

## BIASED VOLUNTARY DISCLOSURE

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# Biased Voluntary Disclosure\*

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**Abstract:** We provide a bridge between the voluntary disclosure and the earnings management literature. Voluntary disclosure models focus on managers' discretion in deciding whether or not to provide **truthful** voluntary disclosure to the capital market. Earnings management models, on the other hand, concentrate on managers' discretion in deciding how to bias their **mandatory** disclosure. By analyzing managers' disclosure strategy when disclosure is voluntary and not necessarily truthful, we show the robustness of voluntary disclosure theory to the relaxation of the standard assumption of truthful reporting. We also demonstrate the sensitivity of earnings management theory to the commonly made mandatory disclosure assumption.

**Keywords:** Financial accounting; Asymmetric information; Voluntary disclosure; Reporting bias; Earnings management.

**JEL classification:** D82; G14; M41; M43.

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## ***1. Introduction***

Managers of publicly traded firms often face disclosure decisions that comprise two intertwined tiers of discretion. First, they have to decide whether or not to voluntarily disclose their private information. Second, upon disclosure, they have to decide if and how to bias the disclosed content relative to the true underlying private information. Both tiers of managerial disclosure discretion have been extensively studied in the literature, though in isolation from each other. The voluntary disclosure literature focuses on managerial discretion in deciding whether to voluntarily provide disclosure, assuming disclosure to be truthful.<sup>1</sup> The earnings management literature, on the other hand, concentrates on managerial discretion in determining the reporting bias, assuming disclosure is mandatory.<sup>2</sup> In this paper, we offer a juncture that links these two streams in the disclosure literature. We analyze a double-tier disclosure decision and explore non-trivial interactions between the discretion of managers in deciding whether or not to provide a voluntary disclosure and their discretion in deciding how to bias their disclosure if provided.

The analysis is based on a model that depicts a single-period reporting game where a privately informed manager of a publicly traded firm exercises discretion over the voluntary disclosure of falsifiable information to capital market investors. The model combines the key features of the classical voluntary disclosure settings of Verrecchia (1983) and Dye (1985) with

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<sup>1</sup> See, for example, Grossman and Hart, (1980), Grossman (1981), Milgrom (1981), Verrecchia (1983), Dye (1985), Darrough and Stoughton (1990), Wagenhofer (1990), Shin (1994), Kirschenheiter (1997), Nagar (1999), Fishman and Hagerty (2003), Einhorn (2005), Pae (2005), Suijs (2007), Langberg and Sivaramakrishnan (2008).

<sup>2</sup> See, for example, Dye (1988), Stein (1989), Fischer and Verrecchia (2000), Kirschenheiter and Melumad (2002), Dye and Sridhar (2004), Fischer and Stocken (2004), Ewert and Wagenhofer (2005), Guttman, Kadan and Kandel (2006).

those of the earnings management setups of Stein (1989) and Fischer and Verrecchia (2000), relaxing the assumption of truthful reporting embedded in traditional models of voluntary disclosure and the assumption of mandatory disclosure embedded in conventional models of reporting bias. Surprisingly, we show that in equilibrium the manager's decision on whether to voluntarily provide disclosure is independent of the reporting bias decision. Hence, the results drawn from extant models of voluntary disclosure are insensitive to relaxing the standard truthful disclosure assumption. We thus conclude that the reliance on the standard truthful reporting assumption, often regarded by researchers as a significant weakness of voluntary disclosure models, does not detract from the generality and robustness of extant voluntary disclosure theory. At the same time, our analysis shows that the equilibrium reporting bias is sensitive to relaxing the mandatory disclosure assumption, but it nevertheless converges to the bias derived in extant mandatory disclosure models for sufficiently high realizations of the manager's private information.

The paper proceeds as follows. The next section presents our disclosure model. The equilibrium in the model is derived and analyzed in section 3. The final section summarizes and offers concluding remarks. Proofs appear in the appendix.

## **2. *Model***

Our model depicts a single-period reporting game between a privately informed manager of a publicly traded firm and capital market investors. The firm's manager, who has the incentives to influence the firm's market price, exercises discretion over the voluntary disclosure of falsifiable information to the risk-neutral investors. The manager's disclosure strategy is, therefore, based on her rational expectations about the pricing rule applied by investors. Investors, in turn, invoke their rational expectations regarding the manager's disclosure strategy

in pricing the firm. The model combines the conventional setting of voluntary disclosure with the conventional setting of reporting bias, relaxing both the assumption of truthful reporting embedded in extant models of voluntary disclosure and the assumption of mandatory disclosure embedded in extant models of reporting bias. The remainder of this section details the notation and assumptions underlying the model, which are all common knowledge unless otherwise indicated.

The firm's uncertain equity value is represented by the random variable  $\tilde{v}$ , which is normally distributed with mean  $\mu$  and variance  $\sigma^2$ . The random variable  $\tilde{s}$  represents a noisy signal of the underlying firm value  $\tilde{v}$ , which follows the structure  $\tilde{s} = \tilde{v} + \tilde{\varepsilon}$ , where  $\tilde{\varepsilon}$  is an independent normally distributed noise term with zero mean and variance  $\sigma_\varepsilon^2$ . With some probability, the manager privately observes the signal  $\tilde{s}$ . As in the voluntary disclosure model of Dye (1985), the investors are uncertain about the endowment of the manager with the private signal  $\tilde{s}$ . The investors' beliefs about the manager's information endowment are described by a binary random variable  $\tilde{t}$ , which is distributed over the support  $T = \{0, 1\}$ , where  $\tilde{t} = 1$  describes an informed manager and  $\tilde{t} = 0$  describes an uninformed manager. Investors ascribe a probability  $0 < \tau < 1$  to the scenario that the manager is privately informed ( $\tilde{t} = 1$ ) and a probability  $1 - \tau$  to the scenario that the manager is uninformed ( $\tilde{t} = 0$ ). An informed manager can voluntarily provide disclosure about her private signal  $\tilde{s}$  to the capital market investors. As in the voluntary disclosure model of Verrecchia (1983), disclosure is associated with a cost  $\lambda > 0$ .<sup>3</sup> We further assume, like Dye (1985), that an uninformed manager cannot credibly claim to be uninformed and cannot provide any disclosure.<sup>4</sup>

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<sup>3</sup> We incorporate in our model two features that are commonly used in voluntary disclosure models – a disclosure cost in the spirit of Verrecchia (1983) and an uncertain information arrival in the spirit of Dye (1985). These two

We generalize Verrecchia (1983) and Dye (1985) by relaxing the truthful reporting assumption that subsequently became common in the entire voluntary disclosure literature. We allow the manager to bias her voluntary report of the signal  $\tilde{s}$ , but at a cost.<sup>5</sup> As is common in the earnings management literature (e.g., Fischer and Verrecchia, 2000; Dye and Sridhar, 2004; Ewert and Wagenhofer, 2005; Guttman, Kadan and Kandel, 2006), we assume a quadratic biasing cost function. Specifically, when the manager observes a realization  $s$  of the signal  $\tilde{s}$  and chooses to voluntarily report  $r$ , her reporting bias of  $r - s$  is associated with a cost  $c(r - s)^2$  where  $c > 0$ .

Figure 1 provides a timeline depicting the sequence of events in the model. At the beginning, investors establish their prior beliefs about the firm's value  $\tilde{v}$ , the manager's information endowment and her private signal  $\tilde{s}$ . Then, with a probability  $\tau$ , the manager

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features serve to enrich our model but we emphasize that our main insights can be drawn from analyzing a reduced form of the model where only one of the two features is incorporated.

<sup>4</sup> Preventing disclosure by an uninformed manager, which is a standard assumption in conventional truthful reporting settings, is less obvious in our setting, where disclosure is not restricted to be truthful. This assumption is applicable in situations where the manager can falsify information but cannot manufacture information. For example, the manager can falsify the results (and interpretations) of a market analysis, a lab test or an FDA report, but cannot manufacture such information in their absence. Even when an uninformed manager can manufacture the information, she will not be able to manufacture evidence in order to defend and substantiate her disclosure in court. Preventing disclosure by an uninformed manager is equivalent in our setting to assuming that the costs associated with disclosure by an uninformed manager are prohibitively high or that the reporting bias – as formally introduced in the next paragraph – is calculated in such a case from a very low level. In particular, the equilibrium outcomes that emerge from the model remain intact if we allow disclosure by an uninformed manager and assume that the reporting bias in this case is calculated from the lowest realization of the signal  $\tilde{s}$  that is disclosed in equilibrium. That is, in court an uninformed manager has only to substantiate that her report is better than the lowest report that currently exists in the market (and incur the related reporting bias costs).

<sup>5</sup> The assumption of costly misreporting is typical of earnings management models (e.g., Dye, 1988; Stein, 1989; Fischer and Verrecchia, 2000), distinguishing them from cheap talk models, where misreporting is costless (e.g., Crawford and Sobel, 1982; Melumad and Shibano, 1991; Stocken, 2000), and precluding “babbling” equilibria, in which no information is conveyed. Biases in reporting can be associated with a variety of costs. When such biases involve the carrying out of inefficient real transactions, they are associated with the cost of distorting value. In other cases, they might be associated with litigation costs, reputation erosion costs, costs that emerge from conflicts with auditors and audit committees, and the costs of reducing future reporting flexibility.

privately observes the realization of the signal  $\tilde{s}$ . Based on the content of her private signal  $\tilde{s}$ , if received, the manager decides whether to disclose her private signal to the capital market and chooses the optimal reporting bias in case of disclosure. Now, the firm's price is set in the market, given all available public information. Finally, at the end of the period, the firm's value  $\tilde{v}$  is realized and becomes common knowledge, making the content of the signal  $\tilde{s}$  no longer relevant.<sup>6</sup>

[FIGURE 1]

The set of the manager's disclosure alternatives is represented by  $A = \{\phi\} \cup \mathfrak{R}$ , where  $\phi$  describes the alternative of not providing any disclosure and any  $r \in \mathfrak{R}$  describes the content of a voluntary report of the manager (if provided). Using this notation, the function  $D: T \times \mathfrak{R} \rightarrow A$  represents the manager's disclosure strategy, where  $D(1, s) \in A$  is the informed manager's disclosure decision given the realization  $s \in \mathfrak{R}$  of her private signal  $\tilde{s}$ , and  $D(0, s) = \phi$  is the inevitable decision of an uninformed manager for any  $s \in \mathfrak{R}$ . The pricing rule applied by investors is represented by the function  $P: A \rightarrow \mathfrak{R}$ , where  $P(r) \in \mathfrak{R}$  is the market price of the firm under disclosure of a report  $r \in \mathfrak{R}$  and  $P(\phi) \in \mathfrak{R}$  is the market price of the firm in the absence of disclosure. We represent the investors' expectations regarding the manager's disclosure strategy  $D$  by the function  $\hat{D}: T \times \mathfrak{R} \rightarrow A$ , and similarly represent the manager's expectations about the investors' pricing rule  $P$  by the function  $\hat{P}: A \rightarrow \mathfrak{R}$ .

