measures the deviation of the reported components r_1 and r_2 from the relationship $r_2 = m + \lambda r_1$, and it is the sum of two unobservable values: $a_2 - m - \lambda a_1$ and $r_2 - a_2 - \lambda(r_1 - a_1)$. The former is the realization of the value-independent variable $\tilde{\varepsilon}$. The latter is the realization of $\left(\frac{\alpha_2}{2c_2} - \lambda\frac{\alpha_1}{2c_1}\right)\tilde{x}$ and is thus a linear function of the realization x of the independent random variable $\tilde{x}$. Knowing this, investors deduce that any departure of $r_2 - m - \lambda r_1$ from zero could be due to either the economic noise $\tilde{\varepsilon}$ or the noise $\tilde{x}$ embedded in the reporting bias. By allowing them to diminish some of the noise $\tilde{x}$ that underlies the bias, the observable deviation measure $r_2 - m - \lambda r_1$ serves investors as an imperfect indicator of the reporting bias. Therefore, as long as $\lambda \neq c_1/c_2$, the firm's market price comprises the deviation measure $r_2 - m - \lambda r_1$ in addition to the aggregated accounting report $r = r_1 + r_2$ (which is the one and only relevant signal in the benchmark case), even though $r_2 - m - \lambda r_1$ is the realization of a random variable that is independent of the firm's value $\tilde{a}$.[13] This results in an equilibrium

[13] The decomposition of the aggregate accounting measure into two components allows us to employ only one accounting ratio (and thus only one deviation measure) in our analysis. The intuition can, however, be extended to ratio analysis that is based on a vast array of accounting ratios.

market price $P(r_1, r_2) = \alpha_0 + \alpha_1 r_1 + \alpha_2 r_2$ that places different weights, α_1 and α_2, on the two components, r_1 and r_2, of the accounting report.

To further understand the role that the estimator $r_2 - m - \lambda r_1$ plays in the pricing rule after controlling for the aggregate accounting report $r = r_1 + r_2$, it is convenient to distinguish between negative and positive values of the proportion λ. A negative λ can be interpreted as a proportion between two accounting components that are likely to have opposite signs, such as a proportion between an income item and an expense item or a proportion between an asset item and a liability item. A positive λ, on the other hand, represents a proportion between two accounting components that are likely to share the same sign, such as a proportion between two income items or between two assets. Reporting manipulations are likely to distort both types of proportions but for different reasons. The distortion in negative proportions is due to the managerial wish to bias the absolute value of the two accounting components in opposite directions. Suppose, for example, that the manager seeks to inflate earnings and therefore tends to bias incomes upward and expenses downward in absolute values. Since the two earnings components are shifted in opposite directions, their original proportion must be distorted. When considering positive values of λ, we get the same result but for a different reason. In this case, the manager wishes to bias the two accounting components in the same direction, but she might have different degrees of leeway in manipulating the two components. As a result, the bias in the more manageable component is likely to be larger in magnitude and not proportional to the bias in the less manageable component, and this works to distort the proportion between the two components.

Since the costs ratio c_1/c_2 is positive by definition, any negative λ must satisfy $\lambda \neq c_1/c_2$. It thus follows that reporting manipulations always leave a trace of evidence in a disaggregated accounting report that is decomposed into components with opposite signs, regardless of the exact degree of leeway in managing the different accounting components. When λ is negative, even if the manager exercises the same degree of discretion in managing the accounting components, her wish to bias them in opposite directions generates a reporting bias that distorts the fundamental proportion between the two components.[14] Under such circumstances, the observable deviation measure $r_2 - m - \lambda r_1$ is always indicative of the reporting bias, regardless of the biasing costs c_1 and c_2. This sheds light on the merits of disaggregating accounting measures into components with opposite signs (for example, decomposing earnings into income and expense components). However, the dependence of the equilibrium outcomes on λ takes a rather complicated shape when λ is negative, which does not enable us to provide clear-cut comparative statics results for negative values of λ. This is due to the existence of two forces that are at work when λ is negative. The first is the difference in the manager's

[14] To demonstrate this argument, it is useful to consider the case $\lambda = -c_1/c_2$. It follows from $|\lambda| = c_1/c_2$ that the manager has the ability to bias two accounting components in a proportional manner, whereas $\lambda < 0$ implies that the two components are likely to have opposite signs. Here, even though the reporting bias is allocated proportionally across the two accounting components, it shifts the absolute value of the two components in opposite directions. Explicitly, the reporting bias shifts the positive component upward, while shifting the absolute value of the negative component downward, resulting in a distortion of their original proportion.

incentives with regard to the direction of the bias in the two accounting components and the second is the difference in the manager's ability to bias the two components. This is unlike the case of a positive λ, where the manager wishes to bias the two accounting components in the same direction, so that the only force at work is the difference in her ability to bias them. We therefore can provide conclusive comparative statics results only when λ is positive.

Holding the reporting bias constant, accounting disaggregation improves the information environment, as it enables the investors to better adjust for the reporting bias when pricing the firm relative to the benchmark case of an aggregate reporting regime. Proposition 4 indicates that this conclusion continues to hold even when the reporting bias is endogenously derived. It further demonstrates how the power of accounting disaggregation in improving the public information environment depends on the parameters m, λ, and s^2, which determine the stochastic relationship $\tilde{a}_2 = m + \lambda\tilde{a}_1 + \tilde{\varepsilon}$ between the accounting components $\tilde{a}_1$ and $\tilde{a}_2$.

Proposition 4 *The inequality* $\mathrm{var}\left(\tilde{a}\,\middle|\,\tilde{a}_1 + \frac{\alpha_1}{2c_1}\tilde{x},\quad \tilde{a}_2 + \frac{\alpha_2}{2c_2}\tilde{x}\right) \leq \mathrm{var}\left(\tilde{a}\,\middle|\,\tilde{a} + \left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)\tilde{x}\right)$ *always holds, where equality exists if and only if* $\lambda = c_1/c_2$. *For positive values of* λ, $\mathrm{var}\left(\tilde{a}\,\middle|\,\tilde{a}_1 + \frac{\alpha_1}{2c_1}\tilde{x},\quad \tilde{a}_2 + \frac{\alpha_2}{2c_2}\tilde{x}\right)$ *is initially increasing in* λ, *reaching a maximum at* $\lambda = c_1/c_2$, *and then decreasing in* λ. *It is also increasing in* s^2 *and independent of* m.

Proposition 4 shows that the variance of $\tilde{a}$ conditional on a disaggregated accounting report is lower than the variance of $\tilde{a}$ conditional on an aggregate report, though it is still greater than zero (which is the variance of $\tilde{a}$ conditional on a truthful reporting). It also suggests that this role of accounting disaggregation in improving the public information environment is more effective when the distance between λ and c_1/c_2 increases and when s^2 decreases. The more λ departs from c_1/c_2, the greater the gap in the managerial ability to manage the two accounting components, and the larger the disproportion in their reporting bias. As a result, the deviation $r_2 - m - \lambda r_1$ of the reported accounting components from their fundamental relationship becomes more indicative of the reporting bias, so the investors can better detect the bias and adjust for it, and they thus find the accounting report to be more informative. This highlights the advantages of decomposing accounting measures into components that may be equally signed but are very different in the extent to which their reporting can be manipulated, such as cash-based and accrual-based earnings items. When s^2 decreases and the relationship between $\tilde{a}_1$ and $\tilde{a}_2$ becomes less noisy, the deviation measure $r_2 - m - \lambda r_1$ becomes more informative about the bias because it is less influenced by economic noises. Consequently, the accounting report becomes more informative to the market in evaluating the firm as s^2 decreases. This result points to the merits of breaking accounting measures into components that are tightly proportional to each other due to their fundamental economic nature, such as the decomposition of the gross profit into sales and cost of sales.

