

PARTIAL CROSS OWNERSHIP AND TACIT  
COLLUSION UNDER COST ASYMMETRIES

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# Partial cross ownership and tacit collusion under cost asymmetries\*

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## Abstract

We examine the effects that passive investments in rival firms have on the incentives to collude when firms have asymmetric marginal costs. We first show that unilateral investments by the most efficient firm in rivals may not only facilitate collusion but also raise the collusive price. We also show that the most efficient firm prefers to invest in its most efficient rival and only if this investment is insufficient to sustain collusion will it begin to invest in less efficient rivals. We then consider multilateral passive investments in rivals and show that an increase in such investments never hinders tacit collusion and we establish necessary and sufficient conditions for such investments to strictly facilitate tacit collusion.

**JEL Classification:** D43, L41

**Keywords:** partial cross ownership, repeated Bertrand oligopoly, asymmetric costs, tacit collusion, maverick firm

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# 1 Introduction

There are many cases in which firms acquire their rivals' stock as passive investments that give them a share in the rivals' profits but not in the rivals' decision making. These investments are often multilateral; examples of industries that feature complex webs of partial cross ownerships are the Japanese and the U.S. automobile industries (Alley, 1997), the global airline industry (Airline Business, 1998), the Dutch Financial Sector (Dietzenbacher, Smid, and Volkerink, 2000), the Nordic power market (Amundsen and Bergman, 2002), and the global steel industry (Gilo, Moshe, and Spiegel, 2006). While horizontal mergers are subject to substantial antitrust scrutiny and are often opposed by antitrust authorities, passive investments in rivals were either granted a de facto exemption from antitrust liability or have gone unchallenged by antitrust agencies in recent cases (Gilo, 2000).<sup>1</sup> This lenient approach towards passive investments in rivals stems from the courts' interpretation of the exemption for stock acquisitions "solely for investment" included in Section 7 of the Clayton Act.

In an earlier paper (Gilo, Moshe, and Spiegel, 2006) we began to investigate the merits of this lenient approach of courts and antitrust agencies towards passive investments in rivals. We showed that partial cross ownership (PCO) arrangements can facilitate tacit collusion among rival firms though cases exist in which such investments have no effect on the incentive of firms to collude. In particular we shows that when firm  $r$  increases its stake in a rival firm  $s$ , then collusion is never hindered, and that it will be surely facilitated if and only if (i) each firm in the industry holds a stake in at least one rival, (ii) the *maverick firm*

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<sup>1</sup>For example, to the best of our knowledge, Microsoft's investments in the nonvoting stocks of Apple and Inprise/Borland Corp. were not challenged by antitrust agencies while Gillette's 22.9% stake in Wilkinson Sword was approved by the DOJ after the DOJ was assured that this stake would be passive (see *United States v. Gillette Co.* 55 Fed. Reg. at 28,312). The FTC approved TCI's 9% stake in Time Warner which at the time was TCI's main rival in the cable TV industry and even allowed TCI to raise its stake in Time Warner to 14.99% in the future, after being assured that TCI's stake would be completely passive (see *Re Time Warner Inc.*, 61 FR 50301, 1996). The FTC also agreed to a consent decree approving Medtronic Inc.'s almost 10% passive stake in SurVivaLink, one of the only two rivals of Medtronic's subsidiary in the automated External Defibrillators market (In *Re Medtronic, Inc.*, FTC File No. 981-0324, 1998).

in the industry (the firm with the strongest incentive to deviate from a collusive agreement)<sup>2</sup> has a direct or an indirect stake in firm  $r$ ,<sup>3</sup> and (iii) firm  $s$  is not the industry maverick. These results were established however under the assumption that firms are symmetric and have the same marginal cost functions. In the current paper, we relax this assumption and examine the effect of PCO on the incentives of asymmetric firms to collude. This is obviously an important question since most industries feature cost asymmetries among firms.

To address this question we posit an infinitely repeated Bertrand oligopoly model in which firms have asymmetric marginal costs and acquire some of their rivals' (nonvoting) shares. This simple setting allows us to deal with the complexity generated by multilateral PCO. This complexity arises since under multilateral PCO arrangements, the profit of each firm, both under collusion as well as under deviation from collusion, potentially depends on the whole set of PCO in the industry and not only on the firm's own stake in rivals. Another advantage of this model is that PCO does not affect the equilibrium in the one shot case and therefore does not have any unilateral competitive effects. This allows us to focus on the effect of PCO on the ability of firms to engage in tacit collusion. We say that PCO arrangements facilitate tacit collusion if they expand the range of discount factors for which tacit collusion can be sustained.

In the first part of the paper we consider the case where only the most efficient firm in the industry invests in rivals. We show that even unilateral PCO by this firm may facilitate a collusive scheme in which all firms charge the same collusive price and divide the market equally among them. Due to cost asymmetries, each firm has a different monopoly price on which it wishes to collude. We assume that the collusive price is a compromise between the monopoly prices of the different firms. We show that when the most efficient firm invests in rivals, the collusive price would increase relative to the case where there are no PCO

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<sup>2</sup>The Horizontal Merger Guidelines of the US Department of Justice and FTC define maverick firms as "firms that have a greater economic incentive to deviate from the terms of coordination than do most of their rivals," see [www.usdoj.gov/atr/public/guidelines/horiz\\_book/hmg1.html](http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html). For an excellent discussion of the role that the concept of maverick firms plays in the analysis of coordinated competitive effects, see Baker (2002).

<sup>3</sup>Firm  $i$  has an indirect stake in firm  $r$  if it either has a stake in a firm that has a stake in firm  $r$ , or if it has a stake in a firm that has a stake in a firm that has a stake in firm  $r$ , and so on.

arrangements. Moreover, we show that the most efficient firm in the industry prefers to first invest in its most efficient rival both because this is the most effective way to promote tacit collusion and because such investment leads to a collusive price that is closer to the most efficient firm's monopoly price. Only if investment in the most efficient rival is insufficient to sustain a market-sharing scheme will the most efficient firm begin to invest in less efficient rivals. Less efficient firms do not wish to invest in rivals since such investments raise their collusive profits, thus implying that it is sufficient to give these firms smaller market shares in order to induce them to collude.

