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EXUBERANCE AND INVESTMENT BOOMS

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Working Paper No 24/2011

December 2011

**Research No. 03710100**

This paper was partially financed by the Henry Crown Institute of Business Research in Israel.

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# Information Manipulation, Rational Exuberance and Investment Booms<sup>1</sup>

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May 31, 2011

<sup>1</sup>We thank Daron Acemoglu, Mark Armstrong, Ken Arrow, Salvadore Barbera, Ken Binmore, Philip Bond, Andres Caravajal, Michael Fishman, Michael Gallmeyer, Tom George, Itay Goldstein, Roger Guesnerie, Peter Hammond, Milton Harris, Ron Kaniel, Marcus Miller, Dilip Mookherjee, Adriano Rampini, K. Sivaramakrishnan, S. Viswanathan, Myrna Wooders, and seminar participants at University of Houston, Texas A&M University, the Duke-UNC Corporate Finance Conference, the Utah Winter Finance Meetings, the Western Finance Association Meetings, and the Marie Curie Conference (in the honor of Peter Hammond) at the University of Warwick for useful comments or discussions on the issues addressed in this paper.

## Abstract

We present a model of investment booms in rational and frictionless capital markets where there is systematic overinvesting (relative to the efficient capital allocation) in low productivity firms even asymptotically because of strategic information manipulation by privately informed insiders. However, such an allocation is constrained-efficient with respect to the informational imperfections and limited commitment by capital markets regarding investment policies that are inefficient *ex post*. For an open set of parameters investors endogenously get "stuck" at an overoptimistic level of (posterior) expectations on the industry productivity, and the useful information of managers of newly entering firms ceases to get incorporated in their investment decisions, even though there is optimal incentive contracting through optimal wage contracts and renegotiation-proof investment plans. Then, even in the limit, learning on the true industry productivity may not be complete. Our model helps explain in a Bayes-rational framework the historically observed confluence of innovations, strategic information manipulation by insiders, and investment booms that result, in the long run, in industry-wide overcapacity.

**Keywords:** Overinvestment; Information manipulation; Contracting; Learning

**JEL Codes:** G32, D23

# 1 Introduction

Investment in growth opportunities generated by innovations and development of new economic opportunities is central to the evolution of industries and economic growth (Schumpeter, 1942; Romer, 1990). However, such growth opportunities often engender a persistent build-up in investment leading to overinvestment or overcapacity — relative to the efficient capital allocation levels *ex post* — in industries created (or affected) by the innovation. Two prominent recent examples are the investment booms in the telecommunications industry during 1996-2000 (driven by the internet innovation) and in the housing sector in the U.S. during 2002-2006 (driven by financial innovations in the derivatives markets); the former left a glut of fiber-optic capacity, while the latter resulted in over-building of housing stock. Indeed, there is a litany of such apparent conjunctions of new economic opportunities and overinvestment in history. These include the rise and the fall of the South Sea Trading Company in the 18th century; the development of the railroad industries in Britain in 1830s and in the US in the 1860s; and the growth of power Utilities in the U.S. in the 1920s.<sup>1</sup>

These episodes of intense investment activity are often attributed to investor irrationality, driven by “spontaneous optimism” or “animal spirits” (Keynes, 1936) or “irrational exuberance” (Greenspan, 1996) or as an irrational over-reaction to innovations (Shiller, 2000).<sup>2</sup> But a historical analysis also highlights the crucial role of manipulation of investors’ beliefs by self-interested and informed insiders through overly-optimistic representations of financial performance and economic prospects that are often at variance with the actual performance. For example, Sidak (2003, page 207) argues that “WorldCom’s false internet traffic reports and accounting fraud encouraged overinvestment in long-distance capacity and internet backbone capacity [and]... has destroyed billions of dollars of shareholder value in other telecommunications firms.” Similarly, the South Sea Company circulated “...the most extravagant rumors...” (MacKay, 1980, page 54) to attract massive amounts of external capital from public investors. Strategic manipulation of uninformed investors’ beliefs was also

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<sup>1</sup>There is a long literature examining such episodes including Kindleberger (1978), Mackay (1980), and Garber (2001).

<sup>2</sup>For example, Greenspan (1996) asserts that “irrational exuberance” during booms leads investors’ estimation of the expected returns to be “imprudently” high.

rampant in the internet industry in the late 1990's, often abetted by the filing of erroneous financial statements.<sup>3</sup> Such information manipulation is consistent with the view that the problem of asymmetric information between informed insiders and outside investors is aggravated substantially in new industries where capital markets have sparse information and a reliable assessment of economic prospects is difficult.

But models that consider the effects of asymmetric information on investment either conclude that information asymmetries should lead to underinvestment and credit rationing (Myers and Majluf, 1984; Stiglitz and Weiss, 1983; Greenwald, Stiglitz and Weiss, 1984) or argue that information manipulation by informed insiders will have minor investment implications because such strategic disclosures will be discounted by rational market participants (Stein, 1989).<sup>4</sup> And models, such as, Zeira (1987), Rob (1991), and Barbarino and Jovanovic (2007) predict either *underinvestment* relative to the perfect information level or a gradual buildup of investment in which any overcapacity occurs only in the last period, i.e., appears to arise only due to the discrete nature of the model. Finally, Beaudry and Portier (2004) provide a model of Pigouvian cycles where excess investment can occur because agents randomly receive an incorrect positive signal on productivity growth and respond positively to it because these signals are usually precise. But there is no structural explanation there of what generates such “incorrect” signals.

The existing literature therefore does not appear to offer analyses that are consistent with observing long-term overinvestment fueled by possible strategic information manipulation by insiders in an equilibrium framework with rational agents. In particular, overinvestment models based on irrational optimism (or exuberance) or rational behavior based on incorrect signals imply that agents should be enthusiastic and optimistic during the investment buildup and there should be no systematic evidence of malfeasance by informed agents. Yet, historically, during investment buildups there has been considerable *contemporaneous* skepticism expressed of the profit projections by insiders and the wisdom of observed high investment flows (Kindlberger, 1978; Shiller, 2005), and as we noted above, there has been

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<sup>3</sup>It is noteworthy that from January 1997 through June 2002, about 10% of all listed companies announced at least one earnings restatement (Kedia and Philippon, 2005), which on average resulted in a substantial 10% drop in their stock price.

<sup>4</sup>See Stein (2003) for a very useful survey of this literature.

systematic evidence *ex post* of strategic manipulation of market’s beliefs.

Of course, models where strategic information manipulation survives in equilibrium with rational and frictionless capital markets are challenging to construct (cf. Stein, 2003). In this paper, we build on Kumar and Langberg (2009) that presents an equilibrium theory of fraud with rational players based on shareholders’ limited ability to commit *ex ante* to investment policies that are inefficient *ex post* (or after receiving productivity signals from insiders); i.e., inefficient investment policies are not renegotiation-proof (see Bolton and Dewatripont, 2005).<sup>5</sup> In such a situation, inducing truthful information from insiders with low productivity prospects can be very costly (or be incentive inefficient) because it will be used against them in the capital markets. The optimal contract (subject to the commitment constraint) therefore allows low productivity managers to report inflated prospects with a positive probability. But for our purposes the one-shot contracting model of Kumar and Langberg has to be recast to allow dynamic learning and examine the possibility of equilibrium overinvestment even in the long-run.

We examine learning and investment in an industry with unknown capital productivity when there is sequential bilateral contracting between uninformed investors and managers of an infinite sequence of entering firms, when the managers receive private signals on the true industry productivity. Investors provide optimal incentives for informative disclosures through optimal wage contracts and renegotiation-proof investment plans after observing the history of contracting in the industry.<sup>6</sup>

Our principal finding is that for an open set of parameters investors endogenously get “stuck” at an overoptimistic level of (posterior) expectations on the industry productivity and

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<sup>5</sup>In Kumar and Langberg’s (2009) framework the constraint on investment commitment arises endogenously because of the presence of a market for corporate control: transparently inefficient investment policies lead to under-valuation of firms, which can be exploited through a takeover that transfers ownership rights and allows the new owners to make positive expected profits by re-setting the investment policy to the efficient level.

<sup>6</sup>As in models of social learning (Bannerjee, 1992; Bikhchandani et al., 1992; Smith and Sorensen, 1999), individuals in our model face common payoff uncertainty, receive private signals, and sequentially make decisions after observing previous decisions by other informed individuals. But in contrast to these models, we allow communication of private information that is governed through renegotiation-proof bilateral contracts whose design depends on the observed history of contracts and their outcomes. Moreover, our model departs from the literature on dynamic renegotiation-proof contracting with hidden information (Dewatripont, 1988; Laffont and Tirole, 1990; Battaglini, 2007) by examining an infinite sequence of agents with randomly varying but correlated types.

the useful information of managers of newly entering firms ceases to get incorporated in their investment decisions. In particular, with positive probability there is overinvestment (relative to the perfect information efficient level) *even in the limit*: even asymptotically, learning (on the true industry productivity) may not be complete with resultant systematic and long run overinvestment in firms with low productivity and industry-wide overcapacity. Thus, market “exuberance” and industry overcapacity can be consistent with Bayes-rationality and systematic overinvestment can occur even when incentive mechanisms can be designed to elicit information.

The intuition behind our main result is that in designing the optimal contract the net benefits to investors of inducing precise information from managers are negatively related to their productivity expectations; i.e., an optimistic prior on the industry productivity lowers the informational precision of reports from managers with low productivity signals. Indeed, for productivity expectations beyond a threshold level, it is optimal to induce *pooling* (or no information transmission) from low-productivity managers. But in our model productivity expectations vary endogenously over time based on the history of past reports. Thus, a sequence of high productivity reports not only raises the posterior expectations (i.e., generates optimism) but also reduces endogenously the informativeness of managerial disclosures in subsequent periods. In such a situation, the effect of additional high reports on the posterior beliefs progressively weakens and optimistic beliefs are not corrected by the optimal bilateral contracts that govern the new firms entering into the industry.

