VALUING EMPLOYEE STOCK OPTIONS AND RESTRICTED STOCK IN THE PRESENCE OF MARKET IMPERFECTIONS

by

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Valuing employee stock options and restricted stock in the presence of market imperfections*

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ABSTRACT

We develop a new technology for valuing financial assets such as employee stock options and restricted stocks. Our model takes explicit account of the non-diversification of the owner of the asset. The model is an extension of the common binomial pricing model and is relatively easy to implement. This paper explains the issues and uses a database of employee stock options to estimate the model parameters and the value of stock options grants to employees. Using our model, we find that the value of employee stock options on the grant date is approximately 50% of a plain vanilla call option calculated using the Black and Scholes formula.

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1. Introduction

In this paper we introduce a simple model for the valuation of employee stock options (ESO) that takes account of market imperfections. The model, based on a paper by Benninga, Helmantel, Sarig (2005, henceforth BHS) directly incorporates non-marketability into asset valuation and is easy to implement in a binomial framework. A simple extension of the model can also be used to value restricted stock units (RSU).

The extensive literature on employee stock options can be divided into three segments. Our model relates to all three of these segments.

Valuation of ESOs: A significant segment of the literature discusses the value of an employee stock option. The arbitrage-pricing approach of this literature (for example Hull-White 2004, Cvitanić et al. 2007) uses either lattice-based or continuous-time valuation frameworks to value the ESO with its special features. The utility approach of the valuation literature assumes that an employee has a utility function and uses utility-based models to value the ESO (for example Hall and Murphy 2002). Both the arbitrage approach and the utility approach to valuation tend to the conclusion that the Black-Scholes and binomial pricing models overvalue ESOs. However, while the arbitrage strand of the literature results in explicit pricing formulas of ESOs, the utility approach is not as explicit—pricing in this approach is a function of risk aversion and employee income and wealth.

Our model falls into the category of the arbitrage approach models. However, whereas the arbitrage-approach models cited above require somewhat arbitrary assumptions about early exercise, our model endogenizes this decision into the pricing function.

Documenting employee behavior: Another segment of the ESO literature documents actual behavior of the holders of employee stock options. Typically this strand of the literature documents the early-exercise behavior of ESO holders. Huddart and Lang (1996, 2003), Carpenter, Stanton and Wallace (2009) are typical exponents of this part of the literature.

The employee-behavior part of the ESO literature shows clearly that employees tend to early-exercise their options. This behavior contradicts the prediction of standard option-pricing models, in which early exercise of calls is nearly always sub-optimal. Early exercise of ESOs has been attributed to various reasons, typically the difficulty of employees hedging or trading their ESOs, even when the vesting period has passed, because of the long-term nature of the ESO.

This paper is also part of the employee behavior strand of the ESO literature in two ways. First, analytical model explains early exercise of ESOs by pricing the non-diversifiable aspects of the ESO. Second, our large and unique data base of Israeli ESOs enables us to both document early-exercise and calibrate our model's non-diversifiability.

Accounting cost of ESOs: IFRS2 and FAS 123R require an attribution of cost to the grant of employee stock options. Abstracting from philosophical issues of cost versus value[1] the actual implementation of the accounting regulations typically ascribes the ESO cost using a standard valuation model, be it Black-Scholes or one of the other lattice models discussed above. Roughly speaking this literature (of which Chance 2004, Rubinstein 1995, and Hall and Murphy 2002 are the most important articles) discusses whether the accounting cost of an ESO should be its value in a perfect-markets setting or the value incorporating the various option restrictions. Our contribution to the accounting discussion is to provide an explicit pricing model that accounts for non-diversification and is both easily implementable and has some connection to the non-diversification of the ESO holder.

Another approach to this problem is due to Finnerty (2005). Finnerty describes a framework in which an investor buys the ESO rights from an employee. These rights are freely transferable, and the outside investor is fully diversified with respect to the risk associated with each ESO. The price of the ESO rights contract is negotiated between the employee and the investor therefore measures the value of the ESO to the employee. Our model has different perspective: We assume that the employee granted ESOs cannot achieve full diversification, and consider this explicitly in the pricing model.

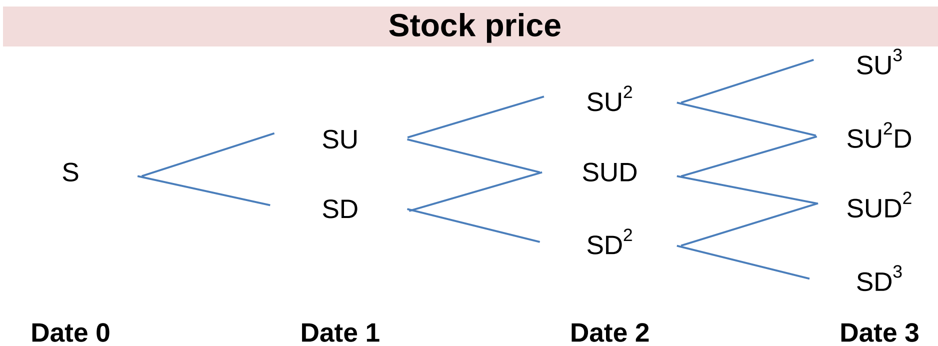
A further contribution of our model is to the pricing of restricted stock units. Current accounting regulations require that RSUs be priced at the market value of the stock on the grant date. Our model prices the RSUs and shows that their true economic value is far below the market value. We thus provide the basis for an "argument" with the IFRS and FASB.

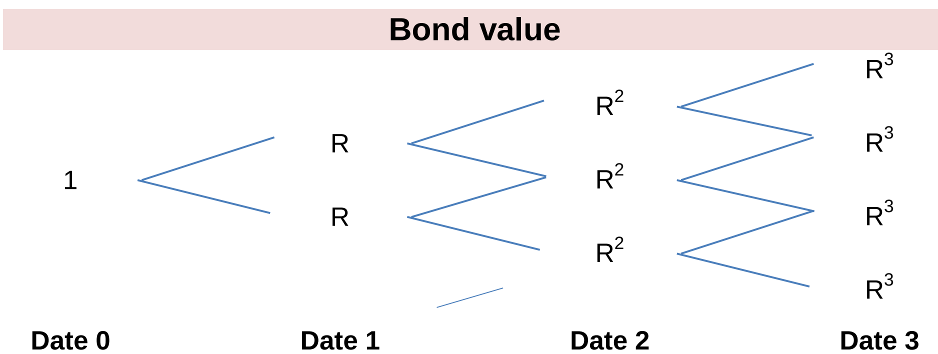
[1] See Chance (2004) and Rubinstein (1995). We find this discussion too philosophical and abstract to be useful.

2. A review of the binomial model

The binomial model of Cox, Ross and Rubinstein (1979) is, after the Black-Scholes (1973) model, the best-known option pricing model. In this section we review this model. The advantages of the binomial model are its simplicity, its easy computability, and (under appropriate assumptions) its congruence with the Black-Scholes framework.