Propositions 2, 3, and 4 may provide guidance to empirical studies in evaluating the currently used indicators of earnings management and in designing new improved indicators. They point to deviations of accounting measures from their expected fundamental ratios as indicators of biases in reporting, highlighting especially three key properties of such indicators that enhance their efficacy in identifying reporting biases: (i) a tight fundamental economic proportion between the accounting measures involved (captured in our model by a low s^2), (ii) dissimilarity in the exposure of the accounting measures involved to biases in reporting (captured in our model by a large distance between λ and c_1/c_2), and (iii) disparity in the signs of the accounting measures involved (captured in our model by a negative λ).[15] Many empirical studies in the accounting literature focus on discretionary accruals as a conventional indicator of earnings management. Discretionary accruals, as commonly computed in the literature based on the modified Jones model (Jones 1991; Dechow et al. 1995), can be roughly viewed as a special case of the indicators suggested by our analysis, because they mostly represent deviations from the expected ratio between accruals and cash revenues, two earnings components that are very different in the extent to which they can be biased. Nevertheless, these two earnings components do not seem to be fundamentally very tightly proportional to each other. This might be the reason behind the relatively modest incremental power of discretionary accruals in explaining stock prices (Guay et al. 1996). Our analysis draws attention to a variety of potential complementary indicators of earnings management, which are based on deviations from ratios that involve accounting items with opposite signs that are fundamentally strongly proportional to each other, such as deviations from various expected profit margins (net profit margin, operating profit margin, gross profit margin).[16]

The role of disaggregated accounting data in detecting reporting manipulations and adjusting for them, as described in Proposition 4, has two countervailing effects on managerial misreporting incentives. On one hand, the manager, recognizing the ability of the market to imperfectly identify and back out reporting manipulations, is more reluctant to manipulate the accounting report in the first place. On the other hand, this albeit limited ability of the market results in a pricing rule that places more weight on the accounting report, increasing the incentives of the manager to bias the report. Proposition 5 indicates that the former force dominates, and thus accounting disaggregation works to decrease the manager's optimal choice of the total reporting bias. It further demonstrates how the power of accounting disaggregation in reducing the magnitude of the total reporting bias $\left(\frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}\right)x$

[15] Empirical examination of how effective irregularities in the accounting report are in indicating earnings management requires the careful design of proxies for the unobservable economic proportion between different accounting items (as captured by the parameter λ in our model). Financial ratios seem natural candidates for this role. Particular attention should be given, however, to the fact that financial ratios are measured using reported (potentially biased) accounting data rather than the underlying unobservable true data. This problem could be mitigated by averaging the firm-specific financial ratios over several periods (provided that biases in reporting mean-revert over time) or alternatively by averaging the data of a certain period across similar firms that operate in the same industry.

[16] Empirical findings in this direction are documented by Amir et al. (2012).

depends on the parameters m, λ, and s^2 underlying the stochastic relationship between the accounting components $\tilde{a}_1$ and $\tilde{a}_2$.[17]

Proposition 5 *The inequality* $\frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2} \leq \frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}$ *always holds, where equality exists if and only if* $\lambda = c_1/c_2$. *For positive values of* λ, $\frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ *is initially increasing in* λ, *reaching a maximum at* $\lambda = c_1/c_2$, *and then decreasing in* λ. *It is also increasing in* s^2*and independent of* m.

Proposition 5 indicates the magnitude of the total reporting bias $\left(\frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}\right)x$ under a disaggregated accounting report is lower than that of the total reporting bias $\left(\frac{\alpha^b}{2c_1} + \frac{\alpha^b}{2c_2}\right)x$ obtained in the benchmark case of an aggregate report, unless $\lambda = c_1/c_2$. This implies that ratio analysis of disaggregated accounting information, via its role in imperfectly detecting the reporting bias, also works to mitigate managerial misreporting propensity. Also, just as accounting disaggregation becomes more effective in detecting earnings management when the distance between λ and c_1/c_2 increases and when s^2 decreases, so does it become more effective in suppressing earnings management. This result again highlights the merits of decomposing accounting measures into components that are very different in the extent to which their reporting can be manipulated or strongly proportional to each other. Proposition 5 yields some interesting empirical predictions regarding the managerial tendency to engage in earnings management. For instance, a lower level of earnings management is predicted for firms that voluntarily provide more detailed disclosures on the components of the financial statements. In a similar vein, a reduction in earnings management is to be expected following the issuance of an accounting standard that mandates additional disaggregated disclosures (e.g., mandating segment disclosures). It also follows from Proposition 5 that opportunistic misreporting practices are expected to be more severe in industries operating in a risky and volatile business environment (where the proportions between accounting measures are likely to be more noisy) than in industries that operate in low risk and stable business environments.

We continue the analysis by discussing welfare considerations. In the context of our model, welfare considerations are interesting only when they pertain to the manager. The only other parties in our model are the risk-neutral investors, who do not lose or benefit ex-ante from the reporting discretion. This does not necessarily imply that the manager is indifferent ex-ante to the option to bias her report, because the model is not a zero-sum game. It has been well established in the literature that managers can be worse off with the option of biasing their accounting reports (Stein 1989). Managers might end up taking costly actions to bias their reporting even

[17] While the bottom line of the accounting report, and also the total bias embedded in it, are not necessarily of general interest when the report is disaggregated, they could be important to certain parties such as regulators, lawyers, or empiricists. In our setting, disaggregation is irrelevant to investors for valuation purposes when the aggregate accounting measure is truthfully reported to them, so its only role is in assisting investors to detect the total bias embedded in the bottom line of the report. In this sense, the total bias can serve in our setting as a measure of the extent to which the accounting reporting is manipulated.

when they know that they cannot fool the market. They are trapped into such inefficient behavior because they take the market's conjectures as fixed, knowing that investors will suspect their report in any case. A similar problem still exists in the presence of exogenous noise that does not allow the investors to perfectly back out the reporting bias. In such circumstances, some types of managers actually benefit from the bias option but at the expense of other types of managers (Fischer and Verrecchia 2000). Proposition 6 demonstrates that, as long as $\lambda \neq c_1/c_2$, accounting disaggregation works to mitigate the problem by reducing the sensitivity of all types of managers to the bias option. It further demonstrates how this role of accounting disaggregation depends on the parameters m, λ, and s^2 underlying the stochastic relationship between the accounting components $\tilde{a}_1$ and $\tilde{a}_2$.