Our analysis appears to be the first to capture the notion of an *investment boom* — where even less profitable firms can obtain large investment funding — in an equilibrium framework with Bayes-rationality, common priors, frictionless capital markets, and optimal contracting subject to plausible commitment constraints.<sup>7</sup> And we show that such booms can lead to systematic overinvestment even in the long run. Thus, our model helps explain the historically observed confluence of innovations, strategic information manipulation by insiders, and firm-level overinvestment that results, in the long run, in industry-wide overcapacity with

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<sup>7</sup>Somewhat ironically, the contracting friction highlighted in our analysis, namely, renegotiation-proofness constraints on investment, would appear to be intrinsic to developed capital markets with active trading in equity ownership.

respect to the efficient capital allocation in relation to the average productivity of firms. Of course, such overinvestment patterns do not always arise, and we identify conditions on primitives — technology, information structure, preferences, and priors — that determine the *ex ante* likelihood of long run overinvestment versus a relatively quick convergence to the efficient investment levels.

We organize the remaining paper as follows. Section 2 specifies the basic model and defines the equilibrium. Section 3 characterizes information manipulation in the optimal contract. Section 4 analyzes learning dynamics in the long run and Section 5 examines overinvestment along the equilibrium path. Section 6 concludes.

## 2 The Model

We consider an emerging industry built on some technological or financial innovation. At  $t = 0$ , nature moves and determines the capital productivity of the industry, but this move is unobservable. Firms then enter the industry sequentially between  $t = 0$  and  $t = 1$  in countably infinite number of stages  $n = 1, 2, \dots$ . Firm  $n$  (that enters at stage  $n$ ) invests  $k_n$  and obtains output  $y_n$  at  $t = 1$ . That is, outputs of all firms are realized simultaneously at  $t = 1$ ; hence, investment by firms occurs prior to the realization of performance.<sup>8</sup>

For simplicity, output has a binary support, i.e.,  $y_n \in \{0, 1\}$  for all  $n$ , and is stochastically related to the investment ( $k_n$ ) through the firm’s capital productivity (or ‘type’),  $s_n \in \{s_h, s_\ell\}$ ,  $1 > s_h > s_\ell$ . Specifically,  $\Pr(y_n = 1) = 2s_n\sqrt{k_n}$ , for  $k_n \in \mathbf{K} \equiv [0, k^{\max}]$ .<sup>9</sup>

The likelihood of economic success of entering firms is positively related to the underlying (and unknown) industry productivity. For parsimony, we represent the industry productivity by a parameter  $\theta \in \Theta = \{\theta_\ell, \theta_h\}$ ,  $0 < \theta_\ell < \theta_h < 1$  that is chosen (by nature) according to the probability  $\beta_1 = \Pr(\theta = \theta_h)$ , which is common knowledge.<sup>10</sup> Further, we model the

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<sup>8</sup>This time-line is consistent with the observation that when an industry emerges and develops from the introduction of new technologies (e.g., railroads, internet, telecommunications), its firms rely less on hard performance measures, such as earnings or sales, and more on their beliefs regarding the economic prospects of the industry (Gort and Klepper, 1982).

<sup>9</sup>The maximal investment level is set at  $k^{\max} = (2s_h)^{-1}$  to assure that the probability of high output remains bounded above by 1.

<sup>10</sup>Our main results are most easily explicated by assuming that  $\theta$  can take only two values. However, the



positive association between firm-specific productivity ( $s_n$ ) and the industry productivity ( $\theta$ ) simply by assuming that, conditional on  $\theta$ , the sequence of firm types is *i.i.d.* and  $\Pr(s_n = s_h \mid \theta) = \theta$ , for all  $n \geq 1$ .

## 2.1 Ownership and Control

Upon entry, each firm  $n$  is “incorporated” and its shares are traded in a frictionless capital (or equity ownership) market among a continuum of risk-neutral and non-atomic investors indexed by  $z \in Z$ . All investors have identical information sets and a common opportunity cost of investment given by the gross return  $R > 1$ . Hence, for simplicity, and without loss of generality, we label the initial group of shareholders of firm  $n$  as the “owner,”  $X_n \subset Z$ .

However, there is a separation of ownership and control. Each firm  $n$  is controlled by a risk-averse manager,  $M_n$ , who receives two types of utility from managing the firm: Utility from consuming the compensation  $w_n$  that is paid at the time of the firm’s liquidation ( $t = 1$ ) and private benefits from control that increase with the size of the capital assets. The latter induce an agency conflict between the managers and the owners on the optimal allocation of investment or capital for any given productivity state (Stulz, 1990; Hart and Moore, 1995).

We assume the power utility function, i.e.,  $u(w_n) = w_n^{1/\gamma}$ ,  $\gamma > 2$ , and represent the private benefits of control by  $b(k_n) = \psi\sqrt{k_n}$ ,  $\psi > 0$ .<sup>11</sup> Therefore, the expected utility of  $M_n$  is  $E(w_n^{1/\gamma}) + \psi\sqrt{k_n}$ .<sup>12</sup> Managers have no initial wealth and enjoy limited liability; hence, wages must be non-negative. Managers’ reservation utilities are normalized to zero, without loss of generality.<sup>13</sup> The owner’s residual payoffs at the time of liquidation, given an investment  $k_n$ , output  $y_n$ , and wage  $w_n$  are therefore  $v(w_n, k_n, y_n) = y_n - w_n - Rk_n$ .

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results will qualitatively hold for more general distributions of  $\theta$  and these details are available upon request.

<sup>11</sup>While risk aversion requires only  $\gamma > 1$ , the slightly stronger restriction on  $\gamma$  is useful in later analysis to compactly characterize the intensity of the agency conflict between managers and investors. Similarly, while we use the above parameterization of  $b(k)$  for tractability, our results will apply for any benefits function  $b(\cdot)$  that is increasing in the asset size of the firm. Such benefit functions have a special relevance in the financial services industry, where managers’ compensation typically includes a component that is proportional to the (asset) size of the fund.

<sup>12</sup>For notational ease, we suppress the manager’s subjective rate of discount for future consumption.

<sup>13</sup>Our results do not rely on each manager having a zero reservation utility, but require that the managers earn information-based rents in equilibrium. We assume that managers do not trade in the equity market.

## 2.2 Information and Contracting

The true industry productivity parameter  $\theta$  is unobservable to both the investors and managers. But there is learning on  $\theta$  from the sequence of firm types  $\{s_n\}_{n=1}^{\infty}$  because they are jointly distributed. However, there is asymmetric information between the investors and the sequence of managers  $M_n$ , each of whom privately observes  $s_n$  at the beginning of stage  $n$ .

We allow the manager to communicate with the owner regarding its private information; i.e., the owner can design an incentive mechanism (or contract)  $\mathbf{C}_n$  to induce information from  $M_n$ . The contract  $\mathbf{C}_n$  generally specifies (i) a noisy or randomized reporting policy for the manager, contingent on its type; (ii) a wage policy that determines the manager's compensation as a function of the manager's report and the liquidation earnings ( $y_n$ ); and, (iii) an investment policy that determines  $k_n$  as a function of the report.

We let  $\pi_n^{jr}$  denote the probability that the manager  $M_n$  reports  $r_n = r$  when the firm's actual type is  $j$ , for  $j, r = \ell, h$ . And conditional on the report  $r$ ,  $k_n^r$  is the owner's investment response, while  $w_n^{r,+}$  (or  $w_n^{r,0}$ ) is the manager's compensation when earnings  $y_n$  are positive (or zero). We summarize the stage  $n$  contract as the profile  $\mathbf{C}_n = \{\boldsymbol{\pi}_n, \mathbf{w}_n, \mathbf{k}_n\}$ , where  $\boldsymbol{\pi}_n = (\pi_n^{\ell\ell}, \pi_n^{hh}) \in \Delta(\Theta)$  — the space of probability measures on  $\Theta$ ;  $\mathbf{k}_n \equiv (k_n^\ell, k_n^h) \in \mathbf{K}^2$ ; and,  $\mathbf{w}_n = (w_n^{r,+}, w_n^{r,0})_{r=\ell}^h \in \mathbf{R}_+^2$ .<sup>14</sup>

A contracting *outcome* for firm  $n$  is the pair  $c_n = (r_n, k_n)$ , which is publicly observable. Thus, at every stage  $n \geq 2$ , the observable history is the profile  $\phi_n = (c_1, \dots, c_{n-1})$  (with  $\phi_1$  is an empty set), and we assume that investors and managers have perfect recall (Kuhn, 1953). Therefore,  $\mathbf{C}_n$  is the mapping  $(\boldsymbol{\pi}_n, \mathbf{w}_n, \mathbf{k}_n)(\phi_n) : \rightarrow \Omega \equiv \Delta(\Theta) \times \mathbf{K}^2 \times \mathbf{R}_+^2$ .

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<sup>14</sup>Notice that the manager's wage contract is not directly contractible on the investment (in the firm or in the industry). As in the incomplete contracts literature (Hart and Moore, 1988), we assume that a complete specification of future investment in the corporation is sufficiently complex to make wage contracts contingent on future investment prohibitively costly to enforce. In fact, managerial compensation contracts are not typically contingent on the external investment in the firm (Kole, 1997).

## 2.3 Change in Ownership and Renegotiation

For each firm  $n$ , the wage contracts  $\mathbf{w}_n$  are enforceable in the sense that managers can move the courts to enforce them even when the ownership of the firm changes.<sup>15</sup> However, the investment  $k_n^r$  may change subsequent to the report  $r_n = r$  if the ownership changes (prior to the investment implementation). This is because investment at any given point in time is legally the domain of the *current* capital owners.

Specifically, at the information set  $\phi_{n+1} = (\phi_n, c_n)$ , but prior to the implementation of  $k_n^r$ , any  $z \in Z$  can make a takeover offer of  $G_{n+1}(z)$  to the owner  $X_n$  for the entire equity of firm  $n$ . If  $X_n$  accepts this offer, then  $z$  becomes the new owner and can alter the pre-announced investment ( $k_n^r$ ) to an alternative investment. However,  $z$  can not alter  $\mathbf{w}_n$  (without the manager's consent). We note that the manager's individual rationality or participation constraint is not violated with the change in investment since wages are non-negative and the reservation utility is zero. Finally, if  $X_n$  rejects the takeover offer, then the pre-announced  $k_n^r$  is implemented.