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<sup>6</sup> Like many disclosure models, our model captures only one disclosure decision. Hence, the analysis does not address issues such as the timing of disclosure or inter-temporal dynamics of subsequent disclosure decisions that could arise in a repeated multi-period disclosure setup (e.g., Stocken, 2000; Einhorn and Ziv, 2008). Nevertheless, despite its single-period nature, the model is applicable to a wide range of business contexts where the time horizon of the manager is relatively short. It is also applicable to a prevalent class of corporate disclosures that become irrelevant over time as more information arrives in the market.

In equilibrium, the disclosure strategy of an informed manager is based on her rational expectations about the market pricing rule, which is, in turn, determined by the investors' rational expectations regarding the manager's disclosure strategy. A perfect Bayesian equilibrium with pure strategies in the model is formally defined as a vector  $(D, \hat{D} : T \times \mathfrak{R} \rightarrow A, P, \hat{P} : A \rightarrow \mathfrak{R})$ , which satisfies three conditions. First, the manager chooses the disclosure decision that maximizes the rationally anticipated market price of the firm subject to the costs associated with disclosure and with the reporting bias upon disclosure. Formally, for any  $s \in \mathfrak{R}$ ,  $D(1, s) = \operatorname{argmax}_{r \in \mathfrak{R}} \{ \hat{P}(r) - \lambda - c(r - s)^2 \}$  if  $\operatorname{argmax}_{r \in \mathfrak{R}} \{ \hat{P}(r) - \lambda - c(r - s)^2 \} \geq \hat{P}(\phi)$  and  $D(1, s) = \phi$  otherwise, whereas  $D(0, s) = \phi$ .<sup>7</sup> Second, risk-neutral investors set the firm's market price to be its expected value conditional on all the available public information, including their rational expectations about the manager's disclosure strategy. That is,  $P(d) = E[\tilde{v} | \hat{D}(\tilde{t}, \tilde{s}) = d]$  for any  $d \in A$ . The third, and last, equilibrium condition requires that both the investors and the manager have rational expectations regarding each other's behavior, or  $\hat{D}(t, s) = D(t, s)$  and  $\hat{P}(d) = P(d)$  for any  $t \in T$ ,  $s \in \mathfrak{R}$  and  $d \in A$ .

Disclosure games typically involve a large number of equilibria, because investors may hold a multitude of beliefs in response to out-of-equilibrium reports. We therefore restrict attention to equilibria with reasonable out-of-equilibrium beliefs that satisfy the D1 criterion of

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<sup>7</sup> In the knife-edge case where  $\operatorname{argmax}_{r \in \mathfrak{R}} \{ \hat{P}(r) - \lambda - c(r - s)^2 \} = \hat{P}(\phi)$ , an informed manager is indifferent between providing and not providing disclosure. We assume that in such a case the manager chooses to provide disclosure. Our assumption eliminates multiple equilibria that stem only from the manager's indifference. To simplify the presentation, we also assume that the disclosure costs ( $\lambda$ ) and the biasing costs ( $c(r - s)^2$ ) are incurred by the manager, but all of our results hold when these costs are (partially or entirely) incurred by the firm.



Cho and Kreps (1987). The D1 criterion requires that any out-of-equilibrium report will be ascribed by the market to managerial types that are most likely to deviate to this report, in the sense that the set of market responses that make each of them better off making such a deviation contains all market responses that make any other managerial type better off or indifferent to the same deviation. This criterion eliminates the multiple pooling, partially-separating and inefficient fully-separating equilibria that typically emerge in conventional signaling models, selecting Riley's (1979) least cost separating equilibrium as the unique equilibrium.<sup>8</sup> It also appears to be useful in selecting the most informative equilibrium as the unique equilibrium in our model. Similar refinement methods are widely (albeit implicitly) employed in the disclosure literature.<sup>9</sup> We thus henceforth use the term equilibrium to denote a perfect Bayesian equilibrium with pure strategies and reasonable out-of-equilibrium beliefs in the sense of the D1 criterion of Cho and Kreps (1987).

Our model comprises the voluntary disclosure setups of Verrecchia (1983) and Dye (1985), as well as a simplified version of the reporting bias model of Fischer and Verrecchia (2000). It coincides with the voluntary disclosure setup of Verrecchia (1983) in the edge case where  $c(b)$  converges to  $+\infty$  for any  $b \in \mathfrak{R}$  (implying truthful reporting) and  $\tau$  converges to 1 (implying the manager is always informed). It similarly coincides with the voluntary disclosure

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<sup>8</sup> In monotonic signaling games, the D1 criterion yields the same unique equilibrium as do stronger refinement criteria such as the divinity and the universal divinity criteria of Banks and Sobel (1987) and the stability criterion of Kohlberg and Mertens (1986). The weaker intuitive criterion of Cho and Kreps (1987) is sufficient to accomplish this refinement only in signaling games with two types of informed player. See Cho and Kreps (1987) and Cho and Sobel (1990) for further details.

<sup>9</sup> For example, Korn and Schiller (2003) show that the classical models analyzed by Grossman and Hart (1980), Grossman (1981) and Milgrom (1981) yield many partial disclosure equilibria, but only the known full disclosure equilibrium satisfies the intuitive criterion (and the stronger D1 criterion) of Cho and Kreps (1987). Also, many earnings management models (e.g., Fischer and Verrecchia, 2000; Dye and Sridhar, 2004; Fischer and Stocken, 2004; Ewert and Wagenhofer, 2005) focus on equilibria with linear strategies, and therefore yield a unique separating equilibrium, which is also the only one that survives the D1 criterion of Cho and Kreps (1987).

setup of Dye (1985) in the edge case where  $c(b)$  converges to  $+\infty$  for any  $b \in \mathfrak{R}$  (implying, again, truthful reporting) and  $\lambda$  converges to 0 (implying costless disclosure). Lastly, when disclosure is assumed to be mandatory and costless ( $\lambda$  converges to 0), and the manager is always informed ( $\tau$  converges to 1), our model coincides with a simplified version of the reporting bias setup of Fischer and Verrecchia (2000) where the investors are capable of perfectly backing out the reporting bias (as also in Stein (1989)).<sup>10</sup> Having roots in both the voluntary disclosure and the reporting bias literature, our setup yields an equilibrium that is a straightforward extension of those derived in earlier research. This forms a convenient basis for analyzing how the manager's misreporting option affects her decision on whether or not to voluntarily provide disclosure, and how the manager's option of not providing disclosure affects her decision regarding her reporting bias upon disclosure. The analysis in the next section demonstrates that the interaction between the managerial options of misreporting and of refraining from disclosure has only limited effect on the manager's overall disclosure behavior.

### 3. *Equilibrium Analysis*

To facilitate the equilibrium analysis, we break down the manager's disclosure decision into two tiers: the informed manager's decision on whether or not to voluntarily provide disclosure, and the subsequent decision about the reporting bias upon a decision to provide disclosure. We first consider two benchmarks that capture each decision tier separately, and then move to our main research goal – understanding the interaction between the two decision tiers. In our first benchmark, we assume truthful disclosure, and thereby capture the conventional models of voluntary disclosure, where the managerial decision on whether or not

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<sup>10</sup> Instead of assuming  $\tau$  converges to 1, we can assume that the manager's information endowment is observable.

to voluntarily provide disclosure is considered in isolation. In the second benchmark, we assume mandatory disclosure, and thereby capture the conventional models of reporting bias, where the managerial decision about the reporting bias is considered in isolation. The analysis of the two benchmarks is given in sections 3.1 and 3.2, and the unrestricted analysis where disclosure is voluntary and not necessarily truthful is presented in section 3.3.

### *3.1 The benchmark of truthful disclosure*

Our first benchmark, denoted by the superscript  $B_1$ , pertains to the case where disclosure is assumed to be truthful. This case provides a natural point of reference as it coincides with traditional models of voluntary disclosure.<sup>11</sup> The literature on corporate voluntary disclosures is conceptually based on the seminal perception of private withheld information as being unraveled by the rational behavior of the market participants (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981). This perception is based on the argument that investors rationally interpret any piece of withheld information as conveying bad news, inducing firms to fully disclose their private information, however unfavorable it may be, in order to distinguish themselves from firms possessing even worse information. A large body of subsequent research aims at understanding the triggers behind the commonly observed tendency of firms to suppress substantial amounts of private information from the capital market. The classical factors known to impede corporate voluntary disclosures are the costs associated with disclosure (Jovanovic, 1982; Verrecchia, 1983) and the uncertainty of investors about the endowment of the firms with the information (Dye, 1985; Jung and Kwon, 1988).<sup>12</sup> Each of

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<sup>11</sup> For surveys of the literature on corporate voluntary disclosure, see Dye (2001) and Verrecchia (2001).

<sup>12</sup> Other explanations for the withholding of information by firms include the proprietary nature of the disclosed information (Darrough and Stoughton, 1990; Wagenhofer, 1990), the correlation between proprietary and

these two factors is sufficient to support a partially revealing equilibrium where firms disclose to investors only sufficiently favorable information and withhold unfavorable information. These earlier results, as well as most of the subsequent theoretical insights on corporate voluntary disclosures, are drawn from models that assume that the underlying information can be credibly disclosed. They are captured, therefore, by the equilibrium outcomes in the benchmark case of truthful disclosure, which are formally presented in Observation 1.