Proposition 6. *For any $x \in \Re$, the ex-ante (before the realization of $\tilde{a}_1$ and $\tilde{a}_2$) benefit or loss to a manager of type x from the option to bias the report is $\left(\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}\right)\left((x-\mu_x)^2 - \mu_x^2\right)$ when the report is disaggregated, and $\left(\frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}\right)\left((x-\mu_x)^2 - \mu_x^2\right)$ in the benchmark case of an aggregate report. The inequality $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2} \leq \frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}$ always holds, where equality exists if and only if $\lambda = c_1/c_2$. For positive values of λ, $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}$ is initially increasing in λ, reaching a maximum at $\lambda = c_1/c_2$, and then decreasing in λ. It is also increasing in $s^2$and independent of m.*

Similar to Fischer and Verrecchia (2000), the option to bias the accounting report could either decrease or increase the manager's welfare. Regardless of whether the report is aggregate or disaggregated, managers with a conventional reporting objective, whose type x is sufficiently close to the mean type μ_x (i.e., $(x-\mu_x)^2 < \mu_x^2$), are worse off with the biasing option. On the other hand, managers with a less usual objective, whose type x is sufficiently far from the mean type μ_x (i.e., $(x-\mu_x)^2 > \mu_x^2$), are better off with the biasing option. In both cases, unless $\lambda = c_1/c_2$, Proposition 6 implies that the absolute value of the manager's ex-ante (before the realization of the accounting measures) benefit or loss from the biasing option is lower under disaggregated reporting as compared with the benchmark case of aggregate reporting. That is, accounting disaggregation reduces the benefit of managers who are better off with the option to bias their reporting but also reduces the loss of managers who are worse off with the biasing option. This implies that disaggregated accounting data work to decrease the sensitivity of all managerial types to the degree of discretion in reporting. Via its power to imperfectly reveal the managerial type, accounting disaggregation impedes misreporting incentives of managers who are otherwise capable of fooling the market, while relaxing the pressure on all other managers to engage in inefficient reporting manipulations.[18] Just as the role of accounting disaggregation in diminishing the market uncertainty

[18] Averaging across all managerial types and using the fact that $E[(x-\mu_x)^2] = \sigma_x^2$, it follows from Proposition 6 that managers benefit on average from the biasing option only when the uncertainty about their type is sufficiently severe (i.e., $\sigma_x^2 > \mu_x^2$) and lose on average otherwise. In both cases, unless $\lambda = c_1/c_2$, accounting disaggregation works to decrease their average benefit or loss.

becomes more effective when the distance between λ and c_1/c_2 increases and when s^2 decreases, so does its role in reducing the managerial sensitivity to the degree of reporting discretion.

4 Concluding remarks

We demonstrate how accounting disaggregation serves users of the financial statements in detecting biases in reporting and thereby also serves as an effective tool for mitigating managers' misreporting incentives. Our analysis also offers guidance to accounting policymakers in selecting among alternative accounting disaggregation procedures. It points to the merits of disaggregating accounting measures into components with opposite signs, such as income and expense components of earnings. It illuminates the advantages of decomposing accounting measures into components that may be equally signed but are very different in the extent to which their reporting can be manipulated, such as pure cash-based earnings items and pure accrual-based earnings items. The analysis further indicates the usefulness of breaking accounting measures into components that are tightly proportional to each other due to their fundamental economic nature, such as sales and cost of sales. Accounting disaggregation procedures that follow these guidelines are expected to provide fertile ground for implementing a process of financial statement analysis that is greatly effective in detecting and mitigating reporting manipulations.

While our study highlights considerations that could be important to accounting standards setters in selecting the proper accounting disaggregation procedure, these considerations obviously should be regarded in the context of other factors that are not analyzed here. They should primarily be viewed in light of the direct informational value associated with alternative accounting disaggregation procedures. They should also be considered in light of the proprietary costs implied by different accounting disaggregation procedures and in light of the likelihood of different kinds of disaggregated data to be voluntarily disclosed by managers even in the absence of mandatory requirements (e.g., Einhorn 2005). It should further be noted that the efficacy of accounting disaggregation in mitigating misreporting is not solely determined by the disaggregation procedures applied. It also depends on the particular business environment in which the accounting disaggregation procedures are carried out. This may imply, for instance, that firms that operate in several business segments might distort the optimal allocation of resources across the various segments to create a more convenient reporting environment (e.g., Einhorn and Ziv 2007). Such potential real effects are not considered in our study and should also be taken into account by accounting standards setters when designing the desirable accounting disaggregation procedures.

We also emphasize that our conclusions only pertain to disaggregated information that details additive line items of an aggregate accounting measure, such as incomes and expenses that are accumulated into the net earnings measure. These conclusions do not necessarily hold with respect to other types of detailed accounting data, such as the historical cost and the fair value measures that underlie the familiar lower of cost or market accounting measurement. In examining such substitutable (non-additive) objective and subjective measures on which a single summary accounting datum is based, Dye and Sridhar (2004) demonstrate circumstances where this kind of detailed accounting data could increase managerial misreporting incentives, rather than suppress them, because more weight is attached to subjective measures.

5 Appendix: Proofs

Using symmetry considerations, we assume throughout the appendix that $-1 < \lambda \leq c_1/c_2$. This assumption does not detract from the generality of the proofs, as $\lambda = \mathrm{cov}(\tilde{a}, \tilde{a}_2)/\mathrm{cov}(\tilde{a}, \tilde{a}_1) > c_1/c_2$ implies $0 < 1/\lambda = \mathrm{cov}(\tilde{a}, \tilde{a}_1)/\mathrm{cov}(\tilde{a}, \tilde{a}_2) < c_2/c_1$, whereas $\lambda < -1$ implies $-1 < 1/\lambda < 0$. So the proofs apply to the case of $\lambda > c_1/c_2$ or $\lambda < -1$ after exchanging the indexes of $\tilde{a}_1$ and $\tilde{a}_2$.

Proof of Observation 1 In the benchmark case, the firm price is a function of the aggregate accounting report $r_1 + r_2$. We look for equilibrium with a linear pricing function and thus assume there exist scalars α_0^b and α_1^b, such that $P^b(r_1, r_2) = \alpha_0^b + \alpha_1^b(r_1 + r_2)$. Accordingly, $\left(B_1^b(a_1, a_2, x), \quad B_2^b(a_1, a_2, x)\right) = \arg\max_{(b_1, \quad b_2)} U^b(b_1, b_2) = x\left(\alpha_0^b + \alpha_1^b(a_1 + b_1 + a_2 + b_2)\right) - c_1 b_1^2 - c_2 b_2^2$. The first-order conditions are $\partial U^b/\partial b_1 = x\alpha_1^b - 2c_1 b_1 = 0$ and $\partial U^b/\partial b_2 = x\alpha_1^b - 2c_2 b_2 = 0$. The function $U^b(b_1, b_2)$ gets its maximal value at $b_1 = B_1^b(a_1, a_2, x) = \frac{\alpha_1^b}{2c_1}x$, $b_2 = B_2^b(a_1, a_2, x) = \frac{\alpha_1^b}{2c_2}x$, where the first-order conditions hold, as do the second-order conditions: $\partial U^b/\partial b_1 \partial b_1 = -2c_1 < 0$, $\partial U^b/\partial b_2 \partial b_2 = -2c_2 < 0$ and $\left(\partial U^b/\partial b_1 \partial b_1\right) \cdot \left(\partial U^b/\partial b_2 \partial b_2\right) - \left(\partial U^b/\partial b_1 \partial b_2\right)^2 = (-2c_1)(-2c_2) - 0 > 0$. The firm price is the expected value of $\tilde{a}$ conditional on the aggregate report $r_1 + r_2$, so $P^b(r_1, r_2) = E\left(\tilde{a} \,\middle|\, \tilde{a} + \tilde{x}\left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right) = r_1 + r_2\right) = \mu + \frac{\sigma^2}{\sigma^2 + \sigma_x^2\left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)^2}(r_1 + r_2 - \mu - \left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)\mu_x)$, implying $\alpha_0^b = (1 - \alpha_1^b)\mu - \alpha_1^b\left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)\mu_x$ and α_1^b is the solution to the equation

$$\alpha_1^b = \frac{\sigma^2}{\sigma^2 + \sigma_x^2\left(\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}\right)^2} \tag{1}$$