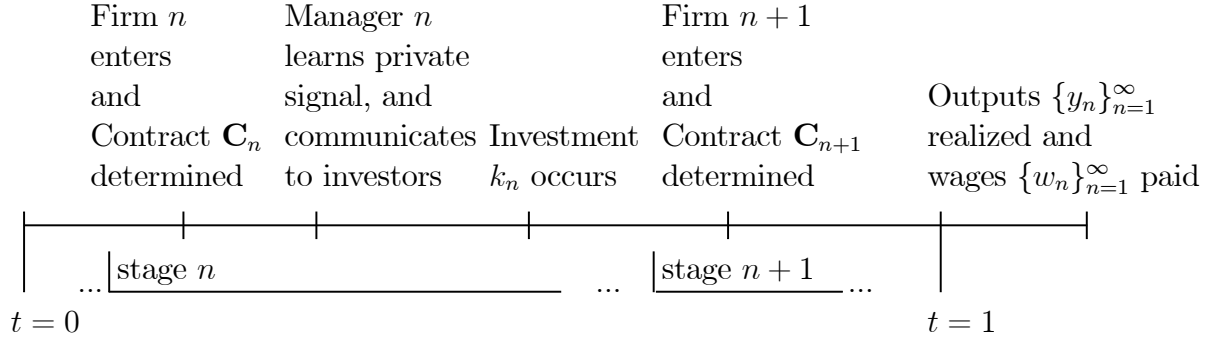
In sum, at the point of designing  $\mathbf{C}_n$ , the owner  $X_n$  can not credibly pre-commit to investment menus that are not *renegotiation-proof* across changes in ownership. That is, any admissible  $k_n$  must be such that, conditional on the (public) report  $r_n$ , there is no profitable opportunity in revising the investment by effecting a change of ownership. We will formalize this notion of renegotiation-proof investment rules momentarily.

## 2.4 Timing Conventions

The timing conventions of the model are summarized in Figure 1 below.

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<sup>15</sup>Indeed, there is much evidence that CEOs are able to successfully enforce their employment contracts, especially the payment of large severance payments, in the event of job termination and the sale of the firm (Murray, 2006; Lublin and Thurm, 2006). In particular, the enforceability of executive bonus compensation contracts even when firms are in financial distress (for e.g., CitiBank and AIG) became a major issue in the recent financial crisis.



**Figure 1**

## 2.5 Equilibrium

We are confronted with a contracting problem with adverse selection but when the principal has limited commitment with respect to the investment response to informed agent's communications. It is well-known that the revelation principle (e.g., Myerson, 1979) fails to hold in such a setting and one cannot restrict attention in general to truth-telling contracts (e.g., Bester and Strausz, 2007). We will characterize the sequence of managerial communications and investment decisions using the Perfect Bayesian Equilibrium (PBE) solution concept (e.g., Fudenberg and Tirole, 1991). In our setting, a PBE will require that contracts and managerial reporting strategies are sequentially rational, and that at each stage players use Bayes rule whenever possible to update their beliefs on  $\theta$ , based on the equilibrium reporting strategies.

Then, fix some stage  $n$ , history  $\phi_n$ , and contract  $\mathbf{C}_n$ . If the manager of type  $j = \ell, h$  uses a reporting strategy  $\pi_n^j \in \Delta(\Theta)$ , then his expected utility is:

$$U_n(j, \pi_n^j | \mathbf{w}_n, \mathbf{k}_n) = \sum_{r=\ell}^h \pi_n^{jr} \left\{ 2s_j \sqrt{k_n^r} [u(w_n^{r,+}) - u(w_n^{r,0})] + u(w_n^{r,0}) + \psi \sqrt{k_n^r} \right\} \quad (1)$$

A reporting strategy  $\pi_n^j \in \Delta(\Theta)$  is *incentive compatible* with respect to  $(\mathbf{w}_n, \mathbf{k}_n)$  for a manager of type  $j$  if  $U_n(j, \pi_n^j | \mathbf{w}_n, \mathbf{k}_n) \geq U_n(j, \pi_n' | \mathbf{w}_n, \mathbf{k}_n)$  for every  $\pi_n' \in \Delta(\Theta)$ .

Next, we formalize the notion of renegotiation-proof investment policies. For each  $z \in Z$  and for every  $1 \leq \tau \leq n$ , let  $\Pi_\tau(z) = (\pi_1^j(z), \dots, \pi_\tau^j(z))_{j=\ell}^h$ , be a given double-profile of reporting strategies imputed to both possible types of the sequence of managers  $(M_1, \dots, M_\tau)$ .

Then  $\beta_n(z) = \Pr(\theta = \theta_h \mid \phi_n, \Pi_{n-1}(z))$  are this investor's posterior beliefs on  $\theta$  prior to the communication from (the current manager)  $M_n$ . Then, by Bayes' rule, the investor's posterior expected productivity of firm  $n$  conditional on the report  $r_n = r$  is:

$$E(s_n \mid r, \phi_n, \Pi_n(z)) = \frac{\beta_n(z)\pi_n^{hr} s_h + (1 - \beta_n(z))\pi_n^{\ell r} s_\ell}{\beta_n(z)\pi_n^{hr} + (1 - \beta_n(z))\pi_n^{\ell r}}, \quad r \in \{\ell, h\} \quad (2)$$

And the investor's posterior value of the firm, given  $\mathbf{C}_n$  and the report  $r$  is:

$$V_n(r, \mathbf{C}_n; z) = 2E(s_n \mid r, \phi_n, \Pi_n(z))\sqrt{k_n^r}[1 - w_n^{r,+} + w_n^{r,0}] - w_n^{r,0} - Rk_n^r \quad r \in \{\ell, h\} \quad (3a)$$

Thus, this investor has an opportunity to increase the value of the firm *ex post* (i.e., following the report  $r$ ) if there exists some  $\hat{k}_n^r \neq k_n^r$  such that:<sup>16</sup>

$$\begin{aligned} V_n(r, \hat{k}_n^r, \mathbf{w}_n; z) &\equiv 2E(s_n \mid r, \phi_n, \Pi_n(z))\sqrt{\hat{k}_n^r}[1 - w_n^{r,+} + w_n^{r,0}] - w_n^{r,0} - R\hat{k}_n^r \\ &> V_n(r, \mathbf{C}_n; z) \end{aligned} \quad (4)$$

If (4) applies, then a profitable renegotiation opportunity exists (for  $z$ ) by acquiring the firm from the owner ( $X_n$ ) and altering the investment plan to  $\hat{k}_n^r$  (while still adhering to the binding wage contract  $\mathbf{w}_n$ ). We will say that  $\mathbf{C}_n$  is *renegotiation-proof* for investor  $z$  relative to  $(\phi_n, \Pi_n(z))$  if there exists no  $\hat{k}_n^r \neq k_n^r$ ,  $r \in \{\ell, h\}$  that is value-improving in the sense of (4).

A PBE is then specified by a sequence of contract mappings  $\{(\boldsymbol{\pi}_n, \mathbf{w}_n, \mathbf{k}_n)(\phi_n) : \rightarrow \Omega\}_{n=1}^\infty$  and productivity expectation mappings  $\{\mu_n(\phi_n) : \rightarrow [\theta_\ell, \theta_h]\}_{n=1}^\infty$  such that:

1. For each  $n$  and given any  $\phi_n$ , all players compute  $\mu_n(\phi_n)$  by applying Bayes rule to the history of observed reports and consistent with the profile of equilibrium reporting

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<sup>16</sup>More formally, we should write  $V_n(r, \hat{k}_n^r, \mathbf{w}_n; z) > \max\{V_n(r, \hat{k}_n^r, \mathbf{w}_n; X_n), V_n(r, \mathbf{C}_n; z)\}$ . Of course, along the equilibrium path all players impute common reporting strategies to the sequence of managers. Thus, at the information set  $\phi_n$ , all investors assess the same value for firm  $n$  for any given  $\mathbf{C}_n$  and conditional on any report  $r_n = r$ . Hence, if  $\mathbf{k}_n$  is renegotiation-proof for some investor  $z$ , then it is also renegotiation-proof for all investors  $z' \in Z$ . Therefore, the correspondence of sequentially rational contracts is independent of the choice of the owner(s). Furthermore, because shares are traded in frictionless and competitive markets, at every information set there is a competitive valuation such that all investors are just indifferent between purchasing and not purchasing equity in the firm.

strategies  $\Pi_{n-1}$ , i.e.,  $\mu_n(\phi_n) = \beta_n(\theta_h - \theta_\ell) + \theta_\ell$  where  $\beta_n = \Pr(\theta = \theta_h \mid \phi_n, \Pi_{n-1})$  is the Bayes-consistent posterior probability, and

2. For each  $n$  and given any  $\phi_n$ ,  $\mathbf{C}_n(\phi_n)$  is *sequentially rational* with respect to  $(\phi_n, \Pi_n)$ ; i.e., it maximizes the expected utility of any  $X_n$  with posterior beliefs  $\beta_n$ , subject to the constraints that (i)  $\boldsymbol{\pi}_n$  is incentive compatible and (ii)  $\mathbf{k}_n$  is renegotiation-proof with respect to  $(\phi_n, \Pi_n)$ .