In the second part of the paper, we turn to multilateral PCO arrangements. In that case, cost asymmetries raise the complexity of the analysis considerably because the most efficient firm (firm 1) earns a positive profit even after the collusive agreement breaks down. Consequently, an increase in a firm  $i$ 's direct or indirect stake in firm 1 has conflicting effects on firm  $i$ 's incentive to collude. On one hand, a larger (direct or indirect) stake in firm 1 makes firm  $i$  less eager to deviate from collusion, because firm  $i$  obtains a larger share in the collusive profit of firm 1. But on the other hand, the increased stake of firm  $i$  in firm 1 also gives it a larger share in the profit of firm 1 once the collusive agreement breaks down. This second effect weakens the incentive of firm  $i$  to collude.

Despite these complications, we are able to show that an increase in the stake of firm  $r$  in firm  $s$  never hinders collusion and it will strictly facilitate collusion if and only if (i) the industry maverick has a direct or indirect stake in firm  $r$ , and (ii) firm  $s$  is not the industry maverick. When either (i) or (ii) fails to hold, the increase in firm  $r$ 's stake in firm  $s$  does not affect tacit collusion. These results extend our earlier results in Gilo, Moshe, and Spiegel (2006) and show that the results when firms have symmetric cost functions generalize to the asymmetric costs case.

Apart from Gilo, Moshe, and Spiegel (2006), we are aware of only one other paper, Malueg (1992), that studies the coordinated effects of PCO. His paper differs from ours in several ways as he considers a repeated symmetric Cournot game in which firms hold identical stakes in one another, and moreover, in his paper, it is effectively the controllers rather than the firms that hold stakes in rivals. This difference is important because investments by controllers do not feature the complex chain-effect interaction between the profits of rival

firms which is a main focus of our paper. Other papers that look at the competitive effects of PCO include Reynolds and Snapp (1986), Bolle and Güth (1992), Flath (1991, 1992), Reitman (1994), and Dietzenbacher, Smid, and Volkerink (2000). These paper however examine the unilateral effects of PCO arrangements in the context of static oligopoly models.<sup>4</sup>

The rest of the paper is organized as follows: Section 2 examines the effect of PCO on the ability of firms to achieve the fully collusive outcome in the context of an infinitely repeated Bertrand model with asymmetric firms. Section 3 examines the case where only the most efficient firm in the industry invests in rivals. Section 4, examines multilateral PCO arrangements. We conclude in Section 5.

## 2 Tacit collusion absent PCO

We examine the coordinated competitive effects of PCO in the context of an infinitely repeated Bertrand oligopoly model with  $n \geq 2$  firms. We assume that the  $n$  firms produce a homogenous product using a constant returns to scale technology and face a downward sloping demand function  $Q(p)$ . In every period, the  $n$  firms simultaneously choose prices and the lowest price firm captures the entire market. In case of a tie, consumers randomize among the set of lowest price firms, so each such firm gets an equal share of the total sales. The firms however have different marginal costs: let  $c_i$  be the (constant) marginal cost of firm  $i$  and assume  $c_1 < c_2 < \dots < c_n$ . That is, higher indices represent higher cost firms. The profit of firm  $i$  when it serves the entire market at a price  $p$  is given by

$$y_i(p) = Q(p)(p - c_i). \tag{1}$$

We assume that  $y_i(p)$  is quasi-concave and hence has a unique global maximizer,  $p_i^m$ . Since  $c_1 < c_2 < \dots < c_n$ , then  $p_1^m < p_2^m < \dots < p_n^m$ , where  $p_i^m$  is the monopoly price from firm  $i$ 's

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<sup>4</sup>See also Bresnahan and Salop (1986) and Kwoka (1992) for a related analysis of static models of horizontal joint ventures. Alley (1997) and Parker and Röller (1997) provide empirical evidence on the effect of PCO on collusion. Alley (1997) finds that failure to account for PCO leads to misleading estimates of the price-cost margins in the Japanese and U.S. automobile industries. Parker and Röller (1997) find that cellular telephone companies in the U.S. tend to collude more in one market if they have a joint venture in another market.

point of view.<sup>5</sup> That is, higher cost firms prefer higher monopoly prices. To ensure that all firms are effective competitors, we will make the following assumption:

**Assumption 1:**  $p_1^m > c_n$ .

When the stage game is infinitely repeated, firms may be able to engage in tacit collusion. The fact that different firms have different monopoly prices raises the obvious question of which price would they coordinate on in a collusive equilibrium? If side payments were possible, firms would clearly let firm 1, which is the most efficient firm, serve the entire market at a price  $p_1^m$  (e.g., firms 2, ...,  $n$  would all set prices above  $p_1^m$  and would make no sales). The firms will then use side payments to share the monopoly profit

$$y_1^m \equiv y_i(p_1^m) = Q(p_1^m)(p_1^m - c_1).$$

We rule out this possibility by assuming that side payments are not feasible, say due to the fear of antitrust prosecution.

Instead, we consider a collusive scheme led by firm 1. According to this scheme, firm 1 sets a price  $\hat{p}$ , which is some compromise between the monopoly prices of the various firms, i.e.,  $p_1^m \leq \hat{p} \leq p_n^m$ . All firms adopt  $\hat{p}$  and consumers randomize among them.<sup>6</sup> Consequently, each firm  $i$  serves  $\frac{1}{n}$  of the market and its collusive profit in every period is  $\frac{\hat{y}_i}{n}$ , where

$$\hat{y}_i \equiv y_i(\hat{p}) = Q(\hat{p})(\hat{p} - c_i), \quad i = 1, \dots, n. \quad (2)$$

Since by assumption,  $c_1 < c_2 < \dots < c_n$ , we have  $\hat{y}_1 > \hat{y}_2 > \dots > \hat{y}_n$ .