As the left side of Eq. (1) is increasing in α_1^b from $-\infty$ to $+\infty$, whereas its right side is decreasing in α_1^b, there exists a unique solution α_1^b to the equation. □

Proof of Propositions 2 and 3 We look for equilibrium with a linear pricing function and thus assume there exist scalars α_0, α_1, and α_2, such that $P(r_1, r_2) = \alpha_0 + \alpha_1 r_1 + \alpha_2 r_2$. Accordingly, $(B_1(a_1, a_2, x), \quad B_2(a_1, a_2, x)) = \arg\max_{(b_1, b_2)} U(b_1, b_2) = x(\alpha_0 + \alpha_1(a_1 + b_1) + \alpha_2(a_2 + b_2)) - c_1 b_1^2 - c_2 b_2^2$. The first-order conditions are $\partial U/\partial b_1 = x\alpha_1 - 2c_1b_1 = 0$ and $\partial U/\partial b_2 = x\alpha_2 - 2c_2b_2 = 0$. The function $U(b_1, b_2)$ gets its maximal value at $b_1 = B_1(a_1, a_2, x) = \frac{\alpha_1}{2c_1}x$, $b_2 = B_2(a_1, a_2, x) = \frac{\alpha_2}{2c_2}x$, where the first-order conditions hold, as do the second-order conditions: $\partial U/\partial b_1\partial b_1 = -2c_1 < 0$, $\partial U/\partial b_2\partial b_2 = -2c_2 < 0$ and $(\partial U/\partial b_1\partial b_1)\cdot(\partial U/\partial b_2\partial b_2) - (\partial U/\partial b_1\partial b_2)^2 = (-2c_1)(-2c_2) - 0 > 0$. The firm price is the expected value of $\tilde{a}$ conditional on the reports r_1 and r_2, which can be replaced by $r_1 + r_2$ and $r_2 - \lambda r_1$ as $\lambda \neq -1$. Let $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ and $w = \frac{\alpha_2}{2c_2} - \lambda\frac{\alpha_1}{2c_1}$ and note that the firm price is $P(r_1, r_2) = E(\tilde{a} \,|\, \tilde{a} + \tilde{x}v = r_1 + r_2, m + \tilde{\varepsilon} + \tilde{x}w = r_2 - \lambda r_1)$. Since $\tilde{\varepsilon}$ and $\tilde{x}$ are independent of $\tilde{a}$, the price equals $\mu + \frac{\sigma^2}{\sigma^2 + v^2 VAR(\tilde{x}\,|\, m+\tilde{\varepsilon}+\tilde{x}w = r_2 - \lambda r_1)}(r_1 + r_2 - \mu - v\,E(\tilde{x}\,|\, m + \tilde{\varepsilon} + \tilde{x}w = r_2 - \lambda r_1))$ or $\mu + \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}\left(r_1 + r_2 - \mu - v\left(\mu_x + \frac{\sigma_x^2}{s^2/w^2+\sigma_x^2}((r_2 - \lambda r_1 - m)/w - \mu_x)\right)\right)$. This implies

$$\alpha_1 = \frac{\sigma^2\left(1 + \lambda\frac{\sigma_x^2 vw}{s^2+\sigma_x^2 w^2}\right)}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}} \tag{2}$$

$$\alpha_2 = \frac{\sigma^2\left(1 - \frac{\sigma_x^2 vw}{s^2+\sigma_x^2 w^2}\right)}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}} \tag{3}$$

The coefficient α_0 can be written as a function of the coefficients α_1 and α_2, since $\alpha_0 = \frac{\sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}\mu + \frac{\sigma^2\frac{\sigma_x^2 vw}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}m - \frac{\sigma^2\frac{s^2 v}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}\mu_x$, where $\frac{\sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}} = \left(1 - \frac{\alpha_1+\lambda\alpha_2}{1+\lambda}\right)$, $\frac{\sigma^2\frac{\sigma_x^2 vw}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}} = \frac{\alpha_1-\alpha_2}{1+\lambda}$ and $\frac{\sigma^2\frac{s^2 v}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}} = \frac{\alpha_1+\lambda\alpha_2}{1+\lambda}v - \frac{\alpha_1-\alpha_2}{1+\lambda}w = \frac{\alpha_1^2}{2c_1} + \frac{\alpha_2^2}{2c_2}$. So

$$\alpha_0 = \left(1 - \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}\right)\mu + \frac{\alpha_1 - \alpha_2}{1+\lambda}m - \left(\frac{\alpha_1^2}{2c_1} + \frac{\alpha_2^2}{2c_2}\right)\mu_x \tag{4}$$

Substituting Eqs. (2) and (3) in $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ and $w = \frac{\alpha_2}{2c_2} - \lambda\frac{\alpha_1}{2c_1}$, we get after rearrangement that v and w are the solution of the following two equations:

$$F(v,w) = v - \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}\left(c_1 + c_2 - (c_1 - \lambda c_2)\frac{\sigma_x^2 wv}{s^2+\sigma_x^2 w^2}\right)(2c_1c_2)^{-1} = 0 \tag{5}$$

$$G(v,w) = w - \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}\left(c_1 - \lambda c_2 - (c_1 + \lambda^2 c_2)\frac{\sigma_x^2 wv}{s^2+\sigma_x^2 w^2}\right)(2c_1c_2)^{-1} = 0 \tag{6}$$

Multiplying $F(v,w) = 0$ by $c_1 + \lambda^2 c_2$ and then subtracting $G(v,w) = 0$ multiplied by $c_1 - \lambda c_2$, we get

$$K(v,w) = (c_1 + \lambda^2 c_2)v - (c_1 - \lambda c_2)w - \frac{(1+\lambda)^2}{2} \cdot \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2 + \sigma_x^2 w^2}} = 0 \quad (7)$$

Multiplying $F(v,w) = 0$ by $c_1 - \lambda c_2$ and then subtracting $G(v,w) = 0$ multiplied by $c_1 + c_2$, we get

$$L(v,w) = (c_1 - \lambda c_2)v - (c_1 + c_2)w - \frac{(1+\lambda)^2}{2} \cdot \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2 + \sigma_x^2 w^2}} \cdot \frac{\sigma_x^2 wv}{s^2 + \sigma_x^2 w^2} = 0 \quad (8)$$