Notice that while firm owners incur the costs of eliciting reports, they can not extract any rents from the future firms from this information. Thus, at any  $\phi_n$  when the posterior expectations (or the Bayes-estimate of  $\theta$ ) are  $\mu_n = \mu_n(\phi_n)$ , the sequentially rational contract is the optimal *stage contract* that maximizes the expected net output  $v(w_n, k_n, y_n)$ , i.e.,

$$\mathbf{C}_n(\phi_n) \in \arg \max_{C \in \Omega} \left[ \begin{array}{l} \mu_n \sum_{r=\ell}^h \pi_n^{hr} (2s_h \sqrt{k_n^r} [1 - w_n^{r,+} + w_n^{r,0}] - w_n^{r,0} - Rk_n^r) + \\ (1 - \mu_n) \sum_{r=\ell}^h \pi_n^{\ell r} (2s_\ell \sqrt{k_n^r} [1 - w_n^{r,+} + w_n^{r,0}] - w_n^{r,0} - Rk_n^r) \end{array} \right] \quad (5)$$

subject to the constraints that (1) the investment  $\mathbf{k}_n$  is *ex post* efficient or optimal given the posterior beliefs of the investors based on the manager's report ( $r_n$ ), (2) the manager's reporting strategy  $\boldsymbol{\pi}_n$  is incentive compatible, and (3) managerial compensation is non-negative in each earnings state at  $t = 1$ . Using (5), it will be convenient to write the optimal contract in stage  $n$  as  $\mathbf{C}_n(\mu_n) = (\boldsymbol{\pi}_n(\mu_n), \mathbf{w}_n(\mu_n), \mathbf{k}_n(\mu_n))$ . (We will suppress the dependence on  $\mu_n$  when convenient.)

Along any equilibrium path, owners of firms  $n = 1, 2, 3, \dots$ , design optimal stage contracts (cf. Section 2.5). We therefore first analyze the optimal contract in any stage  $n$ , taking as given the history  $\phi_n$ . We then use the stage optimal contracts to characterize the dynamics of beliefs and investments along the equilibrium path.

### 3 Information Manipulation in the Stage Contract

The optimal stage contract with over-reporting in a similar set-up to ours is derived in Kumar and Langberg (2009).<sup>17</sup> Consequently, we focus on the situation where, for each firm, owners optimally design contracts such that wages are not output contingent, i.e.,  $w_n^{r,+} = w_n^{r,0} = w_n^r$ ,  $r \in \{\ell, h\}$ , and the low-type managers may over-report their firm's productivity, i.e.,  $\pi_n^{\ell h} \in [0, 1]$ , while the high-type managers report truthfully, i.e.,  $\pi_n^{h\ell} = 0$ .<sup>18</sup> Three outcomes are feasible with over-reporting contracts: *truth-telling* when  $\pi_n^{\ell h} = 0$ ; *pooling* when  $\pi_n^{\ell h} = 1$ ; and *noisy revelation* when  $0 < \pi_n^{\ell h} < 1$ . The low-productivity managers' over-reporting probability  $\pi_n^{\ell h}$  therefore represents (the extent of) information manipulation in our analysis.

To determine the equilibrium level of information manipulation it is useful to define:

$$v \equiv \frac{\psi^\gamma (s_h - s_\ell)^{\gamma-2}}{R^{\gamma-1}}. \quad (6)$$

$v$  is positively associated with (the intensity of) the intrinsic agency conflict between informed managers and uninformed owners, and will serve as a measure of the severity of this conflict. To see this, note that one can decompose  $v$  (defined in (6)) into two components: the manager's subjective benefit from inducing higher investment ( $\psi$ ) and the potential investment inefficiency from productivity misrepresentation, which is proportional to  $(s_h - s_\ell)/R$ . Raising  $\psi$  increases the cost of eliciting truthful information from the low-productivity manager, while the potential investment inefficiency from misrepresentation increases with  $(s_h - s_\ell)$  and decreases with the cost of capital ( $R$ ). Indeed, in the extreme cases where  $s_h \rightarrow s_\ell$  or  $R \rightarrow \infty$ , there is no investment misallocation costs from the separation of ownership and control.

We therefore expect that the equilibrium information manipulation will be positively

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<sup>17</sup>In Kumar and Langberg (2009), the agent is risk neutral and has a linear private benefits of control function, i.e.,  $b(k) = \psi k$ . However, the adaptation to the parametric assumptions of this set-up is straightforward.

<sup>18</sup>In general, the optimal mechanism with limited commitment may induce randomized reporting from all agent-types (Laffont and Tirole, 1990; Bester and Strausz, 2007). In a one-shot version of the model studied in this paper, Kumar and Langberg (2009) provide sufficient conditions for the optimal mechanism to induce randomized reporting from the low-type agent and truthful reporting from the high-type agent and for wages not to depend on output realizations.

related to  $v$ . The following result makes this intuition precise: Inducing truth-telling from the low-type manager is optimal if  $v$  is sufficiently low; pooling is optimal when  $v$  is sufficiently high; and, noisy revelation is optimal when  $v$  is in an intermediate range. Specifically, let

$$\underline{v}(\mu_n) \equiv \left(1 + \frac{\gamma(1 - \mu_n)}{\mu_n}\right)^{-1} < \bar{v}(\mu_n) \equiv \mu_n^{-(\gamma-2)} \quad (7)$$

**Proposition 1** *Pooling is optimal if  $v > \bar{v}(\mu_n)$ , while truth-telling is optimal if  $v < \underline{v}(\mu_n)$ . But if  $\underline{v}(\mu_n) \leq v \leq \bar{v}(\mu_n)$ , then noisy revelation is optimal. Furthermore, the equilibrium information manipulation  $\pi_n^{\text{th}}$  is increasing in  $v$  on  $(0, 1)$ .*

The threshold (agency conflict) values  $\underline{v}(\mu_n)$  and  $\bar{v}(\mu_n)$  depend on  $\mu_n$ . That is, the equilibrium information manipulation at any stage depends on the intrinsic agency conflict ( $v$ ) and the (the history-dependent) productivity expectation ( $\mu_n$ ). We clarify next the *interaction* between  $v$  and  $\mu_n$  on the equilibrium information manipulation. For extreme (i.e., very low or very high) levels of agency conflict, there is either truth-telling or pooling for any  $\mu_n$ .

**Proposition 2** *There exist  $0 < v_T < 1 < v_P$  such that, along equilibrium path, for any  $n$  and given any productivity expectations  $\mu_n$ , truth-telling is optimal if  $v < v_T$ , but pooling is optimal if  $v > v_P$ .*

However, in general, the joint effects of  $v$  and  $\mu_n$  on the equilibrium information manipulation is non-monotonic in the following sense: for relatively low values of  $v$ , the information manipulation is negatively related to  $\mu_n$ ; i.e., information manipulation is greater (lower) when the productivity expectations are low (high). But this relationship between  $v$  and  $\mu_n$  reverses for relatively high values of  $v$ , when the equilibrium information manipulation is positively related to the productivity expectations.

**Proposition 3** *Let  $\mu^- \equiv \frac{v\gamma}{v(\gamma-1)+1}$  and  $\mu^+ \equiv v^{-(\frac{1}{\gamma-2})}$ .*

1. *There exists  $v_N^a \in (v_T, 1)$  such that, if  $v_T < v < v_N^a$ , then noisy revelation is optimal when  $\mu_n < \mu^-$ , but truth-telling is optimal if  $\mu_n \geq \mu^-$ .*



2. There exists  $v_N^b \in (1, v_P)$  such that, if  $v_N^b < v < v_P$ , then noisy revelation is optimal when  $\mu_n < \mu^+$ , but pooling is optimal if  $\mu_n \geq \mu^+$ .
3. If  $v_N^a < v < v_N^b$ , then noisy revelation is optimal.

Another way to represent Proposition 3 is that for relatively low (high) levels of the agency conflict the net benefits from inducing truthful revelation increase (decrease) with the productivity expectations. This is because raising  $\mu_n$  has two conflicting effects. It increases the investment response to a favorable managerial report, which tightens the truth-telling incentive constraints and requires a higher wage compensation to induce a given level of information. But raising  $\mu_n$  also increases the likelihood that the industry productivity is high, thereby reducing the expected wage cost. When  $v$  is high, the former effect dominates and higher levels of  $\mu_n$  lead to less informative (and even uninformative) managerial disclosures, but the latter effect dominates when  $v$  is low. Figure 2 sketches the regions in which the optimal stage contract induces truth-telling (or perfect revelation), noisy revelation, and pooling, as a function of beliefs  $\mu$  and the severity of the agency problem  $v$ .

We next address the issue of asymptotic learning equilibrium learning in the long run by accounting for the endogeneity of productivity expectations  $\mu_n$  along the equilibrium path.

## 4 Asymptotic Learning

Along the equilibrium path, the evolution of the posterior productivity expectation  $\mu_n$  can be recursively computed using Bayes' rule. Given  $\mu_n$  and the report  $r_n$ ,  $\mu_{n+1} = E(\theta \mid \mu_n, r_n)$  is:

$$\mu_{n+1} = \begin{cases} \frac{\sigma_n(1-\pi_n^{\ell h}) + \mu_n \pi_n^{\ell h}}{\mu_n(1-\pi_n^{\ell h}) + \pi_n^{\ell h}}, & \text{for } r_n = h \\ \frac{\mu_n - \sigma_n}{1 - \mu_n}, & \text{for } r_n = \ell \end{cases} \quad (8)$$

where  $\sigma_n \equiv E(\theta^2 \mid \phi_n) = (\mu_n - \theta_\ell) \theta_h + \mu_n \theta_\ell$ . (Here, the initial or prior productivity expectation is  $\mu_1 = \beta_1(\theta_h - \theta_\ell) + \theta_\ell$ .)

There is an intuition that if there is “sufficient” information content in each manager’s communication, then investors will eventually learn the true industry productivity despite the noise. On the other hand, if there are some equilibrium paths where the managers’

optimal reporting is only “marginally informative,” then it is possible that investors never learn the true industry productivity. We make this intuition precise in the following result, building on the fact that conditional beliefs obeying Bayes’ law are martingales and applying the Martingale Convergence Theorem (e.g., Billingsley, 1979).

**Theorem 1** *There exist random variables  $\bar{\mu}$  and  $\bar{\sigma}$  such that, with probability 1,  $\mu_n \rightarrow \bar{\mu}$  and  $\sigma_n \rightarrow \bar{\sigma}$  along any equilibrium path. Moreover,*

$$[\bar{\sigma} - \bar{\mu}^2] [1 - \pi_{\ell h}(\bar{\mu})] = 0 \quad a.s. \quad (9)$$

where,  $\pi_{\ell h}(\bar{\mu}) = \lim_{n \rightarrow \infty} \pi_n^{\ell h}(\mu_n)$ .