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<sup>5</sup>Revealed preferences and the fact that  $y_i(\cdot)$  has a unique maximizer imply that  $Q(p_i^m)(p_i^m - c_i) > Q(p_j^m)(p_j^m - c_i)$ , and  $Q(p_j^m)(p_j^m - c_j) > Q(p_i^m)(p_i^m - c_j)$ . Summing up the two inequalities and simplifying, yields  $Q(p_i^m)(c_j - c_i) > Q(p_j^m)(c_j - c_i)$ . Assuming without a loss of generality that  $j > i$ , and recalling that  $Q'(\cdot) < 0$ , it follows that  $p_j^m > p_i^m$ .

<sup>6</sup>That is, we study “pure” price fixing. A more elaborate collusive scheme might also involve market sharing in which case the market shares need not be equal. Such a scheme however requires firms to commit to preassigned output quotas and will be therefore much harder to enforce and easier for antitrust authorities to detect. For analysis of collusion in the context of a Bertrand duopoly with cost asymmetries which involves both price fixing and market market sharing, see Harrington (1991). Unlike in our paper where the collusive scheme is offered by firm 1, in Harrington it is determined by the Nash Bargaining solution.

Although  $\hat{p}$  can exceed firm 1's monopoly price,  $p_1^m$ , it cannot exceed it by too much. To see why, note that firm 1 can always ensure itself a profit of  $y_1(c_2) = Q(c_2)(c_2 - c_1)$  by undercutting  $c_2$  slightly and capturing the entire market.<sup>7</sup> To ensure that firm 1 has an incentive to collude at  $\hat{p}$ , it must be the case that  $\frac{\hat{y}_1}{n} \geq y_1(c_2)$ . Using Assumption 1,  $c_2 < c_n < p_1^m \leq \hat{p}$ ; hence  $\hat{p}$  must be bounded from above by  $\bar{p}$ , where  $\bar{p}$  is implicitly defined by  $\frac{y_1(\bar{p})}{n} = y_1(c_2)$  (see Figure 1). If this were not the case, i.e., if  $\hat{p} > \bar{p}$ , then firm 1 would have been better off deviating to  $c_2$  and capturing the entire market than colluding at  $\hat{p}$ . In other words, the collusive price,  $\hat{p}$ , is such that  $\hat{p} \in [p_1^m, \bar{p}]$ . Before proceeding, we add the following assumption which is illustrated in Figure 1:

**Assumption 2:**  $\bar{p} < p_2^m$ , where  $\bar{p}$  is given by the large root of the equation  $\frac{y_1(\bar{p})}{n} = y_1(c_2)$ .

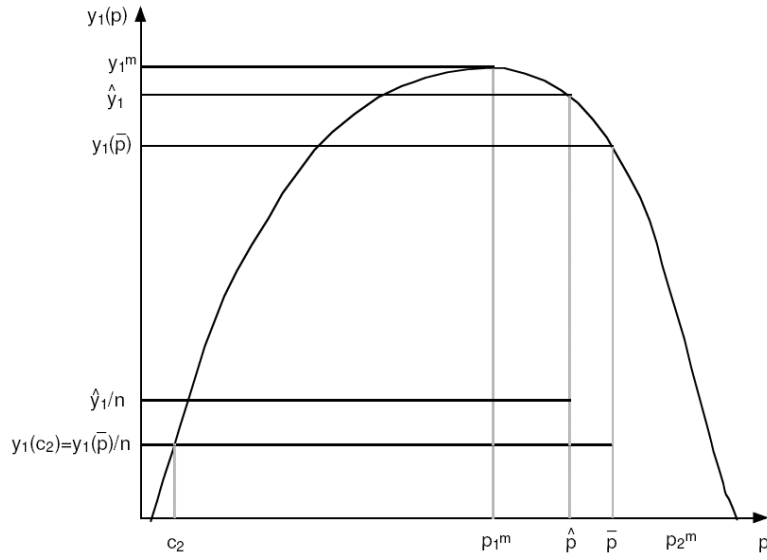


Figure 1: illustrating Assumption 3

Recalling that  $p_1^m < p_2^m < \dots < p_n^m$ , Assumption 2 implies that  $\bar{p} < p_i^m$  for all  $i = 2, \dots, n$ .<sup>8</sup> Since  $\hat{p} \leq \bar{p}$ , it follows that  $\hat{p} < p_i^m$  for all  $i = 2, \dots, n$ : the collusive price is

<sup>7</sup>This strategy can be thought of as the limit strategy in a discrete approximation to our model.

<sup>8</sup>To illustrate, suppose that  $Q(p) = A - p$ . Then,  $p_2^m = \frac{A+c_2}{2}$  and  $\bar{p} = \frac{A+c_1+\sqrt{(A-c_1)^2-4n(A-c_2)(c_2-c_1)}}{2}$ . Assumption 3 is satisfied if  $(A - c_2)((4n - 1)(c_2 - c_1) + c_1 - A) > 0$ . Since  $A > c_2$ , this is equivalent to  $A < (4n - 1)(c_2 - c_1) + c_1$ . Note however that  $A$  cannot be too low since Assumption 2 requires that  $A > 2c_n - c_1$ .



below the monopoly prices of all firms but 1. This implies in turn that the optimal deviation for firm  $i = 2, \dots, n$  is to set a price slightly below  $\hat{p}$ , while the optimal deviation for firm 1 is to set a price  $p_1^m$ . Following any deviation from the collusive scheme (including a deviation by firm 1), firms play a one shot Nash equilibrium; in this equilibrium, firm 1 serves the entire market a price equal to  $c_2$ .<sup>9</sup>

It should be noted that while we focus on collusion among all  $n$  firms, it is at least in principle possible that firm 1 will set a price below the marginal costs of some firms and will thereby exclude them. For instance, if firm 1 sets the collusive price (slightly below)  $c_{j+1}$ , then only firms  $1, \dots, j$  will produce and each firm  $i \leq j$  makes a profit of  $\frac{y_i(c_{j+1})}{j}$ . We rule out this possibility by imposing the following assumption:

**Assumption 3:**  $y_1(c_2) \geq \frac{y_1(c_{j+1})}{j}$  for all  $j \geq 2$ .

Assumption 3, together with the fact that in equilibrium  $\frac{\hat{y}_1}{n} \geq y_1(c_2)$  (otherwise firm 1 does not wish to collude), implies that firm 1 prefers to collude with all  $n - 1$  rivals at  $\hat{p} > c_n$  than collude with only  $j$  rivals by setting a price just below  $c_{j+1}$ .