The equations $F(v,w) = 0$ and $G(v,w) = 0$ are thus equivalent to any pair of the following equations: $F(v,w) = 0$, $G(v,w) = 0$, $K(v,w) = 0$ and $L(v,w) = 0$. We proceed by showing that $F(v,w) = 0$ and $G(v,w) = 0$ imply $v > 0$ and $w \geq 0$, where $w = 0$ only for $\lambda = c_1/c_2$. Suppose by contradiction that $v \geq 0$ and $w < 0$ and note that $\lambda \leq c_1/c_2$ implies that $G(v,w) < 0$—a contradiction. Suppose now by contradiction that $v \leq 0$ and $w \geq 0$ and note that $\lambda \leq c_1/c_2$ implies that $F(v,w) < 0$—a contradiction. Lastly, suppose by contradiction that $v < 0$ and $w < 0$ and note that $L(v,w) = 0$ implies$(c_1 - \lambda c_2)v > (c_1 + c_2)w$, which is equivalent to $(c_1 - \lambda c_2)|v| < (c_1 + c_2)|w|$ or $(c_1 - \lambda c_2)vw < (c_1 + c_2)w^2$ because $v < 0$ and $w < 0$. Thus $c_1 + c_2 - (c_1 - \lambda c_2)\frac{\sigma_x^2 wv}{s^2 + \sigma_x^2 w^2} > c_1 + c_2 - (c_1 + c_2)\frac{\sigma_x^2 w^2}{s^2 + \sigma_x^2 w^2} = (c_1 + c_2)\frac{s^2}{s^2 + \sigma_x^2 w^2} > 0$. However, as $v < 0$, $F(v,w) = 0$ implies $c_1 + c_2 - (c_1 - \lambda c_2)\frac{\sigma_x^2 wv}{\sigma_d^2 + (\sigma_x w)^2} < 0$ —a contradiction. We conclude that $v > 0$ and $w \geq 0$. For $\lambda = c_1/c_2$, $G(v,w) = 0$ iff $w = 0$. For $\lambda < c_1/c_2$, $w > 0$ as $G(v,0) < 0$ for any v. We can therefore focus on nonnegative values of v and w in deriving the values of v and w from the equations $G(v,w) = 0$ and $K(v,w) = 0$. For any nonnegative w, the function $K(v,w)$ is continuous and increasing in v, where $K(0,w)$ is negative and $\lim_{v \to +\infty} K(v,w) = +\infty$. So, for any nonnegative w, there exists a unique positive value of v, denoted $k(w)$, such that $K(k(w),w) = 0$. Since $K(v,w)$ is decreasing in w, $k(w)$ is increasing in w, where $k(0)$ is a positive finite number and $\lim_{w \to +\infty} k(w) = +\infty$. Substituting $v = k(w)$ in $G(v,w) = 0$, we get $G(k(w),w) = 0$. Since $k(w)$ is increasing in w, it follows that $G(k(w),w)$ is increasing in w, where $G(k(0),0)$ is negative and $\lim_{w \to +\infty} G(k(w),w) = +\infty$. So there exists a unique positive value of w that satisfies the equation $G(k(w),w) = 0$, which together with $v = k(w)$ constitutes the unique solution to equations $G(v,w) = 0$ and $K(v,w) = 0$. The equilibrium values of v and w now unequivocally determine the equilibrium pricing coefficients α_0, α_1 and α_2. In the case of $\lambda = c_1/c_2$, $w = 0$, so using Eqs. (1), (2), and (3), we get$\alpha_1 = \alpha_2 = \alpha_1^b$. When $\lambda \neq c_1/c_2$, both v and w are positive, implying $\alpha_1 \neq \alpha_2$. For $-1 < \lambda \leq c_1/c_2$, we get $\alpha_1 > \alpha_2$, so $\frac{c_1\alpha_2}{c_2\alpha_1} < \frac{c_1}{c_2}$. Also, $w > 0$ implies $\frac{c_1\alpha_2}{c_2\alpha_1} > \lambda$. Hence $\lambda < B_2(a_1,a_2,x)/B_1(a_1,a_2,x) = \frac{c_1\alpha_2}{c_2\alpha_1} < \frac{c_1}{c_2}$. □

Proof of Propositions 4 The proof follows from Lemmata 1, 3, and 8, because $\mathrm{var}\left(\tilde{a} \,\middle|\, \tilde{a}_1 + \frac{\alpha_1}{2c_1}\tilde{x}, \tilde{a}_2 + \frac{\alpha_2}{2c_2}\tilde{x}\right) = \sigma^2 - \sigma^2\frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}$ and $\mathrm{var}\left(\tilde{a} \,\middle|\, \tilde{a} + (\frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2})\tilde{x}\right) = \sigma^2 - \sigma^2\alpha_1^b$. □

Proof of Propositions 5 The proof follows from Lemmata 1, 4, and 6. □

Proof of Propositions 6 By Proposition 2, the reporting bias is associated with a cost of $c_1(\alpha_1/2c_1)^2x^2 + c_2(\alpha_2/2c_2)^2x^2$ to the manager, which equals $\left(\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}\right)x^2$. The mean of the price, conditional on x but before the realizations of the accounting measures $\tilde{a}_1$ and $\tilde{a}_2$, is $\alpha_0 + \alpha_1 E(\tilde{a}_1 + \frac{\alpha_1}{2c_1}x) + \alpha_2 E(\tilde{a}_2 + \frac{\alpha_2}{2c_2}x)$, which equals $\mu + \left(\frac{\alpha_1^2}{2c_1} + \frac{\alpha_2^2}{2c_2}\right) \cdot (x - \mu_x)$ using Eq. (4). The manager's utility with the bias option is therefore $x\left(\mu + \left(\frac{\alpha_1^2}{2c_1} + \frac{\alpha_2^2}{2c_2}\right) \cdot (x - \mu_x)\right) - \left(\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}\right)x^2$ or $x\mu + \left(\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}\right) \cdot ((x - \mu_x)^2 - \mu_x^2)$. Her utility without the bias option is $xE(\tilde{a}) = x\mu$. So her benefit (loss) from the bias option is $\left(\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}\right) \cdot ((x - \mu_x)^2 - \mu_x^2)$. In the benchmark case, it is $\left(\frac{1}{4c_1} + \frac{1}{4c_2}\right)\alpha_1^{b2} \cdot ((x - \mu_x)^2 - \mu_x^2)$. The rest of the proof follows from Lemmata 1, 5, and 7. □

Lemma 1 *$\frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2} < \frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}$, $\sigma^2 - \sigma^2 \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda} < \sigma^2 - \sigma^2\alpha_1^b$ and $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2} < \frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}$ for any $\lambda \neq c_1/c_2$.*