Note that, by the definition of  $\sigma_n$  and  $\mu_n$ ,  $\text{Var}_n(\theta \mid \phi_n) = \sigma_n - \mu_n^2$ ; hence,  $\text{Var}_n(\theta \mid \phi_n) \rightarrow (\bar{\sigma} - \bar{\mu}^2)$ . Thus, (9) implies that if the equilibrium path is such that managers’ communications are informative in the limit (i.e.,  $\pi_{\ell h}(\bar{\mu}) < 1$ ), then investors’ productivity expectations are asymptotically consistent and the limiting conditional distribution of  $\theta$  is degenerate with  $\lim_{n \rightarrow \infty} \text{Pr}(\theta = \mu_n \mid \phi_n) = 1$ . Alternatively put, along those equilibrium paths where pooling (i.e.,  $\pi_{\ell h}(\bar{\mu}) = 1$ ) is not optimal for any level of investors’ beliefs, there is *complete* learning (see Aghion et al., 1991) because investors learn the true industry productivity in the limit or  $\text{Var}_n(\theta \mid \phi_n) \rightarrow 0$ . However, for any history in which the communications are uninformative in the limit ( $\pi_{\ell h}(\bar{\mu}) = 1$ ), the uncertainty regarding the industry productivity is not resolved over time and  $\bar{\sigma} - \bar{\mu}^2 = (\theta_h - \bar{\mu})(\bar{\mu} - \theta_\ell)$ .

It follows immediately from Theorem 1 and Proposition 2 that there will be polar outcomes in asymptotic learning when the agency conflict level  $v$  is relatively low and when it is sufficiently high. This is because a sufficient condition for complete learning is that the sequence of managerial communications remain informative, i.e.,  $\pi_{\ell h}(\mu_n) < 1 - \epsilon$  for all  $\mu_n$  and for some  $\epsilon > 0$ , and Proposition 2 indicates that this will occur whenever  $v \leq v_N^b$ . On the other hand, there is complete pooling at every stage if  $v > v_P$ .

**Proposition 4** *If  $v \leq v_N^b$ , then investors obtain a consistent estimate of the unknown industry productivity, i.e., if  $\theta = \theta_j$ ,  $j \in \{\ell, h\}$ , then  $\bar{\mu} = \theta_j$  and  $\bar{\sigma} = \theta_j^2$ . However, if  $v > v_P$ , then there is no learning and  $\bar{\mu} = \mu_1$ .*

By definition (cf. (6)), a low  $v$  value is consistent with low growth potential ( $s_h - s_\ell$ ); low private benefit of control  $\psi$ ; and, a high required rate of return by investors  $R$ . Thus, Proposition 4 suggests that complete learning occurs when it is common knowledge that the innovation has a relatively low growth potential and/or when the cost of external financing is high. However, the rate of convergence of beliefs (or expectations) deserves further scrutiny and, in Figures 3A and 3B, we depict realized paths of productivity expectations when the true  $\theta = \theta_\ell$  and  $v$  sufficiently low. These figures indicate that the rate of convergence is faster for the lower  $v$ .

We turn then to examine the implications when the agency conflict lies in the intermediate range  $v_N^b < v < v_P$ . This range of agency conflicts is consistent with a high growth potential ( $s_h - s_\ell$ ); a greater private benefit of control  $\psi$ ; and, a low cost of capital  $R$ . Theorem 1 and Proposition 3 suggest that in this case there exist equilibrium paths with incomplete learning because the low-type manager's communications are not informative, i.e.,  $\pi_{\ell h}(\mu_n) = 1$ , whenever  $\mu_n > \mu^+$ . But note that if  $\pi_{\ell h}(\mu_n) = 1$  for some  $n$ , then  $\mu_{n+i} = \mu_n$  for all  $i \geq 1$  because investors never receive a low productivity report along the continuation equilibrium path. An immediate implication of this is that:

**Proposition 5** *If  $v_N^b < v < v_P$  and the prior expectations are optimistic, i.e.,  $\mu_1 \geq \mu^+$ , then there is no learning along the equilibrium path and  $\bar{\mu} = \mu_1$ .*

However, if the prior expectations are not optimistic, i.e.,  $\mu_1 < \mu^+$ , then  $\pi_{\ell h}(\mu_1) < 1$  and there is some learning during the first period itself. But along the over-reporting equilibrium path the posterior expectations are monotonic in their priors, i.e., at any stage  $n$ , given  $\mu_n$  and the report  $r_n$ ,  $\mu_{n+1} = E(\theta | \phi_n, r_n) > \mu'_{n+1} = E(\theta | \phi'_n, r_n)$ , whenever  $\mu_n(\phi_n) > \mu_n(\phi'_n)$ . Hence, the posterior expectations are uniformly bounded above by  $\mu^+$ , i.e.,  $\mu_n < \mu^+$  for every  $n \geq 1$ . The reason is that there is pooling if  $\mu_n \geq \mu^+$  so that  $\mu_{n+i} = \mu_n$  for all  $i \geq 1$  if  $\mu_n = \mu^+$ . In words, a sequence of high productivity reports not only raises the posterior expectations (on the industry productivity) but also reduces the informativeness of managerial disclosures in subsequent periods. Thus, as the market becomes more optimistic about the prospects of the industry after a sequence of high productivity reports, the effect of additional high reports on the posterior beliefs progressively weakens.

Therefore, complete learning may not occur if expectations approach  $\mu^+$  asymptotically. In particular, complete learning is not possible when  $\theta = \theta_h$  and investors start with non-optimistic priors ( $\mu_1 < \mu^+$ ). The reason is that complete learning would require  $\mu_n \uparrow \theta_h > \mu^+$ , which is impossible because  $\mu_n < \mu^+$  for every  $n \geq 1$ .

But the asymptotic learning outcomes with non-optimistic priors are richer when  $\theta = \theta_\ell$ . In this case, beliefs either converge to  $\theta_\ell$  or to  $\mu^+$ . In the latter case, markets that start with non-optimistic priors experience a run-up in expectations converging to a level that remains divergent from the true industry productivity. We can go further and examine the *ex ante* probability that the posterior expectations converge to  $\mu^+$  for any (true)  $\theta$  when the initial beliefs are non-optimistic (i.e.,  $\mu_1 < \mu^+$ ) by computing the probability of a run-up in investors' expectations when  $\theta = \theta_\ell$ , namely,  $p(\theta, \mu_1) \equiv \Pr(\bar{\mu} = \mu^+ \mid \theta, \mu_1 < \mu^+)$  for  $\mu_1 < \mu^+$ .

**Theorem 2** *Suppose that  $v_N^b < v < v_P$  and  $\mu_1 < \mu^+$ .*

1. *If  $\theta = \theta_h$ , then  $p(\theta_h, \mu_1) = 1$  (i.e.,  $\mu_n \rightarrow \mu^+$ ).*
2. *If  $\theta = \theta_\ell$ , then there is a run-up in investors' productivity expectations (i.e.,  $\mu_n \rightarrow \mu^+$ ) with probability  $p(\theta_\ell, \mu_1)$  and there is complete learning otherwise (i.e.,  $\mu_n \rightarrow \theta_\ell$ ), where*

$$p(\theta_\ell, \mu_1) = \left( \frac{\theta_h - \mu^+}{\mu^+ - \theta_\ell} \right) \left( \frac{\mu_1 - \theta_\ell}{\theta_h - \mu_1} \right), \text{ for } \mu_1 \in (\theta_\ell, \theta_h). \quad (10)$$

From Propositions 4-2 and Theorem 2 we conclude that unless the agency conflict is very low the asymptotic learning is *incomplete* (with probability 1) whenever the true industry is high, i.e.,  $\theta = \theta_h$  or if investors' prior expectations are optimistic, i.e.,  $\mu_1 \geq \mu^+$ . Thus, whether the industry productivity is actually high, or whether investors prior beliefs suppose it to be so, is irrelevant to the asymptotic learning outcome: in both cases investors do not discern the true productivity even after an infinite sequence of reports from firms in the industry (a.s.).

Moreover, Theorem 2 indicates that with positive probability investors will not asymptotically learn the true state even if the actual industry productivity is low (i.e.,  $\theta = \theta_\ell$ ) and the prior beliefs are relatively pessimistic (i.e.,  $\mu_1 < \mu^+$ ). Using (10), we can compute the

ex ante expectation of the limiting Bayes-estimate of  $\theta$  when the true industry productivity is low ( $\theta = \theta_\ell$ ):

$$E(\bar{\mu} \mid \theta_\ell) = \theta_\ell + (\theta_h - \mu^+) \left( \frac{\mu_1 - \theta_\ell}{\theta_h - \mu_1} \right) \quad (11)$$

Figure 4 depicts realized paths of equilibrium productivity expectations when the true productivity is *low* and  $v_N^b < v < v_P$ . The *a priori* likelihood of converging to beliefs  $\mu^+$  is 13.4% here. Along the equilibrium path in Figure 4A, beliefs are relatively high at first but converge to the true productivity, while in Figure 4B, beliefs monotonically converge to  $\mu^+$ .

The closed form solution for  $p(\theta_\ell, \mu_1)$  in (10) also facilitates comparative dynamics analysis. First, we confirm that  $p(\theta_\ell, \mu_1)$  is increasing in the prior expectation  $\mu_1$ , but decreasing in the agency conflict parameter  $v$  (since  $\mu^+ \equiv v^{-\left(\frac{1}{\gamma-2}\right)}$ ). Moreover, we can derive a link between investors' *ex ante uncertainty* — specifically mean-preserving spreads — regarding  $\theta$  and  $p(\theta_\ell, \mu_1)$ . We capture investors' *ex ante* uncertainty on  $\theta$  by  $\Delta \equiv (\theta_h - \theta_\ell)$ , and write,

$$p(\theta_\ell, \mu_1) = \left( \frac{\mu_1 - \mu^+ + (1 - \beta_1)\Delta}{\mu^+ - \mu_1 + \beta_1\Delta} \right) \left( \frac{\beta_1}{1 - \beta_1} \right) \quad (12)$$

Straightforward computations then show that  $p(\theta_\ell, \mu_1)$  is strictly increasing in  $\Delta$  (while keeping  $\mu_1$  and  $\beta_1$  constant). Thus, if investors' have greater uncertainty *ex ante* regarding the economic potential of the innovation, they are more likely to experience a run-up in expectations. Similarly, one can argue that the *ex ante* probability of a high  $\theta$  (i.e.,  $\beta_1$ ) is negatively related to the riskiness of the innovation. Consistent with this intuition,  $p(\theta_\ell, \mu_1)$  is negatively related to  $\beta_1$  (cf. (12)), *ceteris paribus*.