We assume that the pricing decisions of each firm are effectively made by its controller (i.e., a controlling shareholder) whose ownership stake is  $\gamma_{ii}$ . We are now interested in finding conditions that will ensure that in a subgame perfect equilibrium of the infinitely repeated game, every controller will set  $\hat{p}$  in every period.

Using  $\delta$  to denote the intertemporal discount factor, the condition that ensures that the controller of firm  $i = 2, \dots, n$  does not wish to deviate from the collusive scheme is

$$\gamma_{ii} \frac{\hat{y}_i}{n(1-\delta)} \geq \gamma_{ii} \hat{y}_i. \quad (3)$$

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<sup>9</sup>If we rule out dominated strategies, then in equilibrium,  $p_1 = p_2 = c_2$  and  $p_j \geq c_j$  for all  $j \geq 3$ . The fact that firm 1 serves the entire market can be justified by viewing the equilibrium as the limit to a discrete approximation of the model (where in equilibrium, firm 1 undercuts firm 2 slightly). Thal (2010) also studies Bertrand oligopoly with cost asymmetry and considers harsher punishments, where following deviation by any firm but 1 the prices are as above, but following a deviation by firm 1 from the collusive scheme, prices are  $p_1 = p_i = c_1$  for some  $i \neq 1$  and  $p_j \geq c_j$  for all  $j \neq 1, i$ . Consequently, any firm that deviates from the collusive scheme (including firm 1) makes a profit of 0 in every period following the deviation. This outcome however cannot be seen as the limit to a discrete approximation of the model (firm 1 cannot profitably undercut  $c_1$  slightly) and it requires firm  $i$  to play a weakly dominated strategy.

The left-hand side of (3) is the infinite discounted sum of the share that firm  $i$ 's controller has in firm  $i$ 's collusive profit. The right-hand side of (3) is the controller's share in the one-time profit that firm  $i$  earns in the period in which it undercuts its rivals slightly and captures the entire market. Condition (3) can be rewritten as

$$\delta \geq \widehat{\delta} \equiv 1 - \frac{1}{n}.$$

That is, the controllers of firms  $2, \dots, n$  have an incentive to participate in the collusive scheme provided that they are sufficiently patient. This condition is identical to the well-known condition for tacit collusion in the context of an infinitely repeated Bertrand model with  $n$  identical firms (see e.g., Tirole, 1988, Ch. 6.3.2.1).

As for firm 1, its controller does not wish to deviate from the collusive scheme provided that

$$\gamma_{11} \frac{\widehat{y}_1}{n(1-\delta)} \geq \gamma_{11} \left( y_1^m + \frac{\delta y_1(c_2)}{1-\delta} \right), \quad (4)$$

where  $y_1^m$  is the one-time profit of firm 1 in the period in which it deviates to  $p_1^m$  and captures the entire market, and  $y_1(c_2)$  is the per-period profit of firm 1 in all subsequent periods. Since by definition,  $y_1^m > y_1(c_2)$ , condition (4) can be rewritten as

$$\delta \geq \widehat{\delta}_1(\widehat{p}) \equiv \frac{y_1^m - \frac{\widehat{y}_1}{n}}{y_1^m - y_1(c_2)}. \quad (5)$$

Note that since  $\frac{\widehat{y}_1}{n} > y_1(c_2)$  (otherwise firm 1 does not wish to collude),  $\widehat{\delta}_1 < 1$ . Also note that

$$\widehat{\delta}_1(\widehat{p}) > \frac{y_1^m - \frac{\widehat{y}_1}{n}}{y_1^m} \geq 1 - \frac{1}{n} \equiv \widehat{\delta},$$

where the weak inequality follows because  $y_1^m \geq \widehat{y}_1$ . Since  $\widehat{\delta}_1 > \widehat{\delta}$ , firm 1 is the *maverick firm* in the industry in the sense that it has the strongest incentive to deviate from a collusive agreement. Hence, (5) is a necessary and sufficient condition for the collusive price  $\widehat{p}$  set by firm 1 to be sustained as a subgame perfect equilibrium of the infinitely repeated game. Since  $\widehat{p} \geq p_1^m$ ,  $\widehat{y}_1$  increases as  $\widehat{p}$  is lowered towards  $p_1^m$ , and hence firm 1's controller would prefer to set  $\widehat{p} = p_1^m$  and thereby maximize his infinite discounted stream of collusive profits while relaxing constraint (5).

**Proposition 1:** *Absent PCO by firms, firm 1 is the industry maverick and its controller would like to set the collusive price equal to  $p_1^m$ . Collusion at  $p_1^m$  can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that  $\delta \geq \widehat{\delta}_1(p_1^m)$ .*

Having established the critical discount factor above which collusion can be sustained, we now examine how it is affected by the marginal costs of the  $n$  firms.