**Observation 1.** *In the benchmark case of truthful disclosure, there exists a unique perfect Bayesian equilibrium with pure strategies and out-of-equilibrium beliefs that satisfy the D1 criterion. The equilibrium,  $e^{B1}=(D^{B1}, \hat{D}^{B1} : T \times \mathfrak{R} \rightarrow A, P^{B1}, \hat{P}^{B1} : A \rightarrow \mathfrak{R})$ , is characterized by a disclosure threshold  $s^{B1} \in \mathfrak{R}$ , satisfying for any  $s \in \mathfrak{R}$  and  $r \geq s^{B1}$ :*

$$D^{B1}(0, s) = \hat{D}^{B1}(0, s) = \phi, \quad D^{B1}(1, s) = \hat{D}^{B1}(1, s) = \begin{cases} \phi & \text{if } s < s^{B1} \\ s & \text{otherwise} \end{cases},$$

$$P^{B1}(\phi) = \hat{P}^{B1}(\phi) = \frac{1 - \tau}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} \mu + \frac{\tau \text{prob}(\tilde{s} \leq s^{B1})}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} E[\tilde{v} | \tilde{s} \leq s^{B1}] \text{ and}$$

$P^{B1}(r) = \hat{P}^{B1}(r) = E[\tilde{v} | \tilde{s} = r]$ . *The disclosure threshold  $s^{B1}$  is increasing in  $\lambda$  and decreasing in  $\tau$ , converging to  $-\infty$  when  $\lambda$  converges to 0 and  $\tau$  converges to 1.*

Observation 1 presents the standard upper-tailed disclosure strategy implied by conventional models of voluntary disclosure (including Verrecchia, 1983; Dye, 1985), according to which an informed manager voluntarily discloses only sufficiently high realizations of her private information that exceed a threshold  $s^{B1}$ . Observation 1 further indicates that when  $\lambda$

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nonproprietary private information (Dye, 1986), the inability of unsophisticated investors to understand firms' disclosures (Fishman and Hagerty, 2003), the limited ability of managers to predict the market response to disclosure (Nagar, 1999; Suijs, 2007), the uncertainty of investors about the reporting objectives of managers (Einhorn, 2007), and the unbalanced structure of information (Einhorn and Ziv, 2007).

converges to 0 (implying costless disclosure) and  $\tau$  converges to 1 (eliminating the market uncertainty about the manager's information endowment), the benchmark of truthful disclosure yields the classical full disclosure result of Grossman and Hart (1980), Grossman (1981) and Milgrom (1981). We emphasize that the equilibrium presented in Observation 1 is the one and only equilibrium that the benchmark of truthful disclosure yields when the out-of-equilibrium beliefs are refined to be reasonable (the same being true of traditional models of voluntary disclosure). For the sake of uniformity among the different cases considered, we always apply the D1 criterion to restrict the out-of-equilibrium beliefs, even though the benchmark case of truthful disclosure yields the same equilibrium outcomes under the weaker intuitive criterion.<sup>13</sup>

### *3.2 The benchmark of mandatory disclosure*

Our second benchmark, denoted by the superscript  $B2$ , pertains to the case where disclosure is assumed to be mandatory. To allow for the enforcement of mandatory disclosure, we further assume in the second benchmark case that the manager's information endowment is observable. This case provides another interesting point of reference as it coincides with traditional models of reporting bias. Biases in reporting have been considered in the literature within either cheap talk models where misreporting is costless (e.g., Crawford and Sobel, 1982) or earnings management models where misreporting is costly (e.g., Stein, 1989; Fischer and Verrecchia, 2000). We relate in our analysis to models where misreporting is costly. Stein (1989) introduces one of the most notable models of costly misreporting, demonstrating that managers can be worse off with the option of biasing their accounting reports. Managers may

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<sup>13</sup> Korn and Schiller (2003) explain that when no restrictions are made on the out-of equilibrium beliefs, additional equilibria emerge because misreporting may be part of an off-equilibrium path (although truthful disclosure is assumed in equilibrium). They further show that the commonly made implicit restriction of truthful disclosure in off-equilibrium reports is equivalent to applying the intuitive criterion of Cho and Kreps (1987).

end up taking costly actions to bias their reporting even when they know that they are unable to fool the market. They are trapped into such inefficient behavior because they take the market's conjectures as fixed, knowing that investors will suspect their report in any case. Following this idea, Fischer and Verrecchia (2000) show that in the presence of an exogenous noise that does not allow the investors to perfectly back out the reporting bias, some types of managers actually benefit from the option of biasing their reports but at the expense of other types of managers. These results, as well as most of the theoretical results on reporting manipulations, are drawn from models that assume that reporting is mandatory. They are reflected, therefore, by the equilibrium outcomes in the benchmark case of mandatory disclosure (with observable information endowment), which are formally presented in Observation 2.

**Observation 2.** *In the benchmark case of mandatory disclosure, there exists a unique perfect Bayesian equilibrium with pure strategies and out-of-equilibrium beliefs that satisfy the DI*

*criterion. The equilibrium,  $e^{B^2} = (D^{B^2}, \hat{D}^{B^2} : T \times \mathfrak{R} \rightarrow A, P^{B^2}, \hat{P}^{B^2} : A \rightarrow \mathfrak{R})$ , is characterized*

*by a constant reporting bias  $b^{B^2} = \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ , satisfying for any  $s, r \in \mathfrak{R}$ :*

$$D^{B^2}(0, s) = \hat{D}^{B^2}(0, s) = \phi, \quad D^{B^2}(1, s) = \hat{D}^{B^2}(1, s) = s + b^{B^2}, \quad P^{B^2}(\phi) = \hat{P}^{B^2}(\phi) = \mu \text{ and}$$

$$P^{B^2}(r) = \hat{P}^{B^2}(r) = E[\tilde{v} | \tilde{s} = r - b^{B^2}].$$

Observation 2 presents the constant reporting bias strategy that conventionally arises in models of reporting bias. In equilibrium, an informed manager biases her report upward by a constant amount  $b^{B^2}$ . Therefore, the disclosed content unequivocally reveals the underlying true realization of the manager's private information. So, the investors rationally detect the reporting bias and adjust for it when pricing the firm. Due to the normal distribution assumption, the constant reporting bias results in a market pricing rule that is linear not only in the underlying signal, but also in the manager's report.

As in monotonic signaling games, the D1 criterion filters out all pooling and partially-separating equilibria. Under the D1 criterion, a pooling region, which contains different information realizations that result in the same report, cannot survive because a manager with the highest information realization in the pooling region is better off deviating and providing a slightly higher report. If such a report is part of the equilibrium, it serves the highest managerial type in the pooling region to imitate a better managerial type. Otherwise, it serves to separate the highest managerial type from the pooling region, because the market will ascribe such a deviation to this type of manager, who is the one most likely to make such a deviation under reasonable out-of-equilibrium beliefs (in the sense of the D1 criterion). The equilibrium presented in Observation 2 is the one and only separating equilibrium that the benchmark of mandatory disclosure yields, and thus the unique equilibrium that emerges when the out-of-equilibrium beliefs are confined to be reasonable in the sense of the D1 criterion. This is also the unique equilibrium with linear strategies (a reporting bias that is linear in the signal and a market price that is linear in the report), and therefore the one that arises in traditional models of earnings management due to the commonly made linearity assumption. Hence, in the benchmark case of mandatory disclosure, restricting the strategies to be linear is equivalent to restricting the out-of-equilibrium beliefs to satisfy the D1 criterion. This conclusion provides support to the linear empirical frameworks commonly used to link between mandatory accounting information and stock prices. When no restrictions are made on the out-of-equilibrium beliefs and on the strategies' shape, additional equilibria emerge, such as the partially-separating equilibria explored by Guttman, Kadan and Kandel (2006).

### *3.3 The unrestricted analysis*

Having analyzed the equilibrium outcomes in the benchmark cases, we now turn to analyzing the equilibrium where the manager has both the option of not providing any

disclosure and the option of biasing her reporting upon disclosure. Proposition 3 establishes the existence and uniqueness of an equilibrium in the model and characterizes its form. Proposition 3 further demonstrates how the equilibrium outcomes of the model deviate from those of the two benchmarks.

**Proposition 3.** *In the unrestricted case, there exists a unique perfect Bayesian equilibrium with pure strategies and out-of-equilibrium beliefs that satisfy the DI criterion. The equilibrium,*

*$e_0 = (D_0, \hat{D}_0 : T \times \mathfrak{R} \rightarrow A, P_0, \hat{P}_0 : A \rightarrow \mathfrak{R})$ , is characterized by a disclosure threshold  $s_0 \in \mathfrak{R}$*

*and an increasing biasing function  $b_0 : [s_0, +\infty) \rightarrow \mathfrak{R}$ , satisfying for any  $s \in \mathfrak{R}$  and  $r \geq s_0$ :*

$$D_0(0, s) = \hat{D}_0(0, s) = \phi, \quad D_0(1, s) = \hat{D}_0(1, s) = \begin{cases} \phi & \text{if } s < s_0 \\ s + b_0(s) & \text{otherwise} \end{cases},$$

$$P_0(\phi) = \hat{P}_0(\phi) = \frac{1 - \tau}{1 - \tau \text{prob}(\tilde{s} \geq s_0)} \mu + \frac{\tau \text{prob}(\tilde{s} \leq s_0)}{1 - \tau \text{prob}(\tilde{s} \geq s_0)} E[\tilde{v} | \tilde{s} \leq s_0] \text{ and}$$

$$P_0(r) = \hat{P}_0(r) = E[\tilde{v} | \tilde{s} = f_0^{-1}(r)] \text{ where } f_0(s) = s + b_0(s). \text{ The disclosure threshold } s_0 \text{ equals}$$

$s^{B1}$ . The reporting bias  $b_0(s)$  is concave in  $s$ , it reaches the minimal value of zero at  $s = s_0$ ,

increases in  $s$  for any  $s \geq s_0$ , and approaches the asymptote  $b^{B2}$  when  $s$  converges to  $+\infty$ .