The graphical version of this model is well-known:





The binomial model prices asset by establishing two *state prices*: The state price q_U is the per-dollar discount factor for dollars in the "Up" state and the state price q_D is the per-dollar discount factor for the "Down" state. Graphically:

Notice that $q_u + q_d$ price an asset that pays off 1 in each of the future states. The financial meaning of this property is that the sum of the state prices prices a risk-free asset, (i.e., $\frac{1}{R} = q_U + q_D$, where R is one-plus the risk-free interest rate). It can be shown that $q_U = \frac{R-D}{R(U-D)}$, $q_U = \frac{U-R}{R(U-D)}$.

Insert Box 1 about here

3. Public versus private state prices

Our model is based on the assumption that state prices depend on whether an individual can freely dispose or buy the asset under question. If an individual operates in competitive markets, then the asset is priced by "public state prices" $q_U^{public} = \frac{R-D}{R(U-D)}$ and $q_D^{public} = \frac{U-R}{R(U-D)}$ as defined in the previous section. If, however, the holder of the asset is restricted in selling the asset, we assume that a different set of state prices holds. We call these prices the "private state prices" and assume that

$$q_U^{private} = q_U^{public} - \delta$$
$$q_D^{private} = q_D^{public} + \delta$$

The easiest way to think about public versus private state prices is to consider a numerical example. Suppose that $q_U^{public} = 0.3,\ q_D^{public} = 0.6$. This means that an asset paying 1 in the Up state and 0 in the Down state is valued at 0.3 today and that an asset paying 0 in the Up and 1 in the Down is valued at 0.6 today. It also means that the interest rate is 11.11%:

$$r = R - 1 = \frac{1}{q_U^{public} + q_D^{public}} - 1 = \frac{1}{0.9} - 1 = 11.11\%$$

Now suppose that, following our model, restricted assets are priced by private state prices. If δ = 0.02 then:

$$q_U^{private} = 0.3 - \delta = 0.28$$
$$q_D^{private} = 0.6 + \delta = 0.62$$

The private state prices indicate that assets that pay off in the Up state are worth less than if they were traded and assets that pay off in the Down state are worth more. This means that a call option is worth less in the "private market" than in the "public market."[2] We assume that $q_U^{private} + q_D^{private} = 0.9$, so that the interest rate in the private market is the same as that in the public market.

The intuition behind the private state prices is that an individual who holds restricted assets—be they stock or employee stock options—suffers from non-diversification, and that this non-diversification

[2] Of course there is no "private market," but we use this as convenient shorthand.

expresses itself in the individual having a more-than-optimal amount in Up states and less-than-optimal amount in Down states of the world. This interpretation of non-diversification means that the non-diversified individual's state price for the Up state is less than the public price and that her state price for the Down state is more than the public price—if it were possible to diversify, the individual would transfer some of her consumption from the Up to the Down state. Behind this story is the assumption that when the individual is non-diversified, she is more exposed to the underlying asset's volatility. Consider, for example, an employee in XYZ Corp. who has been given employee stock options in XYZ as part of her compensation package. Since the employee's wages are already tied to the fortunes of XYZ, the addition of (restricted) stock options further increases the employee's dependence on the company. If the employee were allowed to optimally diversify by selling the options, she would purchase assets that are negatively correlated with the fortunes of XYZ, paying off more if XYZ ended in a Down state of the world. The fact that she cannot diversify leads her to value Up-state payoffs less than the market and Down-state payoffs more than the market valuation.

We express this non- diversification of the ESO holder by assuming that her private state prices value the Up state of the world is less than the public state prices and that the private state prices value the Down state of the world is more than the public state prices. This is what is meant by the conditions $q_U^{private} = q_U^{public} - \delta,\ q_D^{private} = q_D^{public} + \delta$.

The symmetry in δ means that $q_U^{private} + q_D^{private} = q_U^{public} + q_D^{public} = \frac{1}{R}$ and is meant to express the assumption that the non-diversified individual has the same access to borrowing/lending markets as the diversified individual.[3] Put differently, the derivation of the standard binomial model is made under the assumption of competitive markets, in which the risk free rate R is the same for borrowing and lending purposes. If δ is the same for the Up and Down states, this means that the perspective of the economic agent is changing, but his ability to access the borrowing/lending markets remains the same.

[3] The assumption of equal access can be weakened, but this does not materially affect our theoretical conclusions.

Private state prices lead to early call option exercise

A remarkable result of our public vs. private pricing model is that an individual who prices options using his private state prices will find it optimal to early-exercise a call option. To frame this result, recall that a standard result in option pricing theory is that the market price of call option on a non-dividend paying stock is always higher than the option's intrinsic value.[4] If, however, the individual prices an option using private state prices, this result no longer holds, and there is always a point where the individual would prefer early exercise to continued holding of the option.

Insert Box 2 about here

The spreadsheets on the next page show some sample calculations:

[4] This result is so well-known that it really requires no reference, but the interested reader can confirm it in Hull (2010).

Two numerical examples in Excel

	A	B	C
1	**SIMULATING PUBLIC AND PRIVATE CALL VALUATIONS** Valuing an at-the-money employee stock option (cell B26)		
2	Stock price, S	50.00	
3	Option exercise, X	50.00	
4	Option maturity, T	4	
5			
6	**Stock price process**		
7	μ, mean	15%	
8	σ, standard deviation	25%	
9	r, risk-free	2%	
10	Δt, time interval	0.004	<-- =1/250
11			
12	Up	1.0165	<-- =EXP(mu*delta_t+sigma*SQRT(delta_t))
13	Down	0.9849	<-- =EXP(mu*delta_t-sigma*SQRT(delta_t))
14	R	1.0001	<-- =EXP(rf*delta_t)
15			
16	**Public state prices**		
17	q_U^{Public}	0.4796	<-- =(R_-Down)/(R_*(Up-Down))
18	q_D^{Public}	0.5204	<-- =1/R_-B17
19			
20	**Private state prices**		
21	δ, non-diversification effect	0.01	
22	$q_U^{Private}$	0.4696	<-- =B17-delta
23	$q_D^{Private}$	0.5304	<-- =B18+delta
24			
25	Public option price (~Black-Scholes)	11.5246	<-- =Binomial_eur_call(Up,Down,R_,B2,B3,ROUND(B4/delta_t,0),0)
26	Private option price	4.1541	<-- =Binomial_eur_call(Up,Down,R_,B2,B3,ROUND(B4/delta_t,0),delta)
27	Intrinsic	0.0000	<-- =MAX(B2-B3,0)