**Corollary 1:** *The critical discount factor above which collusion can be sustained,  $\widehat{\delta}_1(p_1^m)$ , decreases with  $c_1$  (tacit collusion is facilitated) increases with  $c_2$  (tacit collusion is hindered) and is independent of  $c_j$  for all  $j \geq 3$ .*

**Proof:** Given that  $\widehat{p} = p_1^m$ ,  $\widehat{y}_1 = y_1^m$ . Substituting in (5) and rearranging terms,

$$\widehat{\delta}_1(p_1^m) = \frac{1 - \frac{1}{n}}{1 - \frac{y_1(c_2)}{y_1^m}}.$$

Noting that  $\widehat{\delta}_1(p_1^m)$  is increasing with  $\frac{y_1(c_2)}{y_1^m}$ , it follows that we can now study the effect of a change in marginal costs on  $\widehat{\delta}_1(p_1^m)$  by studying its effect on  $\frac{y_1(c_2)}{y_1^m}$ . Recalling that  $y_1^m \equiv Q(p_1^m)(p_1^m - c_1)$ , it follows from the envelop theorem that  $\frac{\partial y_1^m}{\partial c_1} = -Q(p_1^m)$ ; hence,

$$\frac{\partial}{\partial c_1} \left( \frac{y_1(c_2)}{y_1^m} \right) = \frac{-Q(c_2)y_1^m + Q(p_1^m)y_1(c_2)}{(y_1^m)^2} < 0,$$

where the inequality follows since  $c_2 < p_1^m$  implies that  $Q(c_2) > Q(p_1^m)$  and since by definition  $y_1^m > y_1(c_2)$ . Consequently,  $\widehat{\delta}_1(p_1^m)$  decreases with  $c_1$ . Moreover, since  $c_2 < p_1^m$ , then  $y_1(c_2)$  increases with  $c_2$ , so  $\widehat{\delta}_1(p_1^m)$  also increases with  $c_2$ . Finally, it is easy to see that  $\widehat{\delta}_1(p_1^m)$  is independent of  $c_3, \dots, c_n$ . ■

### 3 Tacit collusion with unilateral PCO by firm 1

In this section we examine the competitive effects of unilateral PCO investments by firm 1 in rival firms. The competitive effects of multilateral PCO arrangements are considered in Section 4. We will now use  $\widehat{\delta}_1(\widehat{p})$  (the critical discount factor above which the a collusive scheme led by firm 1 can be sustained) as our measure of the ease of collusion; accordingly,

will say that PCO facilitates tacit collusion if it lowers  $\widehat{\delta}_1(\widehat{p})$ , and will say that PCO hinders tacit collusion if raises  $\widehat{\delta}_1(\widehat{p})$ .<sup>10</sup>

Specifically, assume that firm 1 invests in rivals and let  $\alpha_{12}, \dots, \alpha_{1n}$  be its ownership stakes in firms 2, ...,  $n$ . Since the collusive profit of each firm  $i$  is  $\frac{\widehat{y}_i}{n}$ , it follows that firm 1's infinite discounted stream of profits under collusion is

$$\frac{\widehat{y}_1 + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n(1-\delta)}.$$

If firm 1's controller deviates from the collusive scheme, then all rivals make a profit of zero, so the optimal deviation for the controller is to  $p_1^m$ , which is the price that maximizes  $\widehat{y}_1$ . Following the deviation, firms play the one shot Nash equilibrium, so firm 1's controller sets a price equal to  $c_2$ , and all rivals make zero profits. Hence, the resulting payoff of firm 1's is

$$y_1^m + \frac{\delta y_1(c_2)}{1-\delta},$$

exactly as in the absence of PCO. Consequently, the condition that ensures that firm 1's controller does not wish to deviate from the collusive scheme is now given by

$$\gamma_{11} \left( \frac{\widehat{y}_1 + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n(1-\delta)} \right) \geq \gamma_{11} \left( y_1^m + \frac{\delta y_1(c_2)}{1-\delta} \right), \quad (6)$$

or

$$\delta \geq \widehat{\delta}_1^{po}(\widehat{p}) \equiv \frac{y_1^m - \frac{\widehat{y}_1 + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n}}{y_1^m - y_1(c_2)}. \quad (7)$$

Notice that  $\widehat{\delta}_1^{po}(\widehat{p})$  is decreasing with each  $\alpha_{1i}$ : the larger the stakes of firm 1 in rival firms, the stronger is firm 1's incentive to collude. The reason is that the collusive payoff of firm 1 increases when it invests in rivals, while its payoff under deviation is unaffected because rival firms make a profit of 0 when firm 1 deviates, as well as in all future periods. Clearly, firm 1 does not have an incentive to invest in rivals up to the point where  $\widehat{\delta}_1^{po}(\widehat{p})$  drops below  $\widehat{\delta}$  since then firm 1 is no longer the industry maverick and its stakes in rivals no longer facilitates tacit collusion (in our model, PCO are irrelevant unless they facilitate collusion). Hence, we shall assume in the rest of this section that firm 1 remains an industry

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<sup>10</sup>Of course, the infinitely repeated game admits multiple subgame perfect equilibria. We restrict attention to the most collusive equilibrium and focus on  $\widehat{\delta}_1(\widehat{p})$  because this is a standard way to capture the notion of "ease of collusion."

maverick even when it holds PCO stakes in rivals. The following is a sufficient condition for this:

**Lemma 1:** *A sufficient condition for firm 1's to be the industry maverick when firm 1 makes unilateral PCO investments in rivals is that firm 1's profit following a break down in the collusive agreement is at least as high as its stake in the collusive profits of rivals:*

$$y_1(c_2) > \frac{\sum_{i \neq 1} \alpha_{1i} \hat{y}_i}{n-1}. \quad (8)$$

**Proof:** Given the condition in the lemma,

$$\widehat{\delta}_1^{po}(\hat{p}) \geq \frac{y_1^m - \frac{y_1^m + \sum_{i \neq 1} \alpha_{1i} \hat{y}_i}{n}}{y_1^m - y_1(c_2)} \geq \frac{y_1^m - \frac{y_1^m + (n-1)y_1(c_2)}{n}}{y_1^m - y_1(c_2)} = 1 - \frac{1}{n} \equiv \widehat{\delta}.$$

Hence firm 1 is the industry maverick. ■

In the rest of this section we will assume that (8) holds. With this assumption in place, firm 1's controller selects  $\hat{p}$  to maximize the infinite discounted sum of firm 1's collusive profits, given by the left-hand side of (6), subject to (7).