Proof of Lemma 1 As $\lambda \neq c_1/c_2$, it follows from the proof of Proposition 2 that $v, w > 0$, where $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ and $w = \frac{\alpha_2}{2c_2} - \lambda\frac{\alpha_1}{2c_1}$. By Eq. (8), $L(v, w) = 0$. So$(c_1 - \lambda c_2)v > (c_1 + c_2)w$, implying $c_1 + c_2 - (c_1 - \lambda c_2)\frac{\sigma_x^2 wv}{s^2 + \sigma_x^2 w^2} < c_1 + c_2 - (c_1 + c_2)\frac{\sigma_x^2 w^2}{s^2 + \sigma_x^2 w^2} = (c_1 + c_2)\frac{s^2}{s^2 + \sigma_x^2 w^2}$. Thus $F(v, w) > v - \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2 + \sigma_x^2 w^2}} \cdot \frac{s^2}{s^2 + \sigma_x^2 w^2}(c_1 + c_2)(2c_1c_2)^{-1} > v - \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2}(c_1 + c_2)(2c_1c_2)^{-1} = F(v, 0)$. Now, noting that Eq. (1) is equivalent to the equation $F(v^b, 0) = v^b - \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^{b2}}(c_1 + c_2)(2c_1c_2)^{-1} = 0$, where $v^b = \frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}$, we get that $F(v^b, w) > F(v^b, 0) = 0$ for any $w > 0$. Since $F(v, w)$ is increasing in v, it follows from $F(v, w) = 0$ that $v < v^b$. This implies $\sigma_x^2 v^{b2} > \sigma_x^2 v^2 \geq \sigma_x^2 v^2 \frac{s^2}{v^2 + \sigma_x^2 w^2}$ for any w. Also, $0 < \alpha_1^b = \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^{b2}} < \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{v^2 + \sigma_x^2 w^2}} = \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda} < 1$, implying $\sigma^2 - \sigma^2\frac{\alpha_1 + \lambda\alpha_2}{1+\lambda} < \sigma^2 - \sigma^2\alpha_1^b$. Using Eqs. (1), (2), and (3), $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2} = \frac{v\sigma^2 \frac{s^2}{s^2 + \sigma_x^2 w^2}}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2 + \sigma_x^2 w^2}} < \frac{v^b \sigma^2}{\sigma^2 + \sigma_x^2 v^{b2}} = \frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}$. □

Lemma 2 *For $0 \leq \lambda \leq c_1/c_2$, $w = \frac{\alpha_2}{2c_2} - \lambda\frac{\alpha_1}{2c_1}$ decreases in λ.*

Proof of Lemma 2 By the proof of Proposition 2, w is zero for $\lambda = c_1/c_2$ and strictly positive for $\lambda < c_1/c_2$, so we only need to show that there are no two different values of λ with the same value of w. Suppose by contradiction that there exist λ_L and λ_H, such that $\lambda_L < \lambda_H$, for which w gets the same value w_{LH}, and denote by v_L and v_H the corresponding values of $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$. In this case, $F(v_L, w_{LH}; \lambda_L) = F(v_H, w_{LH}; \lambda_H) = 0$ and $G(v_L, w_{LH}; \lambda_L) = G(v_H, w_{LH}; \lambda_H) = 0$ by Eqs. (5) and (6). As $G(v, w; \lambda)$ is increasing in both v and λ (for $\lambda \geq 0$), $\lambda_L < \lambda_H$ implies $v_L > v_H$. As $F(v, w; \lambda)$is increasing in v and decreasing in λ, $v_L > v_H$ and $\lambda_L < \lambda_H$, we get $F(v_L, w_{LH}; \lambda_L) > F(v_H, w_{LH}; \lambda_H)$—a contradiction. □

Lemma 3 *For* $0 \leq \lambda \leq c_1/c_2$, $\sigma^2 - \sigma^2 \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}$ *increases in* λ.

Proof of Lemma 3 Since $\sigma^2 - \sigma^2 \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}$ equals $\sigma^2 - \sigma^2 \alpha_1^b$ for $\lambda = c_1/c_2$ and it is strictly lower than $\sigma^2 - \sigma^2 \alpha_1^b$ for $\lambda < c_1/c_2$ by Lemma 1, we only need to show that there are no two different values of λ with the same value of $\sigma^2 - \sigma^2 \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}$. Suppose by contradiction that there exist λ_L and λ_H, such that $\lambda_L < \lambda_H$, for which $\sigma^2 - \sigma^2 \frac{\alpha_1 + \lambda\alpha_2}{1+\lambda}$ gets the same value, denote by v_L and v_H the corresponding values of $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ and denote by w_L and w_H the corresponding values of $w = \frac{\alpha_2}{2c_2} - \lambda \frac{\alpha_1}{2c_1}$. This implies $\frac{\sigma_x^2 v_L^2}{s^2 + \sigma_x^2 w_L^2} = \frac{\sigma_x^2 v_H^2}{s^2 + \sigma_x^2 w_H^2}$ because $\frac{\alpha_1 + \lambda\alpha_2}{1+\lambda} = \frac{\sigma^2}{\sigma^2 + \sigma_x^2 v^2 \frac{s^2}{s^2 + \sigma_x^2 w^2}}$ by Eqs. (2) and (3). It follows from Lemma 2 that $w_L > w_H$, and thus $\frac{\sigma_x^2 v_L^2}{s^2 + \sigma_x^2 w_L^2} = \frac{\sigma_x^2 v_H^2}{s^2 + \sigma_x^2 w_H^2}$ implies $v_L > v_H$ and $w_L/v_L > w_H/v_H$. So $\frac{\sigma_x^2 v_L w_L}{s^2 + \sigma_x^2 w_L^2} = \frac{\sigma_x^2 v_L^2}{s^2 + \sigma_x^2 w_L^2} \cdot \frac{w_L}{v_L} > \frac{\sigma_x^2 v_H^2}{s^2 + \sigma_x^2 w_H^2} \cdot \frac{w_H}{v_H} = \frac{\sigma_x^2 v_H w_H}{s^2 + \sigma_x^2 w_H^2}$. However, $\lambda_L < \lambda_H$, $\frac{\sigma_x^2 v_L^2}{s^2 + \sigma_x^2 w_L^2} = \frac{\sigma_x^2 v_H^2}{s^2 + \sigma_x^2 w_H^2}$ and $\frac{\sigma_x^2 w_L v_{LH}}{s^2 + \sigma_x^2 w_L^2} > \frac{\sigma_x^2 w_H v_{LH}}{s^2 + \sigma_x^2 w_H^2}$ imply that $v_L = v_L - F(v_L, w_L; \lambda_L) < v_H - F(v_L, w_H; \lambda_H) = v_H$—a contradiction. □

Lemma 4 *For* $0 \leq \lambda \leq c_1/c_2$, $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ *increases in* λ.