Figure 5 depicts the *ex ante* likelihood of converging to beliefs  $\mu^+$  when the true productivity is low. We see that this probability is increasing in  $\mu_1$  and the *ex ante* uncertainty regarding  $\theta$ .

## 5 Equilibrium Overinvestment

In this section, we highlight the implications of noisy revelation for investment over the horizon. First, compared to the complete information optimal investment allocation, noisy revelation by the low-type manager in any stage  $n$  leads to over-investment in firm  $n$  with

probability  $\pi_n^{\ell h}$ . It is straightforwardly computed that in our setting the efficient investment is  $k^{j*} = (s_j/R)^2$ ,  $j = \ell, h$ . On the other hand, for any  $n$  and given any  $\mu_n$ , the equilibrium investment in response to a high productivity report  $r_n = h$  is

$$k_n^h = \left( \frac{\mu_n s_h + (1 - \mu_n) \pi_{\ell h} s_\ell}{R(\mu_n + (1 - \mu_n) \pi_{\ell h})} \right)^2 > \left( \frac{s_\ell}{R} \right)^2$$

Second, information manipulation also has *intertemporal* implications because it increases investors' posterior productivity expectations, which affects investment in future periods as well: directly, because investment of any firm  $j \geq n+1$  depends on the posterior expectations  $\mu_j$ ; and, indirectly because the low-type manager's equilibrium reporting strategy  $\pi_{\ell h}(\mu_j)$  also depends on  $\mu_j$ . More formally, in our model, if investors know the sequence of realized productivities  $(s_n)$ , then the equilibrium investment levels are conditionally independent. However, along the noisy revelation equilibrium path, at any stage  $n$  with expectations  $\mu_n$ , the expected investment in firm  $n$  is

$$E(k_n | \mu_n) = (\mu_n + (1 - \mu_n) \pi_{\ell h}(\mu_n)) \left( k_h(\mu_n) - \left( \frac{s_\ell}{R} \right)^2 \right) + \left( \frac{s_\ell}{R} \right)^2 \quad (13)$$

where  $k_h(\mu_n)$  is the equilibrium investment response to a high productivity report given expectations  $\mu_n$ . Thus, an inflated report by a low-type manager not only induces overinvestment in his firm but, by raising productivity expectations in the future, it also raises the likelihood of overinvestment by subsequent firms.

Third, there is overinvestment in the industry asymptotically if productivity expectations rise because of high productivity reports early on (i.e., when  $n$  is low) but these expectations are not corrected over time. If the level of the agency conflict is not too low and the prior expectations are optimistic ( $\mu_1 > \mu^+$ ), then managerial reports are uninformative because the low-type manager over-reports with probability 1. In this case, there is no learning and all firms invest the same amount  $k_h(\mu_1) > k^{\ell*}$ ; i.e., there is overinvestment if  $\theta = \theta_\ell$ . But if prior expectations are non-optimistic ( $\mu_1 < \mu^+$ ), then asymptotically:

**Proposition 6** *Suppose that  $v_N^b < v < v_P$  and  $\mu_1 < \mu^+$ . If the true industry productivity is low (i.e.,  $\theta = \theta_\ell$ ), then:*

1. With probability  $p(\theta_\ell, \mu_1)$ ,  $\bar{\mu} = \mu^+$  and there exists some  $n^*$  such that investment  $(k_n)$  monotonically increases for all  $n > n^*$  and  $\lim_{n \uparrow \infty} k_n = k_h(\mu^+)$ .
2. With probability  $1 - p(\theta_\ell, \mu_1)$ , there is complete learning; i.e.,  $\bar{\mu} = \theta_\ell$ , investment levels oscillate but are conditionally independent in the limit, and  $\lim_{n \uparrow \infty} E(k_n | \mu_n) < k_h(\mu^+)$ .

Figures 4A and 4B depict the case where industry productivity is low and there is high agency conflict ( $v = 1.33$ ). Figure 4A depicts paths of convergence to the true (low) productivity, but the speed of convergence is lower here (roughly after 150 and 250 observations) relative to the paths of complete learning in Figures 3A and 3B. When complete learning occurs, investment levels in the limit oscillate between  $k_h(\theta_\ell)$  and  $k_\ell^*$  in accordance with Proposition 6. The probability of complete learning in this case is 86.6%, so that with probability 13.4% beliefs do not converge to the true productivity. Figure 4B depicts equilibrium paths of learning and investment when there is an investment bubble: here investment levels monotonically increase after a certain point in time to reach the upper bound  $k_h(\mu^+)$ .

## 6 Summary and Conclusions

Investment in growth opportunities generated by innovations and development of new economic opportunities often engender a persistent build-up in investment leading to overinvestment or overcapacity — relative to the efficient capital allocation levels *ex post*. A historical analysis also highlights the crucial role of manipulation of investors' beliefs by self-interested and informed insiders through overly-optimistic representations of financial performance and economic prospects. However, the existing literature does not appear to offer analyses that are consistent with observing long-term overinvestment fueled by possible strategic information manipulation by insiders in an equilibrium framework with rational agents.

We present a model of investment booms in rational and frictionless capital markets where there is systematic overinvesting (relative to the efficient capital allocation) in low productivity firms even asymptotically because of strategic information manipulation by privately informed insiders. However, such an allocation is constrained-efficient with respect to the in-

formational imperfections and limited commitment by capital markets regarding investment policies that are inefficient *ex post*. For an open set of parameters investors endogenously get "stuck" at an overoptimistic level of (posterior) expectations on the industry productivity, and the useful information of managers of newly entering firms ceases to get incorporated in their investment decisions, even though there is optimal incentive contracting through optimal wage contracts and renegotiation-proof investment plans. Information is therefore not aggregated asymptotically even though there is bilateral contracting for inducing information revelation, with resultant systematic and long run overinvestment in firms with low productivity and industry-wide overcapacity. Thus, market "exuberance," investment booms and long run industry overcapacity can be consistent with Bayes-rationality and common priors and can occur even when incentive mechanisms can be designed to elicit information.



# Appendix

**Proof of Proposition 1:** Note first that the optimal wage-policy sets zero wages when the manager reports the high-productivity state. To see this, suppose that,  $w_n^h > 0$ . Then, since  $k_n^h = \left(\frac{E(\tilde{s}_n | \phi_n, r_n = h, \mathbf{C}_n)}{R}\right)^2 \geq k_n^\ell = \left(\frac{s_\ell}{R}\right)^2$  (from renegotiation proofnes), and  $u(w_n^\ell) - u(w_n^h) = \psi\left(\sqrt{k_n^h} - \sqrt{k_n^\ell}\right)$  (from incentive compatibility), it follows that  $w_n^\ell > u^{-1}\left(\psi\left(\sqrt{k_n^h} - \sqrt{k_n^\ell}\right)\right)$ . However, one can strictly improve the objective function by reducing wages  $w_n^h$  and  $w_n^\ell$  to  $\hat{w}_n^h = 0$  and  $\hat{w}_n^\ell = u^{-1}\left(\psi\left(\sqrt{k_n^h} - \sqrt{k_n^\ell}\right)\right)$ . Thus, at the optimum,

$$u(w_n^\ell) = \psi(\sqrt{k_n^h} - \sqrt{k_n^\ell}). \quad (14)$$

Now, using (14) and the result  $w_n^h = 0$ , the problem (5) can be written in terms of  $u(w_n^\ell) \equiv u$ ,

$$\max_{u \geq 0} \left\{ \mu_n (s_h 2\sqrt{k_n^h} - Rk_n^h) + (1 - \mu_n) \left[ \pi_n^{\ell h} (s_\ell 2\sqrt{k_n^h} - Rk_n^h) + (1 - \pi_n^{\ell h}) (s_\ell 2\sqrt{k_n^\ell} - Rk_n^\ell - u^\gamma) \right] \right\}, \quad (15)$$

where,  $k_n^\ell = \left(\frac{s_\ell}{R}\right)^2$ ,  $k_n^h = \left(\frac{u}{\psi} + \frac{s_\ell}{R}\right)^2$ , and  $\pi_n^{\ell h} = \frac{\mu_n(\psi(s_h - s_\ell) - uR)}{(1 - \mu_n)uR}$ . Differentiating the objective function in (15), we denote by *OBJ*, yields,

$$\begin{aligned} \frac{dOBJ}{du} &= (1 - \mu_n) \left[ \left( -\frac{uR}{2\psi} \right) \left( \frac{2u}{\psi} \right) + u^\gamma \right] \left[ -\frac{\mu_n \psi (s_h - s_\ell)}{(1 - \mu_n) u^2 R} \right] - \gamma u^{\gamma-1} \frac{Ru - \mu_n \psi (s_h - s_\ell)}{Ru} \\ &\propto -\gamma Ru^{\gamma-1} + \mu_n \psi (s_h - s_\ell) \left[ u^{\gamma-2} (\gamma - 1) + \frac{R}{\psi^2} \right]. \end{aligned} \quad (16)$$

In order to express the solution in terms of the manager's reporting strategy  $\pi_n^{\ell h}$  we note that,

$$\frac{dOBJ}{du} = 0 \Leftrightarrow -\gamma + \mu_n \psi (s_h - s_\ell) \frac{1}{Ru} (\gamma - 1) + \mu_n \psi (s_h - s_\ell) \frac{1}{u^{\gamma-1} \psi^2} = 0.$$