**Proposition 2:** *Suppose that firm 1 invests in rivals but still remains the industry maverick. Using  $\hat{p}^*$  to denote the optimal collusive price from firm 1's perspective, the following holds:*

- (i)  $\hat{p}^*$  is increasing with each  $\alpha_{1i}$  and is above firm 1's monopoly price:  $\hat{p}^* > p_1^m$ .
- (ii)  $\widehat{\delta}_1^{po}(\hat{p}^*)$  is decreasing with each  $\alpha_{1i}$  and is below  $\widehat{\delta}_1(p_1^m)$  which is the critical discount factor above which collusion can be sustained absent PCO.
- (iii) PCO in an efficient rival raises  $\hat{p}^*$  by less and lowers  $\widehat{\delta}_1^{po}(\hat{p}^*)$  by more than a similar PCO in a less efficient rival.

**Proof:** (i) Firm 1 chooses  $\hat{p}$  to maximize the left-hand side of (6). Given Assumption 1 and recalling that  $p_1^m < p_2^m < \dots < p_n^m$ , it follows that  $\hat{p}^*$  is increasing with each  $\alpha_{1i}$  and is above  $p_1^m$ .

(ii) Absent PCO, the critical discount factor above which collusion can be sustained is  $\widehat{\delta}_1(p_1^m)$ . Using (5) and (7) it is clear that,

$$\widehat{\delta}_1(p_1^m) > \frac{y_1^m - \frac{y_1^m + \sum_{i \neq 1} \alpha_{1i} y_i(p_1^m)}{n}}{y_1^m - y_1(c_2)} \geq \frac{y_1^m - \frac{\widehat{y}_1^* + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i^*}{n}}{y_1^m - y_1(c_2)} \equiv \widehat{\delta}_1^{po}(\widehat{p}^*),$$

where  $\widehat{y}_i^* \equiv Q(\widehat{p}^*)(\widehat{p}^* - c_i)$  and where the weak inequality follows because by revealed preferences,  $\widehat{y}_1^* + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i^* \geq y_1^m + \sum_{i \neq 1} \alpha_{1i} y_i(p_1^m)$ . To complete the proof, note that by the envelope theorem,

$$\frac{d\widehat{\delta}_1^{po}(\widehat{p}^*)}{d\alpha_{1i}} = -\frac{\frac{\widehat{y}_i}{n}}{y_1^m - y_1(c_2)} < 0.$$

(iii) Since  $c_2 < \dots < c_n$ , it follows that  $\widehat{y}_2^* > \dots > \widehat{y}_n^*$ , implying that PCO by firm 1 in an efficient rival raises  $\widehat{p}^*$  by less and lowers  $\widehat{\delta}_1^{po}(\widehat{p}^*)$  by more than does a similar investment in a less efficient rival. ■

Proposition 2 implies that investments by firm 1 in rivals do not only facilitate tacit collusion by lowering the critical discount factor above which tacit collusion can be sustained but also lead to a higher collusive price. The latter result arises because, due to its investment in rivals, firm 1 is interested in maximizing a weighted average of its own profit and the profits of the firms it invests in. The higher firm 1's investments in rivals, the bigger the weight that firm 1's assigns to the rivals' profits in its objective function. Maximizing the rivals' profits requires a higher monopoly price than the monopoly price from firm 1's own perspective.

If we assume that the capital market is perfectly competitive, then Proposition 2 implies that firm 1 will have an incentive to minimize its investments in rivals subject to being able to facilitate tacit collusion. To see why, note that when the capital market is perfectly competitive, firm 1 would have to pay a fair price for its rivals' shares and will therefore just break even on these shares. Hence the change in the payoff of firm 1's shareholders from investing in rivals will simply be equal to the change in firm 1's direct profit (i.e., excluding firm 1's share in rivals' profits). The latter is maximized at  $p_1^m$ . But since  $\widehat{p} > p_1^m$ , the direct profit of firm 1 decreases when it invests in rivals, so firm 1 will prefer to invest as little as possible in rivals subject to ensuring that the collusive scheme can be sustained.

Proposition 2 also suggests that to the extent that firm 1 invests in rivals at all, it would prefer to invest in its most efficient rival first. The reason for this is that such an investment leads to a collusive price that is closer to firm 1's monopoly price and also expands the range of discount factors above which collusion can be sustained. Only if investment in the most efficient rival is not sufficient to sustain collusion, does firm 1 begin to invest in the next efficient rival.

## 4 Tacit Collusion with multilateral PCO

In this section we turn to the case where all firms potentially invest in rivals. To this end, let  $\alpha_{ij}$  be firm  $i$ 's partial cross ownership stake in firm  $j$  and define the following  $n \times n$  PCO matrix:

$$A = \begin{pmatrix} 0 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & 0 & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & 0 \end{pmatrix}.$$

Row  $i$  in the matrix  $A$  specifies the stakes that firm  $i$  has in all rival firms, while column  $j$  in the matrix  $A$  specifies the stakes that rival firms hold in firm  $j$ . Since apart from rival firms each firm is also held by its controller and possibly by outside stakeholders, the sum of each column of  $A$  is strictly less than 1. Obviously, all diagonal terms in  $A$  are equal to 0.

### 4.1 The accounting profits under PCO

When firms hold stakes in each other, the profit of each firm potentially depends on the profits of *all* other firms in the industry. For instance, firm 1 may get a share  $\alpha_{12}$  of firm 2's profit which may reflect firm 2's share,  $\alpha_{23}$ , in the profit of firm 3, which in turn may reflect firm 3's share,  $\alpha_{31}$ , in the profit of firm 1. To express the profits of the  $n$  firms, let  $s \equiv (s_1, \dots, s_n)$  be a (row) vector of market shares, where  $s_i = \frac{1}{n}$  for all  $i$  under collusion and  $s_i = 1$  if firm  $i$  either deviates or charges the lowest price in the market. Given a price  $p$  and a vector of market shares  $s$ , the profit of firm  $i$ , including its share in the profits of

rivals, is given by

$$\pi_i(p; A) = s_i y_i(p) + \sum_{j \neq i} \alpha_{ij} \pi_j(p), \quad i = 1, \dots, n,$$

where  $y_i(p)$  is given by (1). The profits of the  $n$  firms under PCO arrangements are then implicitly defined by a system of  $n$  equations in  $n$  unknowns. Using matrix notations, we can write this system as