Proof of Lemma 4 Since v equals $v^b = \frac{\alpha_1^b}{2c_1} + \frac{\alpha_1^b}{2c_2}$ in the case of $\lambda = c_1/c_2$ and is strictly lower in the case of $\lambda < c_1/c_2$ by Lemma 1, we only need to show that there are no two different values of λ with the same value of v. Suppose by contradiction that there exist λ_L and λ_H, such that $\lambda_L < \lambda_H$, for which v gets the same value v_{LH}, and denote by w_L and w_H the corresponding values of $w = \frac{\alpha_2}{2c_2} - \lambda \frac{\alpha_1}{2c_1}$. By Lemma 2, $w_L > w_H$. By Eqs. (5) and (8), $F(v_{LH}, w_L; \lambda_L) = F(v_{LH}, w_H; \lambda_H) = 0$ and $L(v_{LH}, w_L; \lambda_L) = L(v_{LH}, w_H; \lambda_H) = 0$. It follows from $v_{LH} - F(v_{LH}, w_L; \lambda_L) = v_{LH} - F(v_{LH}, w_H; \lambda_H)$ and $w_L > w_H$ that $(c_1 - \lambda_L c_2) \frac{\sigma_x^2 w_L v_{LH}}{s^2 + \sigma_x^2 w_L^2} > (c_1 - \lambda_H c_2) \frac{\sigma_x^2 w_H v_{LH}}{s^2 + \sigma_x^2 w_H^2}$. It now follows from $L(v_{LH}, w_L; \lambda_L) = L(v_{LH}, w_H; \lambda_H) = 0$ or $L(v_{LH}, w_L; \lambda_L)/w_L = L(v_{LH}, w_H; \lambda_H)/w_H = 0$ that $(c_1 - \lambda_L c_2) v_{LH}/w_L - \frac{(1+\lambda_L)^2}{2} \cdot \frac{\sigma^2 \sigma_x^2 v_{LH}}{\sigma^2 s^2 + \sigma^2 \sigma_x^2 w_L^2 + \sigma_x^2 v_{LH}^2 s^2}$ equals $(c_1 - \lambda_H c_2)\, v_{LH}/w_H - \frac{(1+\lambda_H)^2}{2} \cdot \frac{\sigma^2 \sigma_x^2 v_{LH}}{\sigma^2 s^2 + \sigma^2 \sigma_x^2 w_H^2 + \sigma_x^2 v_{LH}^2 s^2}$. Since $\lambda_L < \lambda_H$ and $w_L > w_H$, we get that $\frac{(1+\lambda_L)^2}{2} \cdot \frac{\sigma^2 \sigma_x^2 v_{LH}}{\sigma^2 s^2 + \sigma^2 \sigma_x^2 w_L^2 + \sigma_x^2 v_{LH}^2 s^2}$ is lower than $\frac{(1+\lambda_H)^2}{2} \cdot \frac{\sigma^2 \sigma_x^2 v_{LH}}{\sigma^2 s^2 + \sigma^2 \sigma_x^2 w_H^2 + \sigma_x^2 v_{LH}^2 s^2}$, and thus $(c_1 - \lambda_L c_2) v_{LH}/w_L < (c_1 - \lambda_H c_2) v_{LH}/w_H$. Now observe that $(c_1 - \lambda_L c_2) v_{LH}/w_L = (c_1 - \lambda_L c_2)(v_{LH} - F(v_{LH}, w_L; \lambda_L))/(w_L - G(v_{LH}, w_L; \lambda_L))$ and $(c_1 - \lambda_H c_2) v_{LH}/w_H = (c_1 - \lambda_H c_2)(v_{LH} - F(v_{LH}, w_H; \lambda_H))/(w_H - G(v_{LH}, w_H; \lambda_H))$. However, $\lambda_L < \lambda_H$ and $(c_1 - \lambda_L c_2) \frac{\sigma_x^2 w_L v_{LH}}{s^2 + \sigma_x^2 w_L^2} > (c_1 - \lambda_H c_2) \frac{\sigma_x^2 w_H v_{LH}}{s^2 + \sigma_x^2 w_H^2}$ raise a contradiction by implying that $(c_1 - \lambda_L c_2)(v_{LH} - F(v_{LH}, w_L; \lambda_L))/(w_L - G(v_{LH}, w_L; \lambda_L)) > (c_1 - \lambda_H c_2)(v_{LH} - F(v_{LH}, w_H; \lambda_H))/(w_H - G(v_{LH}, w_H; \lambda_H))$. □

Lemma 5 *For* $0 \leq \lambda \leq c_1/c_2$, $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}$ *increases in* λ.

Proof of Lemma 5 Since $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}$ equals $\frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}$ in the case of $\lambda = c_1/c_2$ and is strictly lower in the case of $\lambda < c_1/c_2$ by Lemma 1, we only need to show that there are no two different values of λ with the same value of $\frac{\alpha_1^2}{4c_1}+\frac{\alpha_2^2}{4c_2}$. Suppose by contradiction that there exist λ_L and λ_H, such that $\lambda_L<\lambda_H$, for which $\frac{\alpha_1^2}{4c_1}+\frac{\alpha_2^2}{4c_2}$ gets the same value, denote by v_L and v_H the corresponding values of $v=\frac{\alpha_1}{2c_1}+\frac{\alpha_2}{2c_2}$ and denote by w_L and w_H the corresponding values of $w=\frac{\alpha_2}{2c_2}-\lambda\frac{\alpha_1}{2c_1}$. By Lemmata 2 and 4 $w_L>w_H$ and $v_L<v_H$. By Eq. (5), $F(v_L,w_L;\lambda_L)=F(v_H,w_H;\lambda_H)=0$. Together with $v_L<v_H$ and $w_L>w_H$, this implies that $(c_1-\lambda_L c_2)\frac{\sigma_x^2 w_L v_L}{s^2+\sigma_x^2 w_L^2}>(c_1-\lambda_H c_2)\frac{\sigma_x^2 w_H v_H}{s^2+\sigma_x^2 w_H^2}$. By Eq. (8), $L(v_L,w_L;\lambda_L)=L(v_H,w_H;\lambda_H)=0$ or $L(v_L,w_L;\lambda_L)/w_L=L(v_H,w_H;\lambda_H)/w_H=0$, and thus $(c_1-\lambda_L c_2)v_L/w_L-\frac{(1+\lambda_L)^2}{2}\cdot\frac{\sigma^2}{\sigma^2+\sigma_x^2 v_L^2\frac{s^2}{s^2+\sigma_x^2 w_L^2}}\cdot\frac{\sigma_x^2 v_L}{s^2+\sigma_x^2 w_L^2}$ equals $(c_1-\lambda_H c_2)v_H/w_H-\frac{(1+\lambda_H)^2}{2}\cdot\frac{\sigma^2}{\sigma^2+\sigma_x^2 v_H^2\frac{s^2}{s^2+\sigma_x^2 w_H^2}}\cdot\frac{\sigma_x^2 v_H}{s^2+\sigma_x^2 w_H^2}$. Using Eqs. (2) and (3), we get after rearranging that $\frac{\alpha_1^2}{4c_1}+\frac{\alpha_2^2}{4c_2}=\frac{v\sigma^2\frac{s^2}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}$. So $\frac{v_L\sigma^2\frac{s^2}{s^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_L^2\frac{s^2}{s^2+\sigma_x^2 w_L^2}}=\frac{v_H\sigma^2\frac{s^2}{s^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_H^2\frac{s^2}{s^2+\sigma_x^2 w_H^2}}$. Together with $\lambda_L<\lambda_H$, this implies that $\frac{(1+\lambda_L)^2}{2}\cdot\frac{v_L\sigma^2\frac{s^2}{s^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_L^2\frac{s^2}{s^2+\sigma_x^2 w_L^2}}<\frac{(1+\lambda_H)^2}{2}\cdot\frac{v_H\sigma^2\frac{s^2}{s^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_H^2\frac{s^2}{s^2+\sigma_x^2 w_H^2}}$, and thus $(c_1-\lambda_L c_2)v_L/w_L<(c_1-\lambda_H c_2)v_H/w_H$. Now observe that $(c_1-\lambda_L c_2)v_L/w_L=(c_1-\lambda_L c_2)(v_L-F(v_L,w_L;\lambda_L))/(w_L-G(v_L,w_L;\lambda_L))$ and $(c_1-\lambda_H c_2)v_H/w_H=(c_1-\lambda_H c_2)(v_H-F(v_H,w_H;\lambda_H))/(w_H-G(v_H,w_H;\lambda_H))$. However, $\lambda_L<\lambda_H$ and $(c_1-\lambda_L c_2)\frac{\sigma_x^2 w_L v_L}{s^2+\sigma_x^2 w_L^2}>(c_1-\lambda_H c_2)\frac{\sigma_x^2 w_H v_H}{s^2+\sigma_x^2 w_H^2}$ raise a contradiction by implying that $(c_1-\lambda_L c_2)(v_L-F(v_L,w_L;\lambda_L))/(w_L-G(v_L,w_L;\lambda_L))>(c_1-\lambda_H c_2)(w_L-F(v_H,w_H;\lambda_H))/(w_H-G(v_H,w_H;\lambda_H))$. □