	A	B	C
1	**SIMULATING PUBLIC AND PRIVATE CALL VALUATIONS** Valuing a deep-in-the-money employee stock option (cell B26)		
2	Stock price, S	80.00	
3	Option exercise, X	50.00	
4	Option maturity, T	1	
5			
6	**Stock price process**		
7	μ, mean	15%	
8	σ, standard deviation	25%	
9	r, risk-free	2%	
10	Δt, time interval	0.004	<-- =1/250
11			
12	Up	1.0165	<-- =EXP(mu*delta_t+sigma*SQRT(delta_t))
13	Down	0.9849	<-- =EXP(mu*delta_t-sigma*SQRT(delta_t))
14	R	1.0001	<-- =EXP(rf*delta_t)
15			
16	**Public state prices**		
17	q_U^{Public}	0.4796	<-- =(R_-Down)/(R_*(Up-Down))
18	q_D^{Public}	0.5204	<-- =1/R_-B17
19			
20	**Private state prices**		
21	δ, non-diversification effect	0.02	
22	$q_U^{Private}$	0.4596	<-- =B17-delta
23	$q_D^{Private}$	0.5404	<-- =B18+delta
24			
25	Public option price (~Black-Scholes)	31.1350	<-- =Binomial_eur_call(Up,Down,R_,B2,B3,ROUND(B4/delta_t,0),0)
26	Private option price	19.8878	<-- =Binomial_eur_call(Up,Down,R_,B2,B3,ROUND(B4/delta_t,0),delta)
27	Intrinsic	30.0000	<-- =MAX(B2-B3,0)

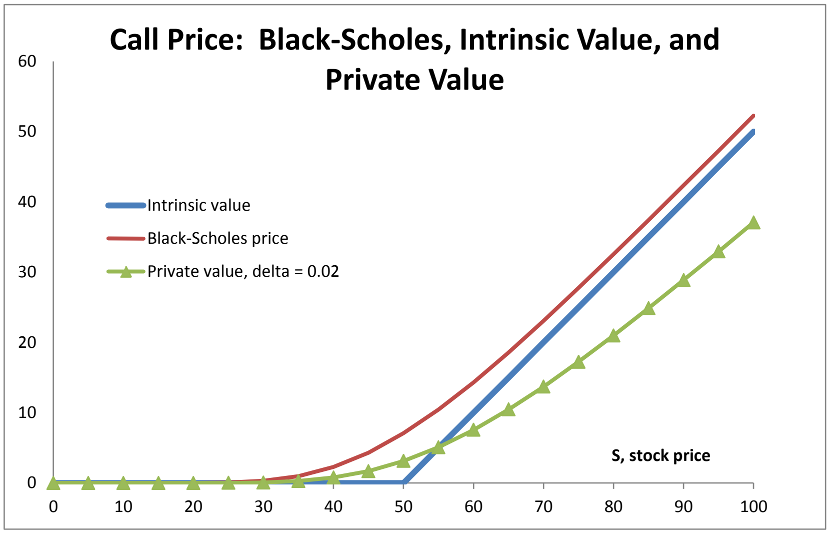
Notes to the examples: These spreadsheets show two typical calculations for the Black-Scholes and the private valuation of an option. In the top example we value at at-the-money option with 4 years to maturity. Given δ = 0.02, the Black-Scholes price (Cell B25) is 11.52 whereas the private valuation is 4.15. This example might be typical of a valuation of the valuation of an employee stock option at issue; it shows the dramatic difference between the valuation of the option were it to be marketable versus its valuation as a private, non-diversifiable security.

In the second example we value a deep-in-the-money ESO with one year to maturity. In this particular example, the private option price is 19.89 (cell B26), which is less than the intrinsic value of 30 (cell B27). Thus—in the absence of the opportunity to sell the option—the holder of the option would prefer to exercise early.

Technical note: The valuation in cells B25 uses a binomial model, which for the small time interval (Δt = 0.004) is very close to the Black-Scholes valuation.

Excel note: All of the tables and Excel example in the paper are included in an Excel notebook available from the authors.

When we graph the Black-Scholes price, the intrinsic value, and the private option price, we clearly see that the private price intersects the intrinsic value:



In the above drawing, this happens when S = 55.16. The operative conclusion: If the holder of an ESO is past the vesting period and the stock price > 55.16, then early exercise of the ESO is optimal.

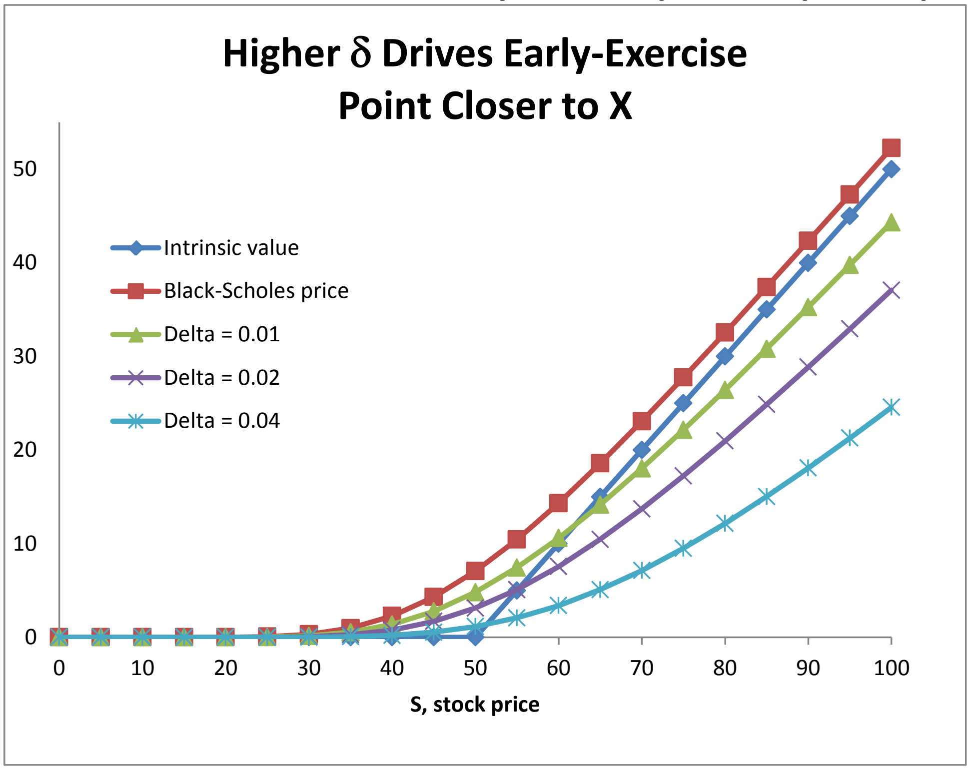
We can easily do sensitivity analysis on these computations to show the following:

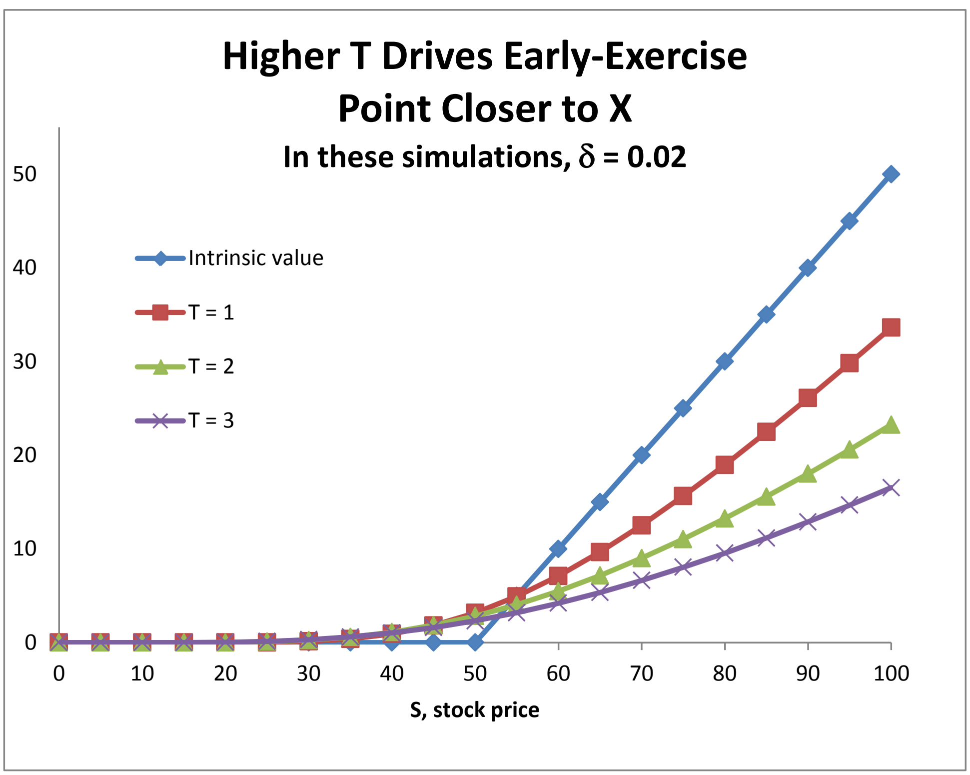
- The crossing point (where intrinsic value = private value) is always greater than S = X.
- The crossing point is a decreasing function of δ. Larger δ implies that the crossing point is closer to S = X.
- The crossing point is a decreasing function of time to option maturity. The larger the option maturity, the closer is the crossing point to S = X.

These points are illustrated in the graphs on the following page .

[Separate page]

The effect of δ and time to maturity T on the public vs private prices





4. Estimating the public versus the private state prices

In this section we calibrate our model and estimate the non-diversification measure δ using a proprietary data set of employee stock option grants and exercise records. This proprietary data set was obtained from Tamir-Fishman & Co., an Israeli based investment house which offers management services of share-based compensation programs. To respect the anonymity of the data, we identify the companies by a two-digit code and report the results on an aggregate level using two-digit SIC code.

The Tamir-Fishman database is comprised of complete histories of stock option grants, vesting structures, option exercises and cancellation events for employees in both private and public firms. The employees in the database work in firms which are located in Israel. These firms include subsidiaries of international companies and Israeli firms traded in foreign stock exchanges. We identify ninety four firms in the database that are either currently public or were public in the past and private firms that were acquired by a public firm and now serve as its subsidiary granted SOPs. These firms are traded in Israel, U.S. and European stock markets.[5]

We process the data according to the following criteria:

- Employees are sometimes forced to exercise their stock options. These forced exercises are usually results of job termination and mergers and acquisitions.[6] We are interested in voluntary exercise records and therefore exclude all exercise records that were made 100 days before or after the employee's job termination date. The 100-day period relates to the common practice that allows employees up to three months to exercise their stock options after they cease working in the company. We exclude 100 days preceding the job termination date to account for the case that the option exercise is part of a plan to cease working in the company.
- In order to price stock options we need to estimate the standard deviation of the underlying stock—a component in setting the value of the state prices as demonstrated in Section 2. We use historical volatility as a proxy of the underlying asset's standard deviation. As a result, we

[5] Since we are analyzing proprietary data, we cannot compare it to other databases or to employee behavior in other countries. We feel confident in our numerical calibration, however, since our data base represents a wide spectrum of firms listed in the U.S., Europe and Israel.

[6] Forced exercise due to job termination or as a result of mergers and acquisitions is a common practice. In case the company did not force early exercise and the employee exercised his option during 100 days from the job termination, we did not exclude his exercise records from our sample.

only relate to options that were granted while the underlying security was traded, and exclude exercise records in case the grant date occurred prior to the company's IPO

- We require a minimum of 14 trading days in a month. The reason is straightforward: Stocks with low trading volume underestimate the volatility measure. Therefore, we exclude exercise records in months that this criterion is violated.
- To refrain from bias in the estimation, we exclude exercise records in case a single exercise record resulted in less than 50 shares.
- A stock option with an exercise price of zero is parallel to restricted stock, since the option holder does not pay any amount upon exercise. Since we focus on ESOs, we exclude exercise records with exercise price lower than 0.1.
- We exclude ESO exercise records that were 100 days before the contractual expiration date of the option. As presented on Section 2, the private pricing model results in endogenous early exercise decision. Employees that exercise their options close to expiration are not suitable candidates for examining early exercise patterns and therefore cause to bias in the non-diversification estimation.

The final sample contains 33,294 employee-by-employee exercise records from sixty five companies. 83% of the final sample firms are traded in US markets, 9.2% in the Israeli market and 7.7% in European stock markets. 20% of the sample firms' shares are traded in more than one exchange. The sample period of the stock option grants is between 1995 and 2009. The sample period of the exercise records is between 1998 and 2009.

ESO parameters and data sources

The Black-Scholes model and the binomial model require six input parameters: The underlying security price, the option's exercise price, expected standard deviation of the underlying asset, risk free rate, time to expiration and dividend yield.[7] We estimate the ESO value on the grant date and on the exercise date. For each ESO exercise record we match the following estimation:

- <u>Historical volatility</u>: We calculate the historical volatility of the underlying security using the daily continuous compounded return. We require a minimum estimation period of 20 trading

[7] ASC 718 (previously FAS 123(R)) also requires that these input parameters will be included in the valuation model of the equity based compensation.

days from the firm's IPO (subject to 14 trading days in a month restriction). We expand the estimation window to 30 days and then use a rolling window estimation of 30 trading days.[8]

The proxy for the expected volatility is the historical volatility on the calculation date. For example, if we calculate the ESO value on the grant date, then the historical volatility of the underlying security on that date serves as the proxy of the expected volatility in the pricing model.

- Risk free rate: We match a risk free rate according to the currency the company's shares are traded. Hence, if the company's shares are traded in the NASDAQ we use the U.S. T-bill rate.
- Time to expiration: We use the original expiration date of the ESO. Due to insufficient data, ESOs exercise records before 2000 were excluded from the sample. In addition, exercise records of ex-employees in which the original expiration date is identical to the last date of exercise were excluded.[9] We also excluded exercise records of ESO grants of less than four years.
- Dividend yield: We assumed a dividend yield of zero for the sample firms. This assumption fits 80% of the sample firms during (and before) the sample period. This property fits new-economy firms, which according to Table 1 below, represents the vast majority of our sample.[10]
- Vesting period: We assume an average vesting period of three years. This assumption is relevant only to the estimation of ESO at the grant date (the option is already vested when the employee exercises it).