**Definition:** Let  $\alpha$  represent the inverse of  $v$ , namely,

$$\alpha \equiv \frac{R^{\gamma-1}}{\psi^\gamma (s_h - s_\ell)^{\gamma-2}}.$$

Thus, since  $\pi_n^{\ell h} = \frac{\mu_n(\psi(s_h - s_\ell) - uR)}{(1 - \mu_n)uR} \Leftrightarrow u = \frac{\mu_n \psi (s_h - s_\ell)}{R(\mu_n + (1 - \mu_n)\pi_n^{\ell h})}$  the optimal reporting strategy is defined

by  $FOC = 0$ , where,

$$FOC \equiv -\gamma + (\gamma - 1)(\mu_n + (1 - \mu_n)\pi_n^{\ell h}) + \alpha\mu_n \left[ 1 + \left( \frac{1}{\mu_n} - 1 \right) \pi_n^{\ell h} \right]^{\gamma-1}. \quad (17)$$

To calculate the second order derivative at the optimum (i.e., for  $u$  such that  $\frac{dOBJ}{du} = 0$ ), we refer back to the derivation of  $\frac{dOBJ}{du}$  in (16).

$$\begin{aligned} \frac{d^2OBJ}{du^2} &\propto -\gamma Ru^{\gamma-2}(\gamma - 1) + \mu_n \psi(s_h - s_\ell) u^{\gamma-3} (\gamma - 1) (\gamma - 2) \\ &= \frac{(\gamma - 1)}{u} [-\gamma Ru^{\gamma-1} + \mu_n \psi(s_h - s_\ell) u^{\gamma-2} (\gamma - 2)] \\ &< \frac{(\gamma - 1)}{u} \left[ -\mu_n \psi(s_h - s_\ell) \frac{R}{\psi^2} \right] \quad (\text{from } \frac{dOBJ}{du} = 0 \text{ and (16)}) \\ &< 0. \end{aligned}$$

Thus, the interior solution to  $\frac{dOBJ}{du} = 0$  is a local maximum and uniqueness follows. Finally, the interior solution  $\pi_n^{\ell h}$  is feasible (i.e.,  $\pi_n^{\ell h} \in [0, 1]$ ) if and only if  $\alpha \in \left[ \mu_n^{\gamma-2}, 1 + \frac{\gamma(1-\mu_n)}{\mu_n} \right] \equiv [\underline{\alpha}(\mu_n), \bar{\alpha}(\mu_n)]$ . Moreover, if  $\alpha > 1 + \frac{\gamma(1-\mu_n)}{\mu_n}$ , then the solution is a truth-telling equilibrium (i.e.,  $\pi_n^{\ell h} = 0$ ), and if  $\alpha < \mu_n^{\gamma-2}$ , then the solution is a pooling equilibrium (i.e.,  $\pi_n^{\ell h} = 1$ ).

To obtain the comparative statics on  $\pi_n^{\ell h}$  for  $\alpha \in (\underline{\alpha}(\mu_n), \bar{\alpha}(\mu_n))$ , we apply the Implicit Function Theorem. To establish the derivative  $\frac{\partial \pi_n^{\ell h}}{\partial \alpha}$  note that  $\frac{\partial FOC}{\partial \alpha} > 0$ ,  $\frac{\partial FOC}{\partial \pi_n^{\ell h}} > 0$ , and therefore  $\frac{\partial \pi_n^{\ell h}}{\partial \alpha} < 0$ . Moreover, it will become useful to analyze the derivative  $\frac{\partial \Delta_n}{\partial \mu_n}$ , where  $\Delta_n \equiv \mu_n + (1 - \mu_n)\pi_n^{\ell h}$  is the probability of a high productivity report in period  $n$  for expectations  $\mu_n$ . Thus, we rewrite the first order condition  $FOC$  as  $G(\mu_n, \Delta_n) = -\gamma + (\gamma - 1)\Delta_n + \alpha\mu_n^{-(\gamma-1)}\Delta_n^{\gamma-1}$ , and conclude that  $\frac{\partial \Delta_n}{\partial \mu_n} = -\frac{\partial G(\mu_n, \Delta_n)}{\partial \mu_n} / \frac{\partial G(\mu_n, \Delta_n)}{\partial \Delta_n} > 0$ . ■

**Proof of Propositions 2 and 3:** These Propositions follow directly from the optimal contract as derived in the proof of Proposition 1 and the following two lemmas. First, we establish a condition on  $\alpha$  under which the optimal contract specifies  $\pi_n^{\ell h} = 1$  or  $\pi_n^{\ell h} = 0$ . Second, we derive the properties of  $\pi_n^{\ell h}$  with respect to  $\mu_n$ .

**Lemma A:** (i) If  $\alpha > 1$ , then there exists a unique  $\mu^- \in (0, 1)$  such that  $\pi_n^{\ell h} = 0$  solves (17); but, there does not exist  $\mu$  such that  $\pi_n^{\ell h} = 1$  solves (17). (ii) If  $\alpha < 1$ , then there exists a unique  $\mu^+ \in (0, 1)$  such that  $\pi_n^{\ell h} = 1$  solves (17); but, there does not exist  $\mu$  such that  $\pi_n^{\ell h} = 0$  solves

(17). (iii) Moreover,  $\mu^- = \frac{\gamma}{\gamma-1+\alpha}$  and  $\mu^+ = \alpha^{\frac{1}{\gamma-2}}$ .

**Proof of Lemma A:** Let  $\mu^-$  be such that  $\pi_n^{\ell h} = 0$  is an interior solution of (17). Then,

$$0 = FOC|_{\mu^-, \pi_n^{\ell h}=0} = -\gamma + (\gamma - 1)\mu^- + \alpha\mu^- \Leftrightarrow \mu^- = \frac{\gamma}{\gamma - 1 + \alpha}.$$

And let  $\mu^+$  be such that  $\pi_n^{\ell h} = 1$  is an interior solution of (17). Then,

$$0 = FOC|_{\mu^+, \pi_n^{\ell h}=1} = -\gamma + (\gamma - 1) + \alpha \left[ \frac{1}{\mu^+} \right]^{\gamma-2} \Leftrightarrow \mu^+ = \alpha^{\frac{1}{\gamma-2}}.$$

Note that  $\mu^- \in (0, 1) \Leftrightarrow \alpha > 1$ , and  $\mu^+ \in (0, 1) \Leftrightarrow \alpha < 1$ . ■

**Lemma B:** (i) If  $\alpha > 1$ , then  $\pi_n^{\ell h} = 0$  for  $\mu_n \geq \mu^-$ , and  $\pi_n^{\ell h} \in (0, \mu^-)$  otherwise. (ii) If  $\alpha < 1$ , then  $\pi_n^{\ell h} = 1$  for  $\mu_n \geq \mu^+$ , and  $\pi_n^{\ell h} \in (0, 1)$  otherwise.

**Proof of Lemma B:** Consider first the case  $\alpha > 1$ . By definition of  $\mu^-$ ,  $FOC|_{\mu^-, \pi_n^{\ell h}=0} = 0$ . Consider any  $\hat{\mu} < \mu^-$ , and suppose to the contrary, that  $\pi_n^{\ell h} = 0$ . Then, from uniqueness it must be that  $FOC|_{\hat{\mu}, \pi_n^{\ell h}=0} \geq 0$ , i.e., the objective function is lower for lower levels of  $u$  or higher levels of  $\pi_n^{\ell h}$  (recall that  $\frac{dOBJ}{du} = 0 \Leftrightarrow FOC = 0$  from (16) and (17) and that the equilibrium relation between  $u$  and  $\pi_n^{\ell h}$  is negative). But,  $FOC|_{\hat{\mu}, \pi_n^{\ell h}=0} = -\gamma + (\gamma - 1)\hat{\mu} + \alpha\hat{\mu} < -\gamma + (\gamma - 1)\mu^- + \alpha\mu^- = 0$ , and we reach a contradiction. Thus, for  $\mu \in (0, \mu^-)$  the solution  $FOC = 0$  satisfies  $\pi_n^{\ell h} > 0$ . Moreover, it follows from  $\alpha > 1$  and Lemma A that  $\pi_n^{\ell h} < 1$  for  $\mu \in (0, 1)$ . Next, to establish that  $\pi_n^{\ell h} = 0$  for  $\mu \in (\mu^-, 1)$ , consider any  $\mu^* > \mu^-$ , and suppose to the contrary that  $\pi_n^{\ell h} \in (0, 1)$ . Hence,  $FOC|_{\mu^*, \pi_n^{\ell h}} = 0$ . But,

$$\begin{aligned} 0 &= FOC|_{\mu^*, \pi_n^{\ell h}} = -\gamma + (\gamma - 1)(\mu^* + (1 - \mu^*)\pi_n^{\ell h}) + \alpha\mu^* \left[ 1 + \left( \frac{1}{\mu^*} - 1 \right) \pi_n^{\ell h} \right]^{\gamma-1} \\ &> -\gamma + (\gamma - 1)\mu^* + \alpha\mu^* > -\gamma + (\gamma - 1)\mu^- + \alpha\mu^- = 0 \Rightarrow \text{contradiction.} \end{aligned}$$

Moreover, still for  $\alpha > 1$ , we show that  $\pi_n^{\ell h} \in [0, \mu^-)$  for all  $\mu \in (0, \mu^-)$ . In particular, for any

$\tilde{\mu} \in (0, \mu^-)$ , suppose by contradiction that  $\pi_n^{\ell h} \in [\mu^-, 1)$ . But,

$$\begin{aligned}
0 &= FOC|_{\tilde{\mu}, \pi_n^{\ell h}} = -\gamma + (\gamma - 1)(\tilde{\mu} + (1 - \tilde{\mu})\pi_n^{\ell h}) + \alpha \left[ \frac{1}{\tilde{\mu}} \right]^{\gamma-2} (\tilde{\mu} + (1 - \tilde{\mu})\pi_n^{\ell h})^{\gamma-1} \\
&> -\gamma + (\gamma - 1)\pi_n^{\ell h} + \alpha \left[ \frac{1}{\tilde{\mu}} \right]^{\gamma-2} (\pi_n^{\ell h})^{\gamma-1} > -\gamma + (\gamma - 1)\pi_n^{\ell h} + \alpha \left[ \frac{1}{\mu^-} \right]^{\gamma-2} (\pi_n^{\ell h})^{\gamma-1} \\
&\geq -\gamma + (\gamma - 1)\mu^- + \alpha \left[ \frac{1}{\mu^-} \right]^{\gamma-2} (\mu^-)^{\gamma-1} = 0 \text{ by definition of } \mu^- \text{ (contradiction)}.
\end{aligned}$$