$$\pi(p; A) = sy(p) + A\pi(p), \quad (9)$$

where  $\pi(p; A) \equiv (\pi_1(p; A), \pi_2(p; A), \dots, \pi_n(p; A))'$  is a (column) vector of total profits and  $y(p) \equiv (y_1(p), y_2(p), \dots, y_n(p))'$  is a (column) vector of direct profits. Since the PCO matrix,  $A$ , is nonnegative and since the sum of each of its columns is strictly less than 1, system (9) is a Leontief system and therefore has unique nonnegative solution (see Berck and Sydsæter, Ch. 21.1 - 21.22, p. 111) defined by

$$\pi(p; A) = Bsy(p), \quad (10)$$

where  $B \equiv (I - A)^{-1}$  is an inverse Leontief matrix; the  $ij$ -th entry in  $B$ , denoted  $b_{ij}$ , specifies the aggregate share that the “real” equityholders of firm  $i$  (i.e., outside equityholders that are not part of the  $n$  firms) have in the accounting profit of firm  $j$ . Following Dorofeenko et al (2008) we will refer to  $b_{ij}$  is the “imputed share” that the “real” equityholders of firm  $i$  have in the profit of firm  $j$ .

Given the important role that the aggregate imputed shares matrix,  $B$ , plays in our analysis, we state the following result whose proof appears in Gilo, Moshe, and Spiegel (2006).

**Lemma 2:** *The aggregate imputed shares matrix  $B$  has the following properties:*

- (i)  $b_{ii} \geq 1$  for all  $i$ , and  $0 \leq b_{ij} < b_{ii}$  for all  $i$  and all  $j \neq i$ .
- (ii) Let  $i$  and  $j$  be two distinct firms. Then,  $b_{ij} = 0$  if and only if firm  $i$  does not have a direct or an indirect stake in firm  $j$ .<sup>11</sup>

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<sup>11</sup>We will say that firm  $i$  has no direct or indirect stake in firm  $j$ , and has no stake in a firm that has a stake in firm  $j$ , and has no stake in a firm that has a stake in a firm that has a stake in firm  $j$  and so on.



- (iii)  $b_{ii} > 1$  if and only if firm  $i$  has a direct or an indirect stake in some firm  $j$  which in turn has a direct or an indirect stake in firm  $i$  (i.e.,  $b_{ij} > 0$  and  $b_{ji} > 0$ ).
- (iv)  $\sum_{j=1}^n (1 - \sum_k \alpha_{kj}) b_{ji} = 1$  for all  $i$ , where  $1 - \sum_k \alpha_{kj}$  is the ownership share of “real” shareholder in the firm  $j$  and  $(1 - \sum_k \alpha_{kj}) b_{ji}$  is the imputed share of these shareholders in firm  $i$ .

To interpret Lemma 2, recall that  $b_{ij}$  is the aggregate imputed share that the real equityholders of firm  $i$  have in the accounting profit of firm  $j \neq i$  through the direct or indirect cross ownership of firm  $i$  in firm  $j$  and  $b_{ii}$  is the aggregate imputed share that the real equityholders of firm  $i$  have in the accounting profit of their own firm. Part (i) of Lemma 2 says that  $b_{ij} < b_{ii}$  for all  $i$  and all  $j \neq i$ . Part (ii) of the lemma says that the real equityholders of firm  $i$  will get a share in the profit of a rival firm  $j$  if and only if firm  $i$  has a direct or indirect stake in firm  $j$ . Part (iii) of the lemma says that if firm  $i$  has a direct or an indirect stake in some rival firm  $j$  and this firm in turn has a direct or an indirect stake in firm  $i$ , then the aggregate imputed share that a real equityholder of firm  $i$  will have in firm  $i$  will exceed 1. In other words, a 1% stake in firm  $i$  will give a “real” equityholder of firm  $i$  more than a 1% share in the firm’s profit. The reason for this surprising property is that multilateral cross ownership arrangements create a multiplier effect that results in an overstatement of the firms’ cash flows.<sup>12</sup> Part (iv) of the lemma ensures however that the aggregate imputed shares of “real” equityholders in each firm  $i$  sum up to 1. Hence, while the accounting profits of firms will overstate the total cash flows, the aggregate payoff of all real equityholders will sum up exactly to the total cash flows. Absent collusion, firm 1 monopolizes the market by undercutting  $c_2$  slightly, so the vector of market shares is  $s = (1, 0, \dots, 0)$  and  $y(p) = (y_1(c_2), 0, \dots, 0)'$ .<sup>13</sup> By (10) then, the equilibrium vector of profits,  $\pi^N = (\pi_1^N, \pi_2^N, \dots, \pi_n^N)'$ , is such that

$$\pi_j^N = b_{j1} y_1(c_2), \quad j = 1, \dots, n. \quad (11)$$

<sup>12</sup>See Dietzenbacher, Smid, and Volkerink (2000) and Dorofeenko et al (2008) for additional discussion of this effect of PCO.

<sup>13</sup>There are additional Nash equilibria: see Shelegia and Spiegel (2010).