The calculation of the above parameters involved the use of the following data sources: Stock prices were obtained from CRSP, Tel-Aviv stock exchange (TASE) website, Yahoo! Finance and websites of the companies in the sample. Annual risk free rates were obtained from the Federal Reserve Statistical Release website (3 months T-bill) and from websites of central banks in Israel and Europe, such as the Bank of Israel website (MAKAM rates).[11]

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[8] We repeat our estimation using an historical volatility calculation of 126 trading days. The results are similar.

[9] The database is managed in a way that the expiration date changes according to the circumstances. For example, if the employee is no longer employed in the company, the expiration date is updated to three months after the job termination date. This update changes the estimation, so we exclude exercise records of grants that lack the original expiration date.

[10] We present how the inclusion of dividends changes our estimation results in the Excel spreadsheet that accompany this paper.

[11] MAKAM is a zero coupon bond issued by the Bank of Israel, parallel to one-year Treasury bill.

Descriptive statistics for the ESO parameters

In this subsection we provide descriptive statistics for the sample companies and the model parameters. The objective is twofold: First, data on ESO grants in scarce. Providing data about practices of this common equity-based compensation form can be useful to policy makers. Second, there is a benefit from the description of the parameters that are being used to estimate the ESO value.

Table 1 describes the industries of the sample companies according to the two-digit firm-level SIC codes. There is considerable heterogeneity in the firm industries in the sample. In addition, a major part of the firms comprising the dataset are new-economy firms related to computers, software, the internet, telecommunications or networking.[12] These firms represent 69.23% of the firms and 73.83% of the exercise records in the sample.

Table 1: Sample Firms Description

Industry	Two-digit firm-level SIC	Percentage (number of firms)	Percentage (number of employees)
Food And Kindred Products	20	1.54%	0.16%
Paper And Allied Products	26	1.54%	0.73%
Printing, Publishing, And Allied Industries	27	1.54%	0.17%
Chemicals And Allied Products	28	3.08%	0.39%
Industrial And Commercial Machinery And Computer Equipment	35	16.92%	28.71%
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	36	26.15%	38.48%
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	38	7.69%	7.22%
Communications	48	7.69%	7.73%
Wholesale Trade-durable Goods	50	1.54%	0.20%
Depository Institutions	60	1.54%	2.09%
Business Services	73	26.15%	6.64%
Engineering, Accounting, Research, Management, And Related Services	87	4.62%	7.48%

The summary statistics in Tables 2 and 3 present the original time to expiration of the options (in years) and on the remaining time to expiration of the options at the exercise date (in years), respectively. The remaining time until expiration of the option is used to estimate the value of the nondiversification measure δ on the exercise date and the original time to expiration of the option is used to value the option on the grant date. Table 2 presents a quite homogeneous picture: The average contractual option life ranges between eight to ten years, with some options grants for 16 years. Combined with the data of Table 3, it indicates that on average the ESOs in the sample are exercised

[12] New economy firms defined as companies with primary SIC codes 3570, 3571, 3572, 3576, 3577, 3661, 3674, 4812, 4813, 5045, 5961, 7370, 7371, 7372 and 7373 (See Hall and Murphy 2003). As mentioned, we identify the companies using 2-digit SIC codes.

when there are nearly two-thirds to half of the option term remaining. These findings are consistent with the findings of previous studies such as Huddart and Lang (1996) and Carpenter, Stanton and Wallace (2009). The sectors that deviate from this early exercise pattern are the food and kindred products and the paper and allied products (SIC codes 20 and 26, respectively).

Table 2: Time to maturity (in years) of the option

Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	8.087	1.918	16.008	4.003	6.005	10.005
Food And Kindred Products	6.283	0.931	8.005	4.268	5.851	6.923
Paper And Allied Products	5.225	0.839	9.005	4.003	5.003	5.003
Printing, Publishing, And Allied Industries	10.008	0.001	10.008	10.005	10.008	10.008
Chemicals And Allied Products	10.023	0.039	10.181	10.005	10.005	10.008
Industrial And Commercial Machinery And Computer Equipment	7.162	1.571	10.507	4.123	6.003	8.003
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	9.091	1.511	16.008	4.003	7.164	10.008
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	5.785	1.628	10.008	4.003	5.000	5.003
Communications	9.751	0.572	10.008	5.849	10.005	10.005
Wholesale Trade-durable Goods	9.312	1.270	10.433	6.499	9.501	10.005
Depository Institutions	5.741	0.761	7.247	4.003	6.000	6.000
Business Services	9.032	1.844	10.008	4.003	9.871	10.008
Engineering, Accounting, Research, Management, And Related Services	6.938	0.679	10.008	4.044	7.003	7.005
Entire sample, employees	8.066	1.921	16.008	4.003	6.005	10.005
Entire sample, executives (Directors and Officers)	8.684	1.728	10.079	4.003	7.005	10.008
Entire sample, exercise (cash)	8.941	1.809	16.008	4.003	7.045	10.008
Entire sample, SDS (same day sale)	8.080	1.917	16.008	4.003	6.005	10.005

This table reports the time to maturity of the option grants at the grant date. The time to maturity is measured as the number of years between the grant date and the expiration date of the option grant. The summary statistics are computed over all the exercise records in the sample period. The summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Table 3: Remaining time to maturity of the options (in years) at the exercise date

Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	4.669	2.334	9.978	0.274	2.871	6.564
Food And Kindred Products	1.868	0.770	2.975	0.529	1.104	2.555
Paper And Allied Products	1.527	0.768	4.003	0.288	0.852	1.979
Printing, Publishing, And Allied Industries	7.068	0.735	8.553	5.441	6.679	7.512
Chemicals And Allied Products	6.949	1.467	9.373	0.630	6.370	7.904
Industrial And Commercial Machinery And Computer Equipment	3.831	2.036	9.318	0.274	2.373	4.981
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	5.417	2.129	9.948	0.274	4.025	7.167
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	2.417	1.955	9.781	0.274	0.923	2.836
Communications	6.679	1.638	9.948	0.282	5.841	7.879
Wholesale Trade-durable Goods	8.493	1.093	9.663	5.975	8.600	8.992
Depository Institutions	3.101	1.317	6.197	0.395	1.962	4.196
Business Services	5.973	2.194	9.978	0.282	4.460	7.674
Engineering, Accounting, Research, Management, And Related Services	3.506	1.439	9.266	0.282	2.577	4.490
Entire sample, employees	4.630	2.333	9.978	0.274	2.830	6.460
Entire sample, executives (Directors and Officers)	5.749	2.115	9.748	0.477	4.013	7.504
Entire sample, exercise (cash)	5.238	2.399	9.660	0.282	3.586	7.274
Entire sample, SDS (same day sale)	4.664	2.333	9.978	0.274	2.866	6.554

This table provides the summary statistics over the sample period for the remaining term (in years) of the stock option at the exercise date. The remaining term is measured as the difference between the expiration date and the exercise date. The summary statistics is organized by the two-digit firm-level SIC categories ass reported by CRSP.