In Figure 2, we illustrate the manager's equilibrium disclosure strategy, as formally presented in Proposition 3. The horizontal axis describes the range of all possible realizations of the manager's private signal  $\tilde{s}$ , while the vertical axis describes the range of all possible reporting biases. As in the benchmark of truthful disclosure, the manager adopts an upper-tailed disclosure strategy. The disclosure threshold  $s_0$ , which also coincides with the threshold  $s^{B1}$  obtained in the benchmark of truthful disclosure, divides the horizontal axis into two ranges. The range above the threshold contains all the realizations of the signal  $\tilde{s}$  which are disclosed by an informed manager. The range below the threshold contains all the realizations of the signal  $\tilde{s}$  for which an informed manager refrains from disclosure. The increasing concave line



describes the reporting bias  $b_0(s)$  for any realization  $s$  of the private signal  $\tilde{s}$  that belongs to the disclosure range. At the disclosure threshold, the manager provides an unbiased report (that is,  $b_0(s_0) = 0$ ). As the realization  $s$  of the signal  $\tilde{s}$  increases, the manager's reporting bias  $b_0(s)$  increases, approaching the asymptote  $b^{B2}$ , which is the constant reporting bias in the benchmark of mandatory disclosure.

[FIGURE 2]

Proposition 3 yields the conclusion that the standard upper-tailed shape of the disclosure strategy implied by many models of voluntary disclosure, including Verrecchia (1983) and Dye (1985), is robust to the relaxation of the truthful disclosure assumption.<sup>14</sup> In equilibrium, an informed manager chooses to disclose her private signal  $\tilde{s}$  if and only if its realization  $s$  exceeds a certain threshold  $s_0$ . The intuition behind this result is based on the observation that the manager's utility in the absence of disclosure is independent of the actual realization of her private signal (if received), because the signal's realization is unobservable to investors and thus cannot be embedded in the market price of the firm. So, if an informed manager chooses to disclose a certain realization of her private signal, then a manager who observes a higher realization is better off disclosing, as she can provide the same report, which requires a lower reporting bias that is less costly, and thus must yield higher utility than non-disclosure.

Not only is the upper-tailed shape of the voluntary disclosure strategy robust to the relaxation of the truthful disclosure assumption, it further follows from Proposition 3 that even the disclosure threshold itself remains intact after such relaxation (that is,  $s_0 = s^{B1}$ ). This result

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<sup>14</sup> A deviation from the upper-tailed shape can occur only when additional features are incorporated into the model. In a recent working paper, for instance, Beyer and Guttman (2009) show that an interaction between disclosure decisions and operating decisions could result in two distinct regions of disclosure.

is important as it implies that the standard truthful reporting assumption commonly used in voluntary disclosure models does not detract from the generality of the results established on these models. The intuition behind this result is compelling. When the actual realization of the manager's private signal exactly equals the disclosure threshold, the manager is indifferent between providing disclosure and keeping quiet. Disclosure in this case, which actually pertains to the worst information realization that the manager is willing to disclose, must be unbiased. Otherwise, the lowest disclosing manager would be better off deviating from equilibrium by providing an off-equilibrium report that is slightly less biased. This is because she knows that the market will ascribe such a deviation to the lowest disclosing manager, who is the one most likely to make such a deviation under reasonable out-of-equilibrium beliefs (in the sense of the D1 criterion). Thus, she has no incentives to incur additional biasing costs to distinguish herself from (non-existing) lower reporting types. As the equilibrium report of the disclosure threshold is unbiased, the disclosure threshold can be derived by comparing the utility of the manager in the absence of disclosure to her utility upon truthful disclosure of the disclosure threshold. The managerial biasing option thus has no effect whatsoever on the disclosure threshold.<sup>15</sup> This yields the empirical prediction that managers' incentives to voluntarily disclose their private information are not expected to be affected by (i) the extent to which they can misreport their information; (ii) the costs of misreporting; and (iii) regulatory changes in these costs.

While the results extracted from conventional models of voluntary disclosure are completely insensitive to relaxing the standard truthful disclosure assumption, the results drawn

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<sup>15</sup> This result holds only when biasing the report is costly. In the edge case where the biasing costs  $c$  converge to zero, the managerial biasing option precludes any voluntary disclosure and the equilibrium disclosure threshold  $s_0$  converges to  $+\infty$ . When the reporting bias is costless, any disclosing manager will set a bias of  $+\infty$ , rendering the reporting uninformative and making the non-disclosure option preferable for all informed managers because it saves on the disclosure cost  $\lambda$ .

from conventional models that consider reporting bias seem to be slightly sensitive to relaxing the mandatory disclosure assumption. Unlike the constant reporting bias  $b^{B2}$  obtained in the benchmark of mandatory disclosure, Proposition 3 yields an equilibrium reporting bias function  $b_0(s)$  that monotonically increases in the true information content underlying the report. The deviation from the benchmark constant reporting bias emerges because the relaxation of the mandatory disclosure assumption truncates the range of the realizations of the manager's private signal  $\tilde{s}$  for which disclosure occurs. While the range of the disclosed information realizations under mandatory disclosure is unbounded, it becomes bounded from below by the threshold  $s_0$  when disclosure is assumed to be voluntary. The truncation of the range of the disclosed information realizations is such that the disclosure threshold constitutes the worst information realization that the manager is willing to disclose. Therefore, when the actual information realization equals the disclosure threshold, the manager provides an unbiased report. The reports of higher information realizations must contain a positive reporting bias that gradually increases in the information realization, because otherwise it would be beneficial to mimic the report of a certain information realization even when the actual information realization is slightly lower. This result, which follows from the separating equilibrium conditions, is in contrast to the usual thinking that worse information enhances managerial misreporting incentives. Interestingly, although the constant reporting bias  $b^{B2}$  implied by models that consider reporting bias under mandatory disclosure is not robust to relaxing the mandatory disclosure assumption, it nevertheless appears to withstand in the limit as the asymptote to which the equilibrium reporting bias function approaches. Moreover, the equilibrium biasing function presented in Proposition 3 is consistent with that implied by models that consider mandatory disclosure of information originally drawn from a support bounded from below.

The monotonically increasing biasing function implies that the investors are still capable of perfectly inferring the manager’s private information from her report, as in the benchmark of mandatory disclosure. However, while the equilibrium market price is an increasing function of the manager’s report, the non-linearity of the biasing function results in a market pricing rule that is not linear in the manager’s report (even though it is linear in the true information inferred from the report due to the normal distribution assumption). This is in contrast to the benchmark of mandatory disclosure, where the constant reporting bias implies a market pricing rule that is linear in the manager’s report. Our analysis, therefore, alludes to a possible misspecification in applying linear empirical frameworks to depict the relationship between voluntarily disclosed information and stock prices.

To complete the analysis, we consider the knife-edge case where the model yields equilibrium with full disclosure. Our model contains two factors that are well-known in the literature as suppressing voluntary disclosure: disclosure costs as in Verrecchia (1983) and uncertainty of investors about the manager’s information endowment as in Dye (1985). These two factors vanish when  $\lambda$  converges to 0 (implying costless disclosure) and  $\tau$  converges to 1 (implying that the manager is surely informed). Corollary 4 presents the shape of the manager’s equilibrium disclosure strategy in this knife-edge case.

**Corollary 4.** *When  $\lambda$  converges to 0 and  $\tau$  converges to 1, the equilibrium disclosure threshold  $s_0$  converges to  $-\infty$  and the equilibrium reporting bias  $b_0(s)$  equals  $b^{B2}$  for any  $s \in \mathfrak{R}$ .*

The setting of Corollary 4 yields a fully-separating equilibrium where the manager always chooses to provide (invertible) biased disclosure. We thus conclude that the full disclosure result of Grossman and Hart (1980), Grossman (1981) and Milgrom (1981) is also robust to relaxing the truthful disclosure assumption. As full voluntary disclosure is equivalent

to mandatory disclosure, the equilibrium reporting bias in the edge case of Corollary 4 exactly coincides with the benchmark constant bias  $b^{B2}$ . When  $\tau$  converges to 1, but  $\lambda > 0$ , the equilibrium coincides with that of Verrecchia (1983). Similarly, when  $\lambda$  converges to 0, but  $\tau < 1$ , the equilibrium coincides with that of Dye (1985).

The equilibrium presented in Proposition 3 and Corollary 4 is the unique equilibrium in our model, which is supported by reasonable out-of-equilibrium beliefs in the sense of the D1 criterion of Cho and Krep (1987). Besides this equilibrium, the model yields many other equilibria, some of which are separating upon disclosure, but all of them fail to satisfy the D1 criterion. Corollary 5 presents the multiple separating-upon-disclosure equilibria that the model yields when the out-of-equilibrium beliefs are not restricted to be reasonable.