Second, consider the case  $\alpha < 1$ . By definition of  $\mu^+$ ,  $FOC|_{\mu^+, \pi_n^{\ell h}=1} = 0$ . Now, consider any  $\tilde{\mu} < \mu^+$ , and suppose by contradiction that  $\pi_n^{\ell h} = 1$ . Then, it must be that  $FOC|_{\tilde{\mu}, \pi_n^{\ell h}=1} \leq 0$ , i.e., the objective function will decrease if  $u$  increases or  $\pi_n^{\ell h}$  decreases. But,  $FOC|_{\tilde{\mu}, \pi_n^{\ell h}=1} = -1 + \alpha \left[ \frac{1}{\tilde{\mu}} \right]^{\gamma-2} > -1 + \alpha \left[ \frac{1}{\mu^+} \right]^{\gamma-2} = 0$ , and we reach a contradiction. Thus, for  $\mu \in (0, \mu^+)$  the solution  $FOC = 0$  satisfies  $\pi_n^{\ell h} < 1$ . Moreover, it follows from Lemma A that  $\pi_n^{\ell h} > 0$  for  $\mu \in (0, 1)$ . Finally, to show that  $\pi_n^{\ell h} = 1$  for  $\mu \in (\mu^+, 1)$ , consider any  $\tilde{\mu} > \mu^+$  and suppose by contradiction that the solution is interior, i.e.,  $\pi_n^{\ell h} \in (0, 1)$  and  $FOC|_{\tilde{\mu}, \pi_n^{\ell h}} = 0$ . But,

$$\begin{aligned}
0 &= FOC|_{\tilde{\mu}, \pi_n^{\ell h}} = -\gamma + (\gamma - 1)(\tilde{\mu} + (1 - \tilde{\mu})\pi_n^{\ell h}) + \alpha \tilde{\mu} \left[ 1 + \left( \frac{1}{\tilde{\mu}} - 1 \right) \pi_n^{\ell h} \right]^{\gamma-1} \\
&< -\gamma + (\gamma - 1) + \alpha \left[ \frac{1}{\tilde{\mu}} \right]^{\gamma-2} < -\gamma + (\gamma - 1) + \alpha \left[ \frac{1}{\mu^+} \right]^{\gamma-2} = 0 \Rightarrow \text{contradiction}.
\end{aligned}$$

We can summarize that,

$$v_T = \frac{\theta_\ell}{\gamma(1 - \theta_\ell) + \theta_\ell}, v_N^a = \frac{\theta_h}{\gamma(1 - \theta_h) + \theta_h}, v_N^b = \theta_h^{-(\gamma-2)}, v_P = \theta_\ell^{-(\gamma-2)}.$$

■

**Proof of Theorem 1:** The existence of the random variables  $\bar{\mu}$  and  $\bar{\sigma}$  follows from the Martingale Convergence Theorem and the fact that  $\mu_n$  and  $\sigma_n$  are bounded martingales. It follows from from (8) that convergence implies that either (1)  $\bar{\mu} = \frac{\bar{\sigma}(1 - \pi_{\ell h}(\bar{\mu})) + \bar{\mu}\pi_{\ell h}(\bar{\mu})}{\bar{\mu}(1 - \pi_{\ell h}(\bar{\mu})) + \pi_{\ell h}(\bar{\mu})}$  or (2)  $\bar{\mu} = \frac{\bar{\mu} - \bar{\sigma}}{1 - \bar{\mu}}$ . If  $\pi_{\ell h}(\bar{\mu}) < 1$  then (2) is satisfied and there is complete learning, i.e.,  $\bar{\sigma} = \bar{\mu}^2$ , and if  $\pi_{\ell h}(\bar{\mu}) = 1$  then (1) satisfied. ■

**Proof of Proposition 4:** Recall,  $\pi_{\ell h}(\mu)$  denotes the optimal reporting strategy,  $\pi_n^{\ell h}$ , when expectations are  $\mu_n = \mu$ . If  $v < v_N^b$ , then  $\pi_{\ell h}(\mu) < 1$  for all  $\mu \in [\theta_\ell, \theta_h]$ , thus,  $\pi_{\ell h}(\bar{\mu}) < 1$  and it

follows that  $\bar{\sigma} = \bar{\mu}^2$  or equivalently that  $\text{Var}(\theta \mid \phi_n) \rightarrow 0$  and  $\lim_{n \uparrow \infty} \Pr(\theta = \mu_n \mid \phi_n) = 1$ . Now, if  $v > v_P$  then  $\pi_{\ell h}(\mu) = 1$  for all  $\mu \in [\theta_\ell, \theta_h]$  and reports are not informative along the equilibrium path, i.e.,  $\bar{\mu} = \mu_1$ . Finally, if  $v_N^b < v < v_P$  then  $\pi_{\ell h}(\mu_1) = 1$  for initial beliefs  $\mu_1 \in [\mu^+, \theta_h]$  and consequently reports are not informative along the equilibrium path, i.e.,  $\bar{\mu} = \mu_1$ . ■

**Proof of Theorem 2:** For the case  $\mu_1 < \mu^+$  and  $v \in (v_N^b, v_P)$  we know from before that  $\pi_{\ell h}(\mu_1) < 1$ , i.e., reports start out informative. Next, we show that beliefs do not exceed the level  $\mu^+$  with probability one in equilibrium. This follows since  $\mu_{n+1} = E(\theta \mid \mu(\phi_n) = \mu, r_n)$  is increasing in  $\mu$  and  $\mu^+ = E(\theta \mid \mu(\phi_n) = \mu^+, r_n)$ . To show that  $\mu_{n+1} = E(\theta \mid \mu(\phi_n) = \mu, r_n)$  is indeed increasing in  $\mu$  first note that  $\frac{\partial u^*}{\partial \mu} > 0$  where  $u^*$  is the solution to (16). This in turn, through (14), implies that  $\frac{\partial E(s_n \mid \mu(\phi_n) = \mu, r_n = h)}{\partial \mu} > 0$ . But,

$$E(s_n \mid \mu(\phi_n) = \mu, r_n = h) = \Pr(s_n = s_h \mid \mu(\phi_n) = \mu, r_n = h)(s_h - s_\ell) + s_\ell.$$

Therefore,  $\Pr(s_n = s_h \mid \mu(\phi_n) = \mu, r_n = h)$  is increasing in  $\mu$ . Moreover,

$$\begin{aligned} E(\theta \mid \mu(\phi_n) = \mu, r_n = h) &= \Pr(s = s_h \mid \mu(\phi_n) = \mu, r_n = h)E(\theta \mid \mu(\phi_n) = \mu, r_n = h, s_n = s_h) \\ &+ \Pr(s = s_\ell \mid \mu(\phi_n) = \mu, r_n = h)E(\theta \mid \mu(\phi_n) = \mu, r_n = h, s_n = s_\ell) \\ &= \Pr(s = s_h \mid \mu(\phi_n) = \mu, r_n = h)E(\theta \mid \mu(\phi_n) = \mu, s_n = s_h) \\ &+ \Pr(s = s_\ell \mid \mu(\phi_n) = \mu, r_n = h)E(\theta \mid \mu(\phi_n) = \mu, s_n = s_\ell) \end{aligned}$$

Clearly,  $E(\theta \mid \mu(\phi_n) = \mu, s_n = s_h) > E(\theta \mid \mu(\phi_n) = \mu, s_n = s_\ell)$ , and both are increasing in  $\mu$ . Moreover,  $\Pr(s = s_h \mid \mu(\phi_n) = \mu, r_n = h)$  is increasing in  $\mu$ . Therefore,  $E(\theta \mid \mu(\phi_n) = \mu, r_n = h)$  is increasing in  $\mu$ . Finally,  $E(\theta \mid \mu(\phi_n) = \mu, r_n = \ell) = E(\theta \mid \mu(\phi_n) = \mu, s_n = s_\ell)$  is also increasing in  $\mu$ . The above implies that  $\mu_{n+1} = E(\theta \mid \mu(\phi_n) = \mu, r_n)$  is indeed increasing in  $\mu$ . Now, since  $\pi_{\ell h}(\mu^+) = 1$ , and  $E(\theta \mid \mu(\phi_n) = \mu^+, r_n) = \mu^+$ , it follows from the above monotonicity result that  $\mu_{n+1} = E(\theta \mid \mu(\phi_n) = \mu, r_n) < \mu^+$  for all  $\mu < \mu^+$ .

Now, this implies that  $\bar{\mu} = \mu^+$  or  $\bar{\mu} = \tilde{\theta} < \mu^+$ . Consequently that if  $\tilde{\theta} = \theta_h$  then it is only possible that  $\bar{\mu} = \mu^+$ . But, if  $\tilde{\theta} = \theta_\ell$ , then beliefs are either optimistic,  $\bar{\mu} = \mu^+$ , or accurate,

$\bar{\mu} = \theta_\ell$ . Since,  $\mu_1 \equiv E(\theta|\phi_1) = E(\bar{\mu}|\phi_1)$ , it follows that:

$$\begin{aligned}\mu_1 &= E(\bar{\mu}|\phi_1) = \Pr(\tilde{\theta} = \theta_\ell)E(\bar{\mu}|\phi_1, \theta_\ell) + \Pr(\tilde{\theta} = \theta_h)E(\bar{\mu}|\phi_1, \theta_h) \\ &= \left(\frac{\theta_h - \mu_1}{\theta_h - \theta_\ell}\right) [p(\theta_\ell, \mu_1)(\mu^+ - \theta_\ell) + \theta_\ell] + \left(\frac{\mu_1 - \theta_\ell}{\theta_h - \theta_\ell}\right) \mu^+.\end{aligned}$$

■

**Proof of Proposition 6:** Follows directly from the above analysis. ■

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