Table 4 reports the summary statistics of the stock price to the exercise price ratio in the sample. There is a difference in the ratios both across and within sectors. The highest ratios reflect run-ups in the stock market during our sample period. Specifically, these ratios stem from market run-ups during the end of the 1990s and the beginning of 2000. In general, the option exercise patterns present evidence on the persistence of early exercise behavior along with considerable heterogeneity both within and across sectors. These findings are consistent with the findings of previous studies such as Carpenter, Stanton and Wallace (2009) and Bettis, Bigjak and Lemmon (2005).

Table 4: The stock price to exercise price ratio at the exercise date

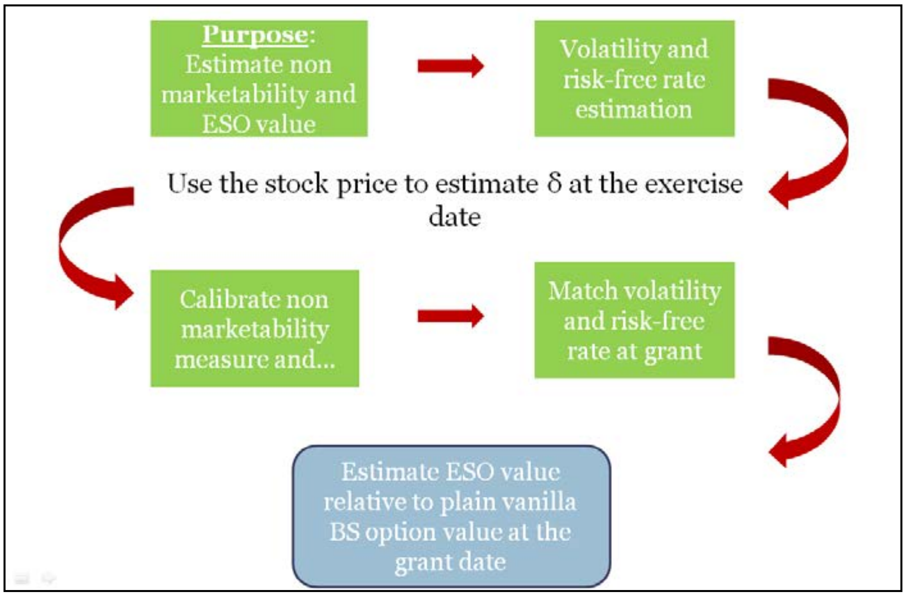
Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	2.877	3.114	39.767	1.001	1.381	3.278
Food And Kindred Products	2.603	0.903	3.972	1.358	1.613	3.393
Paper And Allied Products	2.512	0.962	5.590	1.257	1.868	2.563
Printing, Publishing, And Allied Industries	2.964	0.671	4.339	1.654	2.619	3.500
Chemicals And Allied Products	1.958	0.600	5.152	1.012	1.485	2.359
Industrial And Commercial Machinery And Computer Equipment	3.339	3.314	39.767	1.006	1.551	4.091
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	2.871	3.311	37.758	1.001	1.281	3.414
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	1.892	1.250	30.415	1.010	1.368	1.869
Communications	2.393	2.577	28.980	1.009	1.548	2.525
Wholesale Trade-durable Goods	2.159	0.894	5.531	1.093	1.744	2.196
Depository Institutions	1.562	0.184	1.878	1.019	1.440	1.695
Business Services	3.926	4.476	26.158	1.001	1.471	4.242
Engineering, Accounting, Research, Management, And Related Services	2.128	0.822	8.281	1.028	1.580	2.395
Entire sample, employees	2.860	3.138	39.767	1.001	1.371	3.236
Entire sample, executives (Directors and Officers)	3.342	2.301	25.000	1.033	1.757	4.176
Entire sample, exercise (cash)	4.316	5.774	33.163	1.022	1.412	3.883
Entire sample, SDS (same day sale)	2.865	3.080	39.767	1.001	1.381	3.273

This table provides the summary statistics over the sample period for the ratio of the stock price to exercise price at the exercise date. The summary statistics is organized by the two-digit firm-level SIC categories ass reported by CRSP.

Estimation of the non-diversification measure δ and the ESO value

Our purpose is to estimate the value of the ESO on the grant date. We employ the following procedure: In the first step we estimate the non-diversification measure δ on the exercise date, using the revealed preference approach. We have data on the ESO that the employee was granted, but no information on the employee's characteristics. The only time the employee reveals something about herself is on the day she exercises her stock option. Hence, using the revealed preference approach, we assume that the employee announced that from her point of view, the intrinsic value of the option is higher than the option value. As a result, we set the intrinsic value as the option value and use it to estimate the non-diversification measure δ. Under this estimation procedure, we calculate the input parameters required by the binomial framework on the exercise date (i.e., we use the stock price, the stock's historical volatility, time to maturity of the option, the dividend yield and the risk free rate on the exercise date). Then, using all the above parameters (including the assumed option price), we calculate δ.

In the second step we estimate the ESO value on the ESO's grant date. We use the estimated non-diversification measure (from the first step) and calibrate it into the private pricing model using the input parameters required by the binomial framework on the exercise date (i.e., we use the stock price, the stock's historical volatility, time to maturity of the option, the risk free rate, and the dividend yield on the exercise date. We also use the vesting period of the ESO). The only character of the model which is not time-adjusted is the non-diversification measure that we estimated in the first step, on the ESO's exercise date. The result is as estimation of the ESO value. Then, we calculate the value of a plain vanilla Black-Scholes option on the grant date (using the input parameters we used to calculate the ESO value on the grant date), and divide the private ESO value by the Black-Scholes option value. The outcome is a measure of the value of the ESO to the employee on the grant date. The figure below summarizes the estimation procedure.



The non-diversification measure δ is a specific parameter of the private pricing model and as far as we know, it is the first attempt to estimate it. Applying this model requires data on nontradable securities and financial instruments, and such information is usually non-public information. In the case of ESOs, for example, firms disclose only necessary information required by regulators. Such information relates mainly to executives, and not to non-executive employees. As a result, multinational comparison of δ is not-available.

Table 5 presents the estimation results of the non-diversification measure δ. This estimation is made on the option's (early) exercise date, using data on the stock price on that date, the risk free rate, volatility, dividend yield and exit rate.[13] In addition to industry partitions, we also calculate an average data for non-executive employees relative to executives and to exercise records of employees that

[13] We do not use vesting here because if the stock option can be exercised it means that it is after vesting.

continue to hold the stock after the option's exercise relative to exercise records of employees which sell the stock immediately after the option exercise (cashless exercise). The average non-diversification measure δ in the entire sample equals 0.025, with a similar tendency within the SIC sectors except for the food and kindred products and the paper and allied products (SIC codes 20 and 26, respectively), which have smaller δ values, suggesting that employees in these industries are more diversified than in other industries in the sample. This finding fits the common practice of a higher tendency to grant options in 'new-economy' firms. In addition, according to Table 5 results executives have a lower non-diversification measure relative to non-executive employees. It means that, ceteris paribus, a greater fraction of non-executive employees' wealth is, on average, in assets that cause them to be undiversified compared to executives. A possible explanation is that executives are more aware to the benefits of diversification (for example, Guiso and Jappelli (2008) find strong correlation between measures of financial literacy with the degree of portfolio diversification). It also implies that executives tend to exercise their stock options later (or closer to expiration) than rank and file employees.