**Corollary 5.** *When the out-of-equilibrium beliefs are unrestricted, the model yields multiple perfect Bayesian equilibria with pure strategies that are separating upon disclosure. Any scalar  $k \in [0, b^{B2}]$  is associated with an equilibrium  $e_k = (D_k, \hat{D}_k : T \times \mathfrak{R} \rightarrow A, P_k, \hat{P}_k : A \rightarrow \mathfrak{R})$ , which is characterized by a disclosure threshold  $s_k \in \mathfrak{R}$  and an increasing biasing function*

$b_k : [s_k, +\infty) \rightarrow \mathfrak{R}$ , *satisfying for any  $s \in \mathfrak{R}$  and  $r \geq s_k + k$ :  $D_k(0, s) = \hat{D}_k(0, s) = \phi$ ,*

$$D_k(1, s) = \hat{D}_k(1, s) = \begin{cases} \phi & \text{if } s < s_k \\ s + b_k(s) & \text{otherwise} \end{cases},$$

$$P_k(\phi) = \hat{P}_k(\phi) = \frac{1 - \tau}{1 - \tau \text{prob}(\tilde{s} \geq s_k)} \mu + \frac{\tau \text{prob}(\tilde{s} \leq s_k)}{1 - \tau \text{prob}(\tilde{s} \geq s_k)} E[\tilde{v} | \tilde{s} \leq s_k] \text{ and}$$

$P_k(r) = \hat{P}_k(r) = E[\tilde{v} | \tilde{s} = f_k^{-1}(r)]$  *where  $f_k(s) = s + b_k(s)$ . The disclosure threshold  $s_k$  is*

*strictly increasing in  $k$ , where  $s_0$  equals  $s^{B1}$ . For any  $k \in [0, b^{B2})$ , the reporting bias  $b_k(s)$  reaches the minimal value of  $k$  at  $s = s_k$ , increases in  $s$  for any  $s \geq s_k$ , and approaches the asymptote  $b^{B2}$  when  $s$  converges to  $+\infty$ . When  $k = b^{B2}$ , the reporting bias is always  $b^{B2}$ .*

As indicated by Corollary 5, when the out-of-equilibrium beliefs are not restricted to be reasonable, the model yields a spectrum of equilibria  $\{e_k\}_{k \in [0, b^{B^2}]}$ , which are separating upon disclosure. All these equilibria, except for  $e_0$ , fail to satisfy the D1 criterion. Hence, equilibrium  $e_0$  is supported by the widest set of out-of-equilibrium beliefs, and in this sense it is the most reasonable equilibrium that our model yields. The multiple equilibria presented in Corollary 5 are graphically illustrated in Figure 3. They all follow an upper-tailed disclosure strategy, where the disclosure threshold varies over the bounded range  $[s_0, s_{b^{B^2}}]$  and the reporting bias at the disclosure threshold varies over the bounded range  $[0, b^{B^2}]$ . The equilibrium presented in Proposition 3,  $e_0$ , is the most informative one, as it is associated with the lowest disclosure threshold. It is not necessarily, however, the most efficient one, because it involves more disclosure and thus might be associated with more biasing costs. The equilibrium  $e_{b^{B^2}}$  is characterized by linear strategies – a constant reporting bias of  $b^{B^2}$  and a linear pricing rule. All other equilibria constitute a continuum of increasing, concave and upper-bounded (by the asymptote  $b^{B^2}$ ) bias functions and non-linear pricing functions.

[FIGURE 3]

Korn (2004) studies a model that is similar to the edge case where  $\lambda = 0$  and  $\tau = 1$  in our model (as in Corollary 4), but her focus is the equilibrium with a constant reporting bias and a pricing rule that is linear in the manager's report, which is associated with partial disclosure. In a recent working paper, Kwon, Newman and Zang (2009) analyze the interaction between mandatory and voluntary disclosure using a setting that is similar to the case  $\tau = 1$  in our model, but they also restrict their analysis to an equilibrium with a constant reporting bias and a pricing rule that is linear in the manager's report, and thus get partial disclosure with a disclosure threshold that is strictly higher than the one drawn from traditional models of voluntary

disclosure. The equilibrium analyzed by Korn (2004) and Kwon, Newman and Zang (2009), which is captured in our analysis by the linear equilibrium  $e_{b^{B2}}$ , do not survive when the out-of-equilibrium beliefs are restricted to be reasonable using standard refinement methods. We argue that the equilibrium presented in Proposition 3,  $e_0$ , is the only one that is comparable with the earlier results of the disclosure theory, because most seminal results in the disclosure literature have been (directly or indirectly) obtained under similar refinement methods.

#### **4. *Summary and Conclusions***

Extant theory on corporate voluntary disclosure, which focuses on managerial discretion in deciding whether or not to provide disclosure to the capital market investors, is mostly drawn from the analysis of truthful reporting settings. Truthful reporting is typically justified by the potential litigation costs and reputation erosion costs associated with misreporting. While the truthful reporting assumption seems descriptive of situations where managerial disclosures pertain to past events that may be verifiable to a large extent, it is more difficult to ascribe credibility to statements of management's beliefs and intentions regarding future events. The reliance on the standard truthful reporting assumption is therefore regarded by researchers as a significant weakness of voluntary disclosure theory that generates substantial concerns regarding its validity and robustness. For example, in his comprehensive survey of the voluntary disclosure literature, Verrecchia (2001) alludes to the reliance on the truthful reporting assumption as one of the deficiencies of the voluntary disclosure literature. Although there is also an extensive body of theoretical research on managerial biases and manipulations in disclosing information that cannot be credibly reported, this line of research is mostly based on models that assume that disclosure is mandatory. The prevalent theoretical perception of managerial disclosure decisions thus derives from analyzing the managerial option of refraining from reporting separately from the managerial option of misreporting.

By combining the two layers of managerial disclosure discretion and analyzing the effect of their interaction on the optimal disclosure behavior of managers, our paper offers a bridge that connects two important branches of disclosure theory, tying the voluntary disclosure literature to the literature on reporting bias. The analysis demonstrates that the interaction between the managerial option of misreporting and the managerial option of refraining from disclosure has no effect whatsoever on voluntary disclosure decisions of managers, but it affects, to some extent, the strategy that managers adopt in biasing their reporting. We thus contribute to disclosure theory by reinforcing the robustness of voluntary disclosure theory to the conventional assumption of truthful reporting and in exploring the extent to which the extant theory on reporting bias is sensitive to the conventional assumption of mandatory disclosure.



## Appendix – Proofs

The appendix presents the proofs of Observation 1, Observation 2, Proposition 3, Corollary 4, and Corollary 5. The proofs are based on two lemmata that are stated and proved below.

**Lemma A.** Let  $F(\pi, x) = E[\tilde{v}|\tilde{s} = x] - E[\tilde{v}|\tilde{s} \leq x] + \frac{1-\pi}{1-\pi \text{prob}(\tilde{s} \geq x)} (E[\tilde{v}|\tilde{s} \leq x] - \mu)$  for any

$\pi \in (0,1]$  and  $\square$ . Then,  $F(\pi, x)$  is increasing in  $\square$  and in  $\pi$ , where  $\lim_{x \rightarrow -\infty} F(\pi, x) = -\infty$  for

any  $\pi \in (0,1)$ ,  $\lim_{x \rightarrow -\infty} F(1, x) = 0$  and  $\lim_{x \rightarrow +\infty} F(\pi, x) = +\infty$  for any  $\pi \in (0,1]$ .

**Proof of the Lemma A.** Utilizing the properties of the normal distribution, and using the

notation  $\hat{x} = \frac{x - \mu}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}}$ , we can substitute  $E[\tilde{v}|\tilde{s} = x] = \mu + \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \cdot \hat{x}$ ,

$E[\tilde{v}|\tilde{s} \leq x] = \mu - \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \cdot \frac{z(\hat{x})}{\Phi(\hat{x})}$  and get  $F(\pi, x) = \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \left( \hat{x} + \frac{\pi z(\hat{x})}{1 - \pi(1 - \Phi(\hat{x}))} \right)$ , where  $z$

and  $\Phi$  are, respectively, the probability density function and the cumulative distribution

function for a standard normal variable. Using  $z'(\hat{x}) = -\hat{x}z(\hat{x})$  and  $\Phi'(\hat{x}) = z(\hat{x})$ , we get

$\frac{\partial F}{\partial x} = \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \left( 1 - \frac{\pi z(\hat{x})}{1 - \pi + \pi \Phi(\hat{x})} \left( \hat{x} + \frac{\pi z(\hat{x})}{1 - \pi + \pi \Phi(\hat{x})} \right) \right)$ . It thus follows that  $\frac{\partial F}{\partial x} > 0$  for any

$x \in \mathfrak{R}$  such that  $x + \frac{\pi z(\hat{x})}{1 - \pi + \pi \Phi(\hat{x})} \leq 0$ . It also follows that

$\frac{\partial F}{\partial x} > \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \left( 1 - \frac{z(\hat{x})}{\Phi(\hat{x})} \left( \hat{x} + \frac{z(\hat{x})}{\Phi(\hat{x})} \right) \right)$  for  $x + \frac{\pi z(\hat{x})}{1 - \pi + \pi \Phi(\hat{x})} > 0$ , so using Sampford's (1953)

inequality,  $0 < \frac{z(\hat{x})}{\Phi(\hat{x})} \left( \hat{x} + \frac{z(\hat{x})}{\Phi(\hat{x})} \right) < 1$ , we get  $\frac{\partial F}{\partial x} > 0$ . Also,

$\frac{\partial F}{\partial \pi} = \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \cdot \frac{z(\hat{x})}{(1 - \pi + \pi \Phi(\hat{x}))^2} > 0$ . Using  $\lim_{\hat{x} \rightarrow -\infty} z(\hat{x}) = \lim_{\hat{x} \rightarrow +\infty} z(\hat{x}) = \lim_{\hat{x} \rightarrow -\infty} \Phi(\hat{x}) = 0$  and

$\lim_{\hat{x} \rightarrow +\infty} \Phi(\hat{x}) = 1$ , we get  $\lim_{x \rightarrow -\infty} F(\pi, x) = -\infty$  for any  $\pi \in (0,1)$  and  $\lim_{x \rightarrow +\infty} F(\pi, x) = \infty$  for any  $\pi \in (0,1]$ .

Lastly, using l'Hopital's rule repeatedly, we get

$$\lim_{x \rightarrow -\infty} x\Phi(x) = \lim_{x \rightarrow -\infty} \frac{\Phi(x)}{1/x} = \lim_{x \rightarrow -\infty} \frac{z(x)}{-1/x^2} = \lim_{x \rightarrow -\infty} \frac{x^2}{-1/z(x)} = \lim_{x \rightarrow -\infty} \frac{2x}{z'(x)/z^2(x)} = \lim_{x \rightarrow -\infty} -2z(x) = 0 \text{ and}$$

$$\lim_{x \rightarrow -\infty} \frac{x\Phi(x) + z(x)}{\Phi(x)} = \lim_{x \rightarrow -\infty} \frac{\Phi(x) + xz(x) + z'(x)}{z(x)} = \lim_{x \rightarrow -\infty} \frac{\Phi(x)}{z(x)} = \lim_{x \rightarrow -\infty} \frac{z(x)}{z'(x)} = \lim_{x \rightarrow -\infty} -\frac{1}{x} = 0, \text{ so}$$

$$\lim_{x \rightarrow -\infty} F(1, x) = \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_\varepsilon^2}} \lim_{x \rightarrow -\infty} \frac{x\Phi(x) + z(x)}{\Phi(x)} = 0. \text{ QED.}$$

**Lemma B.** Let  $G(x) = -x - \left(2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2}\right)^{-1} \ln\left(1 - 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} x\right)$  for any  $x < \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ .

Then,  $G(x)$  is decreasing in  $x$  for negative values of  $x$ , reaching a minimum of zero at  $x = 0$ ,

and then increasing in  $x$  for positive values of  $x$ , where  $\lim_{x \rightarrow \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}} G(x) = +\infty$ .

**Proof of the Lemma B.** The function  $G(x)$  is defined only for  $x < \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ . Also,

$$\frac{dG}{dx} = 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} x \left(1 - 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} x\right)^{-1}. \text{ Hence, } \frac{dG}{dx} \text{ is negative when } x < 0, \text{ it is zero when}$$

$x = 0$ , and it is positive when  $0 < x < \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ . Lastly,  $G(0) = 0$  and

$$\lim_{x \rightarrow \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}} G(x) = -\frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)} - \left(2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2}\right)^{-1} \lim_{y \rightarrow 0} \ln(y) = +\infty. \text{ QED.}$$

**Proof of Observation 1.** Due to the truthful reporting assumption, the manager's report upon disclosure equals the true realization of her private signal  $\tilde{s}$ , and thus the firm price upon disclosure with a report  $r \in \mathfrak{R}$  equals  $P^{B1}(r) = E[\tilde{v} | \tilde{s} = r]$ . The positive correlation between the

firm's value  $\tilde{v}$  and the signal  $\tilde{s}$  implies that the price  $P^{B1}(r) = E[\tilde{v} | \tilde{s} = r]$  upon disclosure is an increasing function of the report  $r$ , which equals the true realization  $s$  of the signal  $\tilde{s}$ . Also, the price  $P^{B1}(\phi) = E[\tilde{v} | \hat{D}^{B1}(\tilde{t}, \tilde{s}) = \phi]$  in the absence of disclosure is independent of the actual realization  $s$  of the signal  $\tilde{s}$ . As this shape of the market price rule is rationally inferred by the manager in equilibrium ( $\hat{P}^{B1} = P^{B1}$ ), the disclosure strategy of an informed manager must be an upper-tailed strategy with a minimal threshold  $s^{B1}$ .

As the investors also rationally infer the disclosure strategy of the manager in equilibrium ( $\hat{D}^{B1} = D^{B1}$ ),  $prob(\tilde{s} \geq s^{B1})$  is the investors' estimate of the probability that an informed manager will provide disclosure. Their estimate of the probability that the manager is informed is  $\tau$ . Hence, they attribute a probability  $\tau prob(\tilde{s} \geq s^{B1})$  to disclosure occurrence and a probability  $1 - \tau prob(\tilde{s} \geq s^{B1})$  to its absence. Conditioned on the absence of disclosure,

$\frac{1 - \tau}{1 - \tau prob(\tilde{s} \geq s^{B1})}$  is the probability that the investors attribute to the manager being

uninformed, while  $\frac{\tau (1 - prob(\tilde{s} \geq s^{B1}))}{1 - \tau prob(\tilde{s} \geq s^{B1})}$  is the probability that they attribute to the manager

being informed and choosing to withhold her information. Thus, the firm price in the absence of

disclosure is  $P^{B1}(\phi) = E[\tilde{v} | \hat{D}^{B1}(\tilde{t}, \tilde{s}) = \phi] = \frac{1 - \tau}{1 - \tau prob(\tilde{s} \geq s^{B1})} \mu + \frac{\tau (1 - prob(\tilde{s} \geq s^{B1}))}{1 - \tau prob(\tilde{s} \geq s^{B1})} E[\tilde{v} | \tilde{s} \leq s^{B1}]$ .

As disclosure is truthful, the only cost associated with disclosure is  $\lambda$ . So, the manager's utility upon disclosure of  $r$  is  $P^{B1}(r) - \lambda = E[\tilde{v} | \tilde{s} = r] - \lambda$  for any report  $r \in \mathfrak{R}$ . Her utility in the

absence of disclosure is  $P^{B1}(\phi) = \frac{1 - \tau}{1 - \tau prob(\tilde{s} \geq s^{B1})} \mu + \frac{\tau (1 - prob(\tilde{s} \geq s^{B1}))}{1 - \tau prob(\tilde{s} \geq s^{B1})} E[\tilde{v} | \tilde{s} \leq s^{B1}]$ .

When the realization of her private signal  $\tilde{s}$  equals the threshold  $s^{B1}$ , the informed manager is indifferent between truthfully disclosing or withholding information. This yields the equation

$$E[\tilde{v}|\tilde{s} = s^{B1}] - \lambda = \frac{1 - \tau}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} \mu + \frac{\tau (1 - \text{prob}(\tilde{s} \geq s^{B1}))}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} E[\tilde{v}|\tilde{s} \leq s^{B1}], \text{ which can be rewritten}$$

$$\text{as } E[\tilde{v}|\tilde{s} = s^{B1}] - E[\tilde{v}|\tilde{s} \leq s^{B1}] + \frac{1 - \tau}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} (E[\tilde{v}|\tilde{s} \leq s^{B1}] - \mu) = \lambda \text{ or}$$

$$F(\tau, s^{B1}) = \lambda. \quad (1)$$

By Lemma A, the left side of equation (1) is monotonically increasing in  $s^{B1}$ , converging to  $-\infty$  when  $s^{B1}$  converges to  $-\infty$  and converging to  $+\infty$  when  $s^{B1}$  converges to  $+\infty$ . The right side of the equation is a positive constant, which is independent of  $s^{B1}$ . Therefore, there is only one value of  $s^{B1}$  that solves equation (1). The unique solution  $s^{B1}$  of equation (1) implies the

$$\text{following equilibrium: } D^{B1}(0, s) = \hat{D}^{B1}(0, s) = \phi, \quad D^{B1}(1, s) = \hat{D}^{B1}(1, s) = \begin{cases} \phi & \text{if } s < s^{B1} \\ s & \text{otherwise} \end{cases},$$

$$P^{B1}(\phi) = \hat{P}^{B1}(\phi) = \frac{1 - \tau}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} \mu + \frac{\tau \text{prob}(\tilde{s} \leq s^{B1})}{1 - \tau \text{prob}(\tilde{s} \geq s^{B1})} E[\tilde{v}|\tilde{s} \leq s^{B1}] \text{ and}$$

$$P^{B1}(r) = \hat{P}^{B1}(r) = E[\tilde{v}|\tilde{s} = r] \text{ for any } s \in \mathfrak{R} \text{ and } r \geq s^{B1}. \text{ Since the left side of equation (1) is}$$

increasing in  $\tau$  by Lemma A, whereas the right side of the equation is increasing in  $\lambda$ , the disclosure threshold  $s^{B1}$  is increasing in  $\lambda$  and decreasing in  $\tau$ .

When  $\lambda$  converges to 0 and  $\tau$  converges to 1, equation (1) is reduced to

$$F(1, s^{B1}) = 0. \quad (2)$$

Again using Lemma A, the left side of equation (2) is increasing in  $s^{B1}$ , converging to 0 when  $s^{B1}$  converges to  $-\infty$  and converging to  $+\infty$  when  $t$  converges to  $+\infty$ . Hence, the one and only solution of equation (2) is  $s^{B1} = -\infty$ , implying full disclosure. QED.

**Proof of Observation 2.** As in monotonic signaling games, the D1 criterion filters out all pooling and partially-separating equilibria. Under the D1 criterion, a pooling region, which contains different information realizations that result in the same report, cannot survive because

a manager with the highest information realization in the pooling region is better off deviating and providing a slightly higher report. We thus look for fully-separating equilibria, where the market can adjust for the manager's bias, so the market price of the firm equals

$$P^{BI}(s + b(s)) = E[\tilde{v} | \tilde{s} = s] = \frac{\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \mu + \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot s, \text{ where } b(s) \text{ is the reporting bias for a}$$

realization  $s$  of the signal  $\tilde{s}$ . The biasing function  $b: \mathfrak{R} \rightarrow \mathfrak{R}$  satisfies two conditions for any  $s \in \mathfrak{R}$  and  $\Delta s > 0$ . Denoting  $\Delta b = b(s + \Delta s) - b(s)$ , the first condition is

$$\frac{\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \mu + \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot s - cb(s)^2 \geq \frac{\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \mu + \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot (s + \Delta s) - c(b(s) + \Delta s + \Delta b)^2,$$

implying that it is not beneficial for a manager with an information realization  $s$  to mimic the report of a manager with an information realization  $s + \Delta s$  by choosing a bias of  $b(s) + \Delta s + \Delta b$ .

The second condition is

$$\frac{\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \mu + \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot (s + \Delta s) - c(b(s) + \Delta b)^2 \geq \frac{\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \mu + \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot s - c(b(s) - \Delta s)^2,$$

implying that it is not beneficial for a manager with an information realization  $s + \Delta s$  to mimic the report of a manager with an information realization  $s$  by choosing a bias of  $b(s) - \Delta s$ .

After rearranging that first and second conditions, we get

$$\frac{\Delta b}{\Delta s} \geq \frac{\sqrt{\frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \Delta s / c + b(s)^2} - b(s)}{\Delta s} - 1 \text{ and } \frac{\Delta b}{\Delta s} \leq \frac{\sqrt{\frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \Delta s / c + (b(s) - \Delta s)^2} - b(s)}{\Delta s},$$

respectively. Using l'Hopital's rule,  $\lim_{\Delta s \rightarrow 0} \frac{\sqrt{\frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \Delta s / c + b(s)^2} - b(s)}{\Delta s} - 1$  equals

$$\lim_{\Delta s \rightarrow 0} \frac{\frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)} \left( \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \Delta s / c + b(s)^2 \right)^{-1/2}}{1} - 1 \text{ or } \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)b(s)} - 1. \text{ Again using}$$

l'Hopital's rule,  $\lim_{\Delta s \rightarrow 0} \frac{\sqrt{\frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \Delta s / c + (b(s) - \Delta s)^2} - b(s)}{\Delta s}$  equals

$$\lim_{\Delta s \rightarrow 0} \frac{\frac{1}{2} \left( \frac{\sigma^2}{c(\sigma^2 + \sigma_\varepsilon^2)} - 2(b(s) - \Delta s) \right) \left( \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2} \cdot \Delta s / c + (b(s) - \Delta s)^2 \right)^{-1/2}}{1}, \text{ which is reduced to}$$

$$\frac{1}{2} \left( \frac{\sigma^2}{c(\sigma^2 + \sigma_\varepsilon^2)} - 2b(s) \right) / b(s) \text{ or } \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)b(s)} - 1. \text{ We thus conclude that}$$

$$\frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)b(s)} - 1 \leq \frac{db}{ds} \leq \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)b(s)} - 1, \text{ implying the differential equation}$$

$$\frac{db}{ds} = \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)b(s)} - 1. \quad (3)$$

To derive the biasing function  $b : \mathfrak{R} \rightarrow \mathfrak{R}$ , we now need to solve equation (3). One solution of

the differential equation is the constant biasing function  $b(s) = \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$  for any  $s \in \mathfrak{R}$ .