Lemma 6 *For any $\lambda\neq c_1/c_2$, $v=\frac{\alpha_1}{2c_1}+\frac{\alpha_2}{2c_2}$ increases in s^2.*

Proof of Lemma 6 Since $v<v^b$ for $s^2<\infty$ and $v=v^b$ for $s^2=\infty$ by Lemma 1, we only need to show that there are no two different values of s^2 with the same value of v. Suppose by contradiction that there exist s_L^2 and s_H^2, such that $s_L^2<s_H^2$, for which v gets the same value v_{LH}, and denote by w_L and w_H the corresponding values of w. By Eq. (7), $K(v_{LH},w_L;s_L^2)=K(v_{LH},w_H;s_H^2)=0$. This, together with $s_L^2<s_H^2$, implies that $w_L<w_H$ and $\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}<\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}$. It follows from $\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}<\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}$ that $w_L^2/s_L^2>w_H^2/s_H^2$. Since $w_L<w_H$, we get that $w_L/s_L^2>w_H/s_H^2$. It also follows from $\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}<\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}$ that $\frac{\sigma^2\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_{LH}^2\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}<\frac{\sigma^2\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_{LH}^2\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}$. By Eq. (5), $F(v_{LH},w_L;s_L^2)=F(v_{LH},w_H;s_H^2)=0$, where $F(v_{LH},w_L;s_L^2)$ and $F(v_{LH},w_H;s_H^2)$ can be rewritten as $v_{LH}-\frac{\sigma^2\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_{LH}^2\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}\left(c_1+c_2-\frac{\sigma_x^2 w_L}{s_L^2}((c_1-\lambda c_2)v_{LH}-(c_1+c_2)w_L)\right)(2c_1c_2)^{-1}$ and

$$v_{LH} - \frac{\sigma^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_{LH}^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}} \left(c_1 + c_2 - \frac{\sigma_x^2 w_H}{s_H^2}((c_1-\lambda c_2)v_{LH} - (c_1+c_2)w_H)\right)(2c_1c_2)^{-1},$$

respectively. Since $\frac{\sigma^2 \frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_{LH}^2 \frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}} < \frac{\sigma^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_{LH}^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}$, $w_L/\sigma_{dL}^2 > w_H/\sigma_{dH}^2$ and $w_L < w_H$, we get $v_{LH} = v_{LH} - F(v_{LH}, w_L; s_L^2) < v_{LH} - F(v_{LH}, w_H; s_H^2) = v_{LH}$—a contradiction. □

Lemma 7 *For any $\lambda \neq c_1/c_2$, $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}$ increases in s^2.*

Proof of Lemma 7 Since $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2} < \frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}$ for $s^2 < \infty$ and $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2} = \frac{\alpha_1^{b2}}{4c_1} + \frac{\alpha_1^{b2}}{4c_2}$ for $s^2 = \infty$ by Lemma 1, we only need to show that there are no two different values of s^2 with the same value of $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}$. Suppose by contradiction that there exist s_L^2 and s_H^2, such that $s_L^2 < s_H^2$, with the same value of $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2}$, denote by v_L and v_H the corresponding values of v, and denote by w_L and w_H the corresponding values of w. By Lemma 6, $v_L < v_H$. Using Eqs. (2) and (3), we get $\frac{\alpha_1^2}{4c_1} + \frac{\alpha_2^2}{4c_2} = \frac{v\sigma^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}$, so the manager's benefit (loss) from the bias option is $\frac{v\sigma^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}((x-\mu_x)^2 - \mu_x^2)/2$. A contradiction now arises, as for $(x-\mu_x)^2 > \mu_x^2$ (so that the manager benefits from the bias option) the equilibrium reporting strategy v_L and w_L of the case $s^2 = s_L^2$, when applied to the case $s^2 = s_H^2$, yields a higher benefit than the equilibrium reporting strategy v_H and w_H of the case $s^2 = s_H^2$. This is because $s_L^2 < s_H^2$ implies

$$\frac{v_L\sigma^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_L^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_L^2}} > \frac{v_L\sigma^2 \frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_L^2 \frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}} = \frac{v_H\sigma^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_H^2 \frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}.$$ □

Lemma 8 *For any $\lambda \neq c_1/c_2$, $\sigma^2 - \sigma^2 \frac{\alpha_1+\lambda\alpha_2}{1+\lambda}$ increases in s^2.*

Proof of Lemma 8 As $\sigma^2 - \sigma^2 \frac{\alpha_1+\lambda\alpha_2}{1+\lambda} < \sigma^2 - \sigma^2\alpha_1^b$ for $s^2 < \infty$ and $\sigma^2 - \sigma^2 \frac{\alpha_1+\lambda\alpha_2}{1+\lambda} = \sigma^2 - \sigma^2\alpha_1^b$ for $s^2 = \infty$ by Lemma 1, we only need to show that there are no two different values of λ with the same value of $\sigma^2 - \sigma^2 \frac{\alpha_1+\lambda\alpha_2}{1+\lambda}$. Suppose by contradiction that there exist s_L^2 and s_H^2, such that $s_L^2 < s_H^2$, for which $\sigma^2 - \sigma^2 \frac{\alpha_1+\lambda\alpha_2}{1+\lambda}$ gets the same value, denote by v_L and v_H the corresponding values of $v = \frac{\alpha_1}{2c_1} + \frac{\alpha_2}{2c_2}$ and denote by w_L and w_H the corresponding values of $w = \frac{\alpha_2}{2c_2} - \lambda\frac{\alpha_1}{2c_1}$. This implies $\frac{v_L^2 s_L^2}{s_L^2+\sigma_x^2 w_L^2} = \frac{v_H^2 s_H^2}{s_H^2+\sigma_x^2 w_H^2}$ because $\frac{\alpha_1+\lambda\alpha_2}{1+\lambda} = \frac{\sigma^2}{\sigma^2+\sigma_x^2 v^2 \frac{s^2}{s^2+\sigma_x^2 w^2}}$ by Eqs. (2) and (3). Since $v_L < v_H$ by Lemma 6, this leads to $\frac{v_L s_L^2}{s_L^2+\sigma_x^2 w_L^2} > \frac{v_H s_H^2}{s_H^2+\sigma_x^2 w_H^2}$, which implies $\frac{v_L\sigma^2\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}}{\sigma^2+\sigma_x^2 v_L^2\frac{s_L^2}{s_L^2+\sigma_x^2 w_L^2}} > \frac{v_H\sigma^2\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}{\sigma^2+\sigma_x^2 v_H^2\frac{s_H^2}{s_H^2+\sigma_x^2 w_H^2}}$. This is a contradiction to Lemma 7 because Eqs. (2) and (3) imply $\frac{\alpha_1^2}{4c_1}+\frac{\alpha_2^2}{4c_2}=\frac{v\sigma^2\frac{s^2}{s^2+\sigma_x^2 w^2}}{\sigma^2+\sigma_x^2 v^2\frac{s^2}{s^2+\sigma_x^2 w^2}}$. □

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