Table 5: The non-marketability estimation using the stock price at the exercise date

Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	0.025	0.036	0.474	0.000	0.005	0.030
Food And Kindred Products	0.009	0.007	0.027	0.002	0.004	0.014
Paper And Allied Products	0.010	0.006	0.035	0.003	0.007	0.011
Printing, Publishing, And Allied Industries	0.008	0.004	0.019	0.004	0.005	0.009
Chemicals And Allied Products	0.021	0.031	0.285	0.001	0.007	0.025
Industrial And Commercial Machinery And Computer Equipment	0.020	0.031	0.414	0.000	0.003	0.025
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	0.032	0.044	0.469	0.000	0.006	0.039
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	0.023	0.026	0.319	0.000	0.010	0.025
Communications	0.023	0.029	0.353	0.000	0.008	0.025
Wholesale Trade-durable Goods	0.030	0.028	0.160	0.004	0.016	0.028
Depository Institutions	0.026	0.027	0.299	0.007	0.015	0.026
Business Services	0.023	0.033	0.474	0.000	0.005	0.029
Engineering, Accounting, Research, Management, And Related Services	0.018	0.021	0.291	0.000	0.008	0.021
Entire sample, employees	0.026	0.036	0.474	0.000	0.005	0.031
Entire sample, executives (Directors and Officers)	0.016	0.021	0.328	0.000	0.004	0.022
Entire sample, exercise (cash)	0.027	0.042	0.328	0.000	0.004	0.032
Entire sample, SDS (same day sale)	0.025	0.036	0.474	0.000	0.005	0.030

This table reports the non-marketability estimation at the exercise date. We value the non-marketability using the spesific charachters of each exercise record. Time to maturity is measured as the number of years between the exercise date and the original expiration date of the option grant. Annual risk free rate is adjusted according to the share's currency. Volatility is estimated by historical volatility of the share. The summary statistics are computed over all the exercise records in the sample period and grouped using two-digit firm-level SIC categories as reported in CRSP.

Table 6 presents the estimation results. These estimations calibrate the non-diversification measure from Table 5 with the annual risk free rate, historical volatility, contractual option life, vesting period and dividend yield—all on the grant date. All this data is used to calculate both the ESO private value and the plain vanilla Black-Scholes value on the grant date.

Table 6: ESO private value relative to Black-Scholes value (in percentage) on the grant date						
Industry	**Average**	**SD**	**Max**	**Min**	**1st quartile**	**4th quartile**
Full sample	48.23%	29.62%	99.97%	0.00%	22.01%	74.24%
Food and kindred products	69.06%	16.17%	89.36%	32.05%	57.79%	82.24%
Paper and allied products	68.32%	13.21%	88.35%	25.40%	62.34%	76.02%
Printing, publishing, and allied industries	65.30%	9.82%	76.42%	37.53%	60.05%	73.31%
Chemicals and allied products	50.35%	24.21%	93.75%	0.00%	31.24%	70.02%
Industrial and commercial machinery and computer equipment	57.97%	30.62%	99.97%	0.00%	29.92%	85.37%
Electronic and other electrical equipment and components, except computer equipment	41.21%	30.37%	99.80%	0.00%	13.27%	70.14%
Measuring, analyzing, and controlling instruments; photographic, medical and optical goods; watches and clocks	47.15%	23.59%	98.87%	0.00%	32.45%	60.94%
Communications	43.73%	24.55%	99.17%	0.00%	28.47%	62.45%
Wholesale trade-durable goods	30.92%	18.55%	74.89%	0.00%	21.45%	39.14%
Depository institutions	44.78%	17.30%	73.96%	0.00%	36.69%	56.42%
Business services	48.47%	28.91%	99.46%	0.00%	26.03%	74.18%
Engineering, accounting, research, management, and related services	50.92%	24.96%	97.57%	0.00%	33.22%	68.91%
Entire sample, employees	47.98%	29.68%	99.97%	0.00%	21.68%	74.05%
Entire sample, executives (Directors and Officers)	55.07%	27.00%	97.57%	0.00%	32.96%	78.73%
Entire sample, exercise (cash)	51.28%	32.34%	99.77%	0.00%	22.11%	80.45%
Entire sample, SDS (same day sale)	48.20%	29.59%	99.97%	0.00%	22.00%	74.20%

This table reports the value of the ESO using the private pricing model relative to a plain vanilla Black-Scholes value of the ESO on the grant date. The nonmarketability measure was estimated on the exercise date and calibrated into the model. Time to maturity is measured as the number of years between the grant date and the original expiration date of the option grant. Annual risk-free rate is adjusted according to the share's currency. The volatility is estimated by historical volatility of the stock. The summary statistics are computed over all the exercise records in the sample period, and grouped using two-digit firm-level SIC categories as reported in CRSP.

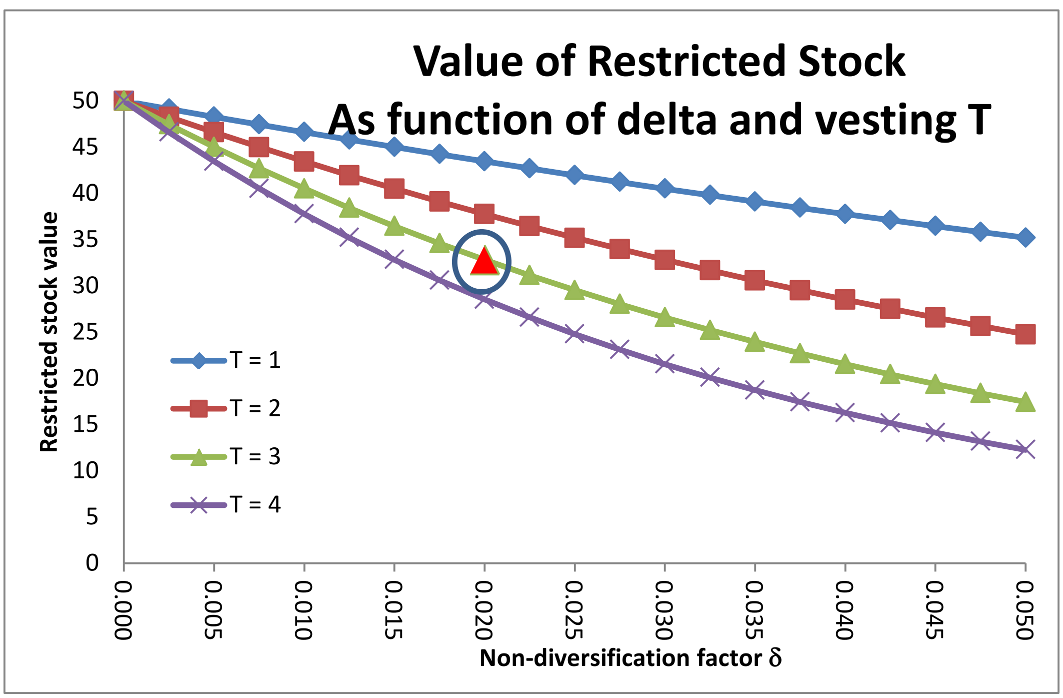
According to our estimation, the private ESO value is, on average, about 50% relative to a plain-vanilla BS value. In the food and kindred products and the paper and allied products (SIC codes 20 and 26, respectively) industries the value is higher—around 65%. Note that the first and fourth quintiles of these industries are also quite high. This finding results from a lower non-diversification measure and from smaller volatilities in these industries. In addition, the lower non-diversification measure of executives results in their higher estimation of ESO value.