Other solutions to the differential equation (3), where  $\frac{db}{ds} \neq 0$ , satisfy the equation

$$\left( \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)b} - 1 \right)^{-1} db = ds \text{ or } 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} b \left( 1 - 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} b \right)^{-1} db = ds. \text{ This implies}$$

$$\int 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} b \left( 1 - 2c \frac{\sigma^2 + \sigma_\varepsilon^2}{\sigma^2} b \right)^{-1} db = \int ds. \text{ So, the biasing function } b(s) \text{ is defined for any}$$

$s \in \mathfrak{R}$  by the implicit equation

$$G(b(s)) = s + \omega, \quad (4)$$

where  $\omega$  could be any scalar. By Lemma B, the function  $G(x)$ , which is defined only for

$x < \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ , gets only non-negative values. So, there is no solution  $b(s)$  to equation (4)

for any  $s < -\omega$ . It follows that the constant biasing function, where  $b(s) = \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$  for

any  $s \in \mathfrak{R}$ , is the only biasing function that exists in equilibrium. The constant reporting bias

$\frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ , denoted  $b^{B2}$ , implies the following fully revealing equilibrium

$$D^{B2}(0, s) = \hat{D}^{B2}(0, s) = \phi, \quad D^{B2}(1, s) = \hat{D}^{B2}(1, s) = s + b^{B2}, \quad P^{B2}(\phi) = \hat{P}^{B2}(\phi) = \mu \text{ and}$$

$$P^{B2}(r) = \hat{P}^{B2}(r) = E[\tilde{v} | \tilde{s} = r - b^{B2}] \text{ for any } s, r \in \mathfrak{R}. \quad \text{QED.}$$

**Proof of Proposition 3.** We first show that the equilibrium disclosure strategy takes the upper-

tailed shape, so that the manager provides disclosure only when the realization of her private

signal is sufficiently high. Suppose by contradiction that there exist  $s_1, s_2 \in \mathfrak{R}$ , such that

$$s_1 < s_2, \quad D(1, s_1) = s_1 + b_1 \text{ and } D(1, s_2) = \phi \text{ where } b_1 \geq 0 \text{ and } s_2 - s_1 \leq b_1. \text{ This implies that}$$

$$P(\phi) \leq P(s_1 + b_1) - \lambda - cb_1^2 \text{ and } P(\phi) \geq \max_{b_2 \in \mathfrak{R}} \{P(s_2 + b_2) - \lambda - cb_2^2\}. \text{ Consequently,}$$

$$\max_{b_2 \in \mathfrak{R}} \{P(s_2 + b_2) - \lambda - cb_2^2\} \leq P(\phi) \leq P(s_1 + b_1) - \lambda - cb_1^2. \text{ However, since } s_1 < s_2, \text{ we get}$$

for  $b_2 = b_1 - (s_2 - s_1)$  that  $P(s_2 + b_2) = P(s_1 + b_1)$  and  $cb_2^2 < cb_1^2$ , which implies

$$P(s_2 + b_2) - \lambda - cb_2^2 > P(s_1 + b_1) - \lambda - cb_1^2 \text{ - a contradiction. The disclosure strategy is thus}$$

characterized by a threshold  $s_0 \in \mathfrak{R}$ , such that  $D(1, s) \neq \phi$  iff  $s \geq s_0$ . Using the same

arguments as in Observation 2, the D1 criterion allows us to focus on equilibria that are

separating upon disclosure, where any two different information realizations that belong to the

disclosure region are associated with two different reports.

We next show that the equilibrium reporting bias at the threshold  $s_0$  is zero. That is,  $b(s_0) = 0$ .

Suppose by contradiction that  $b(s_0) > 0$ . At the threshold  $s_0$ , the informed manager is

indifferent between providing disclosure and keeping quiet, so the manager's utility

$$P(s_0 + b(s_0)) - \lambda - cb(s_0)^2 \text{ upon disclosure equals her utility } P(\phi) \text{ in the absence of disclosure.}$$

Hence,  $E[\tilde{v}|\tilde{s} = s_0] - \lambda - cb(s_0)^2 = P(\phi)$ . For any  $0 < \varepsilon < b(s_0)$ , there exists  $\delta(\varepsilon) > 0$ , such that  $E[\tilde{v}|\tilde{s} = s_0 - \delta(\varepsilon)] - \lambda - c(b(s_0) - \varepsilon)^2 = P(\phi)$ , where  $\delta(\varepsilon)$  is strictly increasing in  $\varepsilon$ .

Therefore, a manager with information realization of  $s_0$  is better off deviating to the out-of-equilibrium report  $s_0 + b(s_0) - \varepsilon$  (where  $0 < \varepsilon < b(s_0)$ ) for any market response

$P(s_0 + b(s_0) - \varepsilon)$  that exceeds  $E[\tilde{v}|\tilde{s} = s_0 - \delta(\varepsilon)]$ . Managers with a slightly lower information realization  $s_0 - \Delta$ , such that  $0 < \Delta < \varepsilon$ , are better off or at least no worse off deviating to the out-of-equilibrium report  $s_0 + b(s_0) - \varepsilon$  only for higher market responses that equal or exceed  $E[\tilde{v}|\tilde{s} = s_0 - \delta(\varepsilon - \Delta)]$ , where  $\delta(\varepsilon - \Delta) < \delta(\varepsilon)$ . Managers with an even lower information realization  $s_0 - \Delta$ , such that  $\Delta \geq \varepsilon$ , are better off or at least no worse off providing the out-of-equilibrium report  $s_0 + b(s_0) - \varepsilon$  only for much higher market reactions (at the level of  $E[\tilde{v}|\tilde{s} = s_0]$  or even higher). Hence, a manager with information realization  $s_0$  is more likely to make a deviation to the out-of-equilibrium report  $s_0 + b(s_0) - \varepsilon$  than managers with lower information realizations. Utilizing the D1 criterion of Cho and Kreps (1987), it follows that the out-of-equilibrium report  $s_0 + b(s_0) - \varepsilon$  must be ascribed by the market to managers with information realization that is at least  $s_0$  for any  $0 < \varepsilon < b(s_0)$ . This implies

$P(s_0 + b(s_0) - \varepsilon) \geq E[\tilde{v}|\tilde{s} = s_0]$ . A contradiction now arises as a manager with an information realization of  $s_0$  can decrease her bias by an amount  $0 < \varepsilon < b(s_0)$  and thereby improve her utility from  $P(\phi)$  to at least  $E[\tilde{v}|\tilde{s} = s_0] - \lambda - c(b(s_0) - \varepsilon)^2$ , which is higher than her equilibrium utility  $E[\tilde{v}|\tilde{s} = s_0] - \lambda - cb(s_0)^2 = P(\phi)$ .

As the reporting bias at the threshold  $s_0$  is zero, we get that the threshold  $s_0$  is the unique solution of equation (5) using the same arguments as in the proof of Observation 1,



$$F(\tau, s_0) = \lambda. \quad (5)$$

This implies that the threshold  $s_0$  equals the benchmark threshold  $s^{B1}$  obtained under the assumption of truthful disclosure as the unique solution of equation (1).

Using the same arguments as in Observation 2, the equilibrium biasing function

$b_0 : [s_0, +\infty) \rightarrow \Re$  upon disclosure satisfies equation (3). It also satisfies  $b_0(s_0) = 0$ . The condition  $b_0(s_0) = 0$  precludes the benchmark constant biasing function. When considering

other solutions to the differential equation (3), where  $\frac{db}{ds} \neq 0$ , and adding the condition

$b_0(s_0) = 0$ , equation (4) is reduced to

$$G(b_0(s)) = s - s_0, \quad (6)$$

where  $s \geq s_0$ . By Lemma B, the left side of equation (6), which is defined only for

$b_0(s) < \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ , gets a minimum of zero at  $b_0(s) = 0$ , and then increases in positive

values of  $b_0(s)$ , converging to  $+\infty$  when  $b_0(s)$  converges to  $\frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$ . The right side of

the equation is non negative for any  $s \geq s_0$ . So, there exists a unique  $b_0(s)$  that solves equation

(6) for any  $s \geq s_0$ . As the right side of the equation is increasing in  $s$ , so is  $b_0(s)$ . The unique

biasing function  $b_0(s)$  is increasing in  $s$ , starting with a zero bias at the threshold  $s_0$  and

converging to the asymptote  $b^{B2} = \frac{\sigma^2}{2c(\sigma^2 + \sigma_\varepsilon^2)}$  when  $s$  converges to  $+\infty$ . QED.

**Proof of Corollary 4.** By the proof of Proposition 3, the threshold  $s_0$  is the unique solution of equation (5). When  $\lambda$  converges to 0 and  $\tau$  converges to 1, equation (5) is reduced to

$$F(1, s_0) = 0. \quad (7)$$

Using Lemma A, the left side of equation (7) is increasing in  $s_0$ , converging to 0 when  $s_0$  converges to  $-\infty$  and converging to  $+\infty$  when  $s_0$  converges to  $+\infty$ . Hence, the one and only solution of equation (7) is  $s_0 = -\infty$ , implying full disclosure. The equilibrium biasing function under full disclosure must equal to that obtained under mandatory disclosure. QED.