Table 6 findings are consistent with predictions of other academic papers regarding ESO value. For example Meulbroek (2001) predicts that in new economy firms (with exhibit higher stock volatility), an undiversified manager would assign lower value to his stock options relative to an un-diversified manager from less volatile industries. Our findings are also consistent with the findings of Ikaheimo et al. (2006), which use the prices of tradable executive stock options, traded at the Helsinki stock exchange after the options are vested (which means these are transferable stock options). By analyzing 27,808 trades, Ikaheimo et al. (2006) show major underpricing of the ESO which can reach over 50% discount relative to Black-Scholes value. Since Ikaheimo et al. (2006) examine tradable stock options, the nonmarketability associated with these options should be less comparing to the standard case of untradeable stock options (which is the case of the stock options in Tamir-Fishman sample). It implies that the untradeable stock options the discount should be higher than the one found by Ikaheimo et al. (2006). In addition, the results ascribe higher option values to executives compared to ESO values to non-executive employees. Overall, these results point out on a relative high discount on equity based compensation.

5. Valuation of restricted stocks units

Restricted stocks are additional form of equity based compensation. In this compensation form, the employee is granted with either right to receive stocks once the vesting requirements are met or with stocks which are restricted until the vesting requirements are met. These compensation forms are called restricted stocks units (RSU) and restricted stock (RS), respectively, with RSU being the more common between the two. The key difference is that since RSU programs deliver the stocks to the employee only after the vesting period while RS programs deliver the (restricted) stocks at the grant date. Thus, RSU holders have no voting rights and usually are not entitled to receive dividend or dividend equivalents.

In this paper we use the private pricing model to value RSU. Since RSUs are not tradable only during the vesting period, we use private state prices with exogenous exit rate during the vesting period, (in which the stocks are non-tradable and the employee is subject to forfeit of the stocks upon job termination). After the vesting period, when the stock is tradable and unrestricted, we use public state prices without any other restrictions. Putting this differently, we basically discount the value of the stock on the vesting date and consider the non-marketability and forfeit restrictions while discounting. The following figure presents the value of a normalized restricted stock as a function of the non-marketability period for different values of δ.



The circled point shows that a restricted stock with a 3-year maturity and a non-diversification factor of δ = 0.02 is worth less than 70% of the value of a non-restricted stock.

6. Summary

The valuation of employee stock options (ESOs) and restricted stock units (RSUs) is problematic for both accountants and finance professionals. Most ESO valuation models use standard valuations based either on Black-Scholes or on lattice approach which have been adjusted to compensate for the special features of typical ESOs. The basis of these valuation models remains, however, the assumption of perfect markets, full employee diversification, and hedgability of the option grants.

In this paper we develop and calibrate a simple model that can account for non-diversification. This non-diversification framework allows us to price ESOs and RSUs. In the case of ESOs, our model provides an endogenous explanation of early ESO exercise that has previously been absent in the literature. Furthermore, the model has empirical content: We show—using a large, proprietary, data base of ESOs—that the model's non-diversification measure can be measured and applied directly to valuations.

The calibration results suggest that standard models overvalue ESOs and restricted stock units. We thus feel that our model has ready applications not just for the valuation of ESOs, but also for their accounting treatment.

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Box 1: Technical details about the binomial model (can be skipped!)

After the Black-Scholes model, the binomial option pricing model is the best-known option pricing framework. The bare bones of the model are explained in Section 2 of the paper. In this box we explain how we use the model for the pricing routines illustrated in the Excel spreadsheet that comes with this paper.

Suppose we are trying to price an option on a stock. Suppose further that the option has exercise price X and that the current price of the stock is S. We divide the interval (0,T) into n subintervals of length Δt: $\left\{0, \Delta t, 2\Delta t, ..., n\Delta t = T\right\}$. The binomial model assumes that in each time period Δt the price of the underlying asset either goes up by a factor U or down by factor D. R is one-plus the interest rate in the economy. U, D, and R are related to the size of the interval Δt, but for simplicity we have repressed this relationship in our notation. For completeness: If U and D are derived from a lognormal process with annual mean μ and standard deviation σ, then $U = \exp\left[\mu\Delta + \sigma\sqrt{\Delta t}\right]$, $D = \exp\left[\mu\Delta - \sigma\sqrt{\Delta t}\right]$, and $R = \exp\left[r\Delta t\right]$.

To prevent arbitrage, it must be that U ≥ R ≥ D. no-arbitrage opportunities exist in the market. Thus at time Δt is the stock price is either *SU* or *SD*. By recursion, the price of the underlying asset at time jΔt is one of the following prices: $\left\{SU^j, SU^{j-1}D, SU^{j-2}D^2, ..., SD^j\right\}$.

The one-period state prices are used to price the returns. We write them as $q_U = \left(R - D\right)/R\left(U - D\right)$ and $q_D = \left(U - R\right)/R\left(U - D\right)$. Given the state prices, we can easily price European or American options. For example, a European call and put on the stock with expiration *T* and strike X can be priced by:

$$Call = \sum_{i=0}^{n} \binom{n}{i} q_U^i q_D^{n-i} Max\left(SU^i D^{n-i} - X, 0\right)$$

$$Put = \sum_{i=0}^{n} \binom{n}{i} q_U^i q_D^{n-i} Max\left(X - SU^i D^{n-i}, 0\right)$$

American options can also be priced by using the state prices.[14]

[14] See, for example, Benninga, *Financial Modeling*, 2008, Chapter 19 for details.

Box 2: Pricing options using private state prices

The pricing of an option using private instead of public state prices is a simple technical matter. Recall from the previous box that the binomial pricing model for an option which is marketable is given by:

$$Call = \sum_{i=0}^{n} \binom{n}{i} \left(q_U^{public}\right)^i \left(q_D^{public}\right)^{n-i} Max\left(SU^i D^{n-i} - X, 0\right).$$

To price a restricted security like an employee stock option, we simply substitute the private state prices for the public prices in the above formula:

$$Restricted\ call = \sum_{i=0}^{n} \binom{n}{i} \left(q_U^{private}\right)^i \left(q_D^{private}\right)^{n-i} Max\left(SU^i D^{n-i} - X, 0\right).$$

Details of our pricing models are in an Excel notebook available from the authors by writing benninga@gmail.com .