**Proof of Corollary 5.** When the out-of-equilibrium beliefs are unrestricted, the bias at the disclosure threshold  $s_0$  is not restricted to be zero. Since the biasing function must satisfy equation (3), and as the function  $G$  is defined only for biases below  $b^{B^2}$ ,  $b(s_0)$  could be any non-negative bias in the range  $[0, b^{B^2}]$ . For any scalar  $k \in [0, b^{B^2}]$ , there exists an equilibrium with a bias of  $k$  at the disclosure threshold. For  $k \in [0, b^{B^2})$ , the implicit equations (1) and (4), which define the disclosure threshold  $s_k$  and the biasing function  $b_k(s)$ , respectively, take the following form after substituting  $b_k(s_k) = k$

$$G(b_k(s)) = s + G(k) - s_k, \quad (8)$$

$$F(\tau, s_k) = \lambda + ck^2. \quad (9)$$

Using the same arguments as in the proof of Proposition 3, equation (8) yields a unique biasing function  $b_k(s)$  and equation (9) yields a unique disclosure threshold  $s_k$  (which is increasing in  $k$ ). For  $k = b^{B^2}$ , we need to replace equation (8) by  $b(s) = b^{B^2}$ . QED.

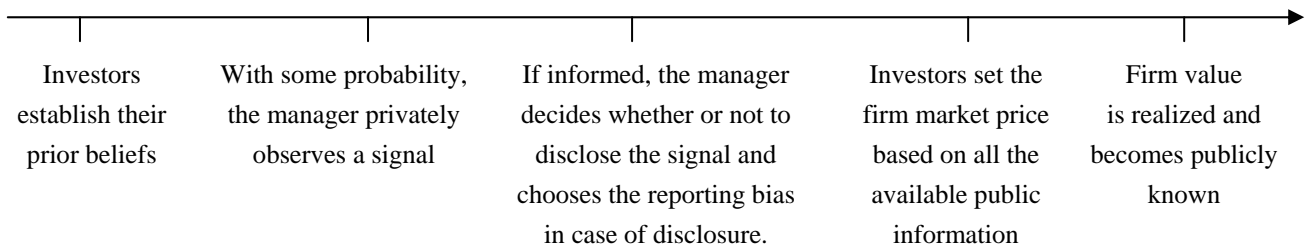
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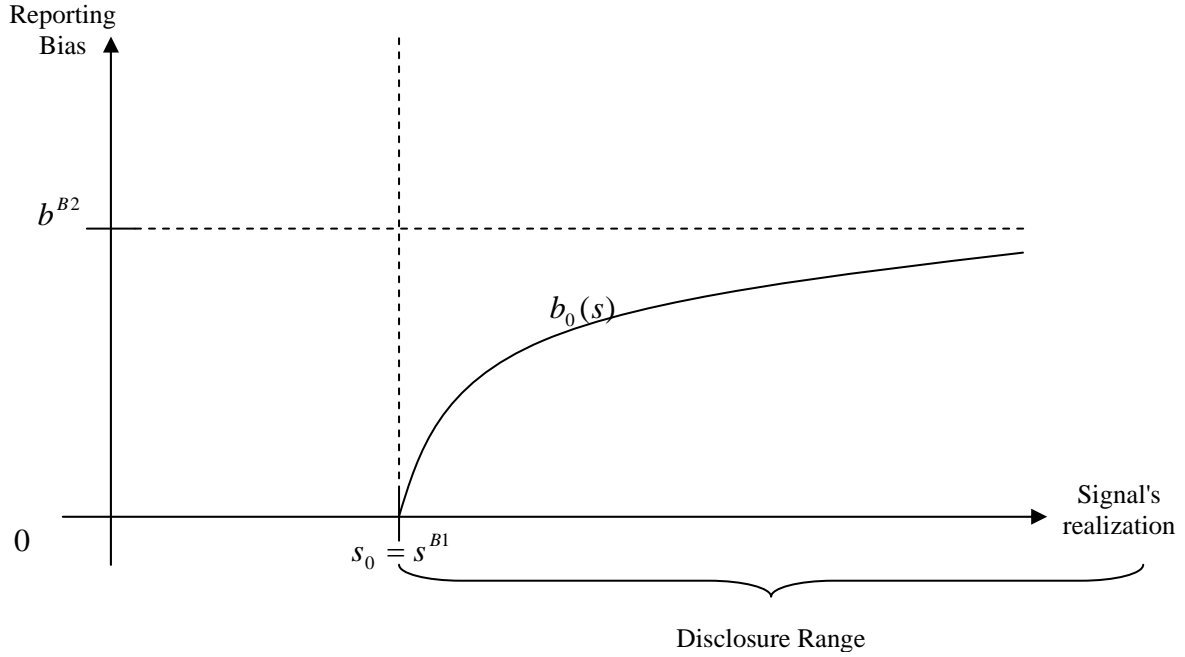
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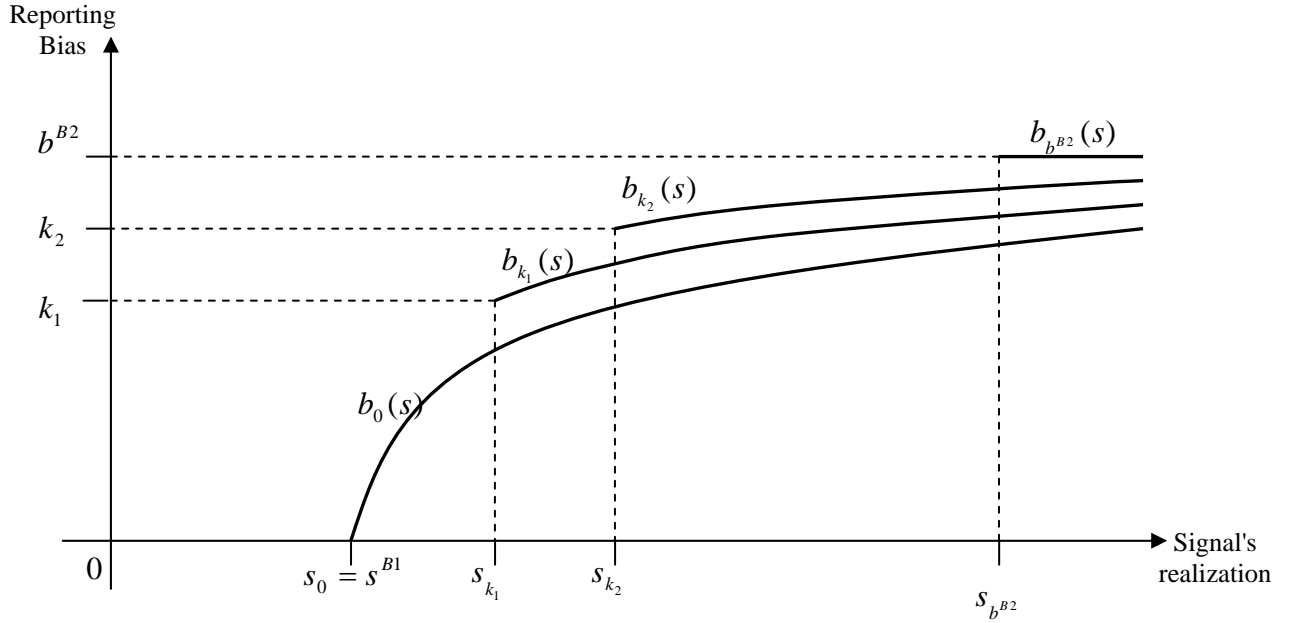
## Figures



**Figure 1.** The figure provides a timeline depicting the sequence of events in the model.



**Figure 2.** The figure illustrates the unique equilibrium disclosure strategy of the manager that the model yields when the out-of-equilibrium beliefs are restricted to be reasonable in the sense of the D1 criterion of Cho and Kreps (1987). The horizontal axis describes the range of all possible realizations of the manager's private signal  $\tilde{s}$ , while the vertical axis describes the range of all possible reporting biases. The disclosure threshold  $s_0 = s^{B1}$  divides the horizontal axis into two ranges. The range above the threshold contains all the realizations of the signal  $\tilde{s}$  which are disclosed by an informed manager. The range below the threshold contains all the realizations of the signal  $\tilde{s}$  for which an informed manager refrains from disclosure. The increasing concave line describes the reporting bias  $b_0(s)$  for any realization  $s$  of the private signal  $\tilde{s}$  that belongs to the disclosure range. At  $s = s_0$ , the reporting bias equals zero (i.e.,  $b_0(s_0) = 0$ ). As the realization  $s$  of the signal  $\tilde{s}$  increases, the manager's reporting bias  $b_0(s)$  increases, approaching the asymptote  $b^{B2}$ .



**Figure 3.** The figure illustrates the spectrum of the equilibrium managerial disclosure strategies that the model yields when the out-of-equilibrium beliefs are unrestricted. The horizontal axis describes the range of all possible realizations of the manager's private signal  $\tilde{s}$ , while the vertical axis describes the range of all possible reporting biases. Each equilibrium is associated with a scalar  $k \in [0, b^{B2}]$ . The figure illustrates four of these equilibria that are associated with the scalars  $0 < k_1 < k_2 < b^{B2}$ . For any  $k = 0, k_1, k_2, b^{B2}$ , the corresponding equilibrium is associated with a disclosure threshold  $s_k$  and a reporting bias function  $b_k(s)$ . The disclosure threshold  $s_k$  divides the horizontal axis into two ranges. The range above the threshold  $s_k$  contains all the realizations of the signal  $\tilde{s}$  which are disclosed by an informed manager. The range below the threshold  $s_k$  contains all the realizations of the signal  $\tilde{s}$  for which an informed manager refrains from disclosure. For  $k = 0, k_1, k_2$ , the reporting bias  $b_k(s)$  equals  $k$  at  $s = s_k$ , and then it increases as the realization  $s$  of the signal  $\tilde{s}$  increases, approaching the asymptote  $b^{b2}$ . For  $k = b^{B2}$ , the reporting bias function  $b_{b^{B2}}(s)$  is the constant  $b^{b2}$ .