## 4.2 Collusion with multilateral PCO

Under collusion, firm 1 sets a collusive price,  $\hat{p}$ , which all rival firms match. Since all  $n$  firms charge  $\hat{p}$ , consumers randomize among them, so the vector of market shares is  $s = (\frac{1}{n}, \dots, \frac{1}{n})$ . Using (10), the (column) vector of collusive profits,  $\hat{\pi} = (\hat{\pi}_1, \hat{\pi}_2, \dots, \hat{\pi}_n)'$ , is given by

$$\hat{\pi} \equiv \pi(\hat{p}; A) = B \frac{\hat{y}}{n},$$

where  $\hat{y} \equiv (\hat{y}_1, \dots, \hat{y}_n)'$ . Using (2), the collusive profit of each firm  $i$  is given by

$$\begin{aligned} \hat{\pi}_i &= \frac{1}{n} \sum_{j=1}^n b_{ij} \hat{y}_j \\ &= \frac{1}{n} \sum_{j=1}^n b_{ij} Q(\hat{p}) (\hat{p} - c_j) \\ &= \frac{\sum_{j=1}^n b_{ij}}{n} Q(\hat{p}) (\hat{p} - \bar{c}_i), \end{aligned} \tag{12}$$

where  $\bar{c}_i \equiv \frac{\sum_{j=1}^n b_{ij} c_j}{\sum_{j=1}^n b_{ij}}$  is the “imputed marginal cost” of firm  $i$  under PCO and is equal to a weighted average of the costs of the  $n$  firms. Since each  $y_i$  is quasi-concave,  $\hat{\pi}_i$  has a unique maximizer denoted  $p_i^m(A)$ . From firm  $i$ 's point of view,  $p_i^m(A)$  is the ideal collusive price. Using a revealed preferences argument, it is easy to show that  $p_j^m(A) > p_k^m(A)$  if and only if  $\bar{c}_j > \bar{c}_k$  (see Footnote 5). That is, firms with higher imputed marginal costs will prefer a higher collusive price. Note that in general  $\bar{c}_i$  can be larger or smaller than  $c_i$ , depending on whether firm  $i$ 's has a large stake in less efficient or in more efficient rivals. Consequently,  $p_i^m(A)$  may either exceed or fall short of firm  $i$ 's monopoly price,  $p_i^m$ . However, given that firm 1 is the most efficient in the industry, then  $p_1^m(A) > p_1^m$  whenever firm 1 it has a stake in at least one rival firm, i.e., whenever  $b_{1j} > 0$  for some  $j \neq 1$ .

It is also worth noting that if firm  $j$  is more efficient than firm  $k$ , i.e.,  $c_j < c_k$ , but firm  $j$  invests in less efficient rivals, while firm  $k$  invests in more efficient rivals, then  $\bar{c}_j > \bar{c}_k$ , and as a result,  $p_j^m(A) > p_k^m(A)$ .<sup>14</sup> In order to simplify the analysis, we will impose the

<sup>14</sup>To illustrate, consider an industry with 4 firms, in which firm 2 holds a share  $\alpha$  in firm 4 and firm 3 holds a share  $\alpha$  in firm 1; otherwise firms do not hold shares in each other. Straightforward calculations show that  $b_{11} = b_{22} = b_{33} = b_{44} = 1$ ,  $b_{24} = b_{31} = \alpha$ , and all other entries in  $B$  are 0's. Consequently,  $\bar{c}_1 = c_1$ ,  $\bar{c}_2 = \frac{c_2 + \alpha c_4}{1 + \alpha}$ ,  $\bar{c}_3 = \frac{c_3 + \alpha c_1}{1 + \alpha}$ , and  $\bar{c}_4 = c_4$ . Clearly then,  $\bar{c}_2 < \bar{c}_3$  if  $\alpha < \frac{c_3 - c_2}{c_4 - c_1}$  and  $\bar{c}_2 > \bar{c}_3$  if  $\alpha > \frac{c_3 - c_2}{c_4 - c_1}$ . For instance, if  $c_1 = 0$ ,  $c_2 = 0.1$ ,  $c_3 = 0.2$ , and  $c_4 = 0.3$ , then  $\bar{c}_2 < (>) \bar{c}_3$  if  $\alpha < (>) 0.33$ .

following assumption:

**Assumption 4:**  $\bar{c}_1 < \bar{c}_j$  for all  $i \neq 1$ .

Assumption 4 states that the imputed marginal cost of firm 1 is the lowest in the industry even when the PCO structure is taken into account. Since  $\bar{c}_1 < \bar{c}_j$ , the ideal collusive price of firm 1 is lower than the ideal collusive prices for all other firms, i.e.,  $p_1^m(A) < p_i^m(A)$  for all  $i \neq 1$ .

Given that firms get a share in the profits of rivals, it is possible, at least in principle, that the controller of some firm  $i$  will prefer to stop producing. Although the controller forgoes the direct profit of his “own” firm,  $\frac{b_{ii}\hat{y}_i}{n}$ , he boosts the market shares of all rival firms from  $\frac{1}{n}$  to  $\frac{1}{n-1}$  and thereby increases the share of firm  $i$  in the profits of rivals from  $\frac{1}{n} \sum_{j \neq i}^n b_{ij}\hat{y}_j$  to  $\frac{1}{n-1} \sum_{j \neq i}^n b_{ij}\hat{y}_j$ . In what follows we will assume that this strategy is never optimal.

**Assumption 5:**  $b_{ii}\hat{y}_i > \frac{1}{n-1} \sum_{j \neq i}^n b_{ij}\hat{y}_j$  for all  $i \neq 1$ .

Assumption 5 ensures that all  $n$  firms prefer to remain active. The assumption does not involve firm 1 since for firm 1 it is always true that  $b_{11}\hat{y}_1 > \frac{1}{n-1} \sum_{j \neq 1}^n b_{1j}\hat{y}_j$  because part (i) of Lemma 2 ensures that  $b_{11} > b_{1j}$  for all  $j \neq 1$  and because  $\hat{y}_1 > \hat{y}_j$  for all  $j \neq 1$ .

Firm 1 chooses the collusive price  $\hat{p}$  in order to maximize its collusive profit  $\hat{\pi}_1 = \frac{1}{n} \sum_{j=1}^n b_{1j}\hat{y}_j$ , subject to constraints that we will specify below and which ensure that no firm wishes to deviate from the collusive scheme. Since we focus on collusion among all  $n$  firms, we will also impose the following assumption which is the analog of Assumption 3:

**Assumption 6:** Firm 1 prefers to monopolize the market by undercutting  $c_2$  slightly than share the market with the  $j-1$  most efficient rivals by undercutting  $c_{j+1}$  slightly:  $b_{11}y_1(c_2) > \frac{1}{j} \sum_{k=1}^j b_{1k}y_k(c_{j+1})$ .

In order for collusion to be sustained, firm 1 must make a higher profit under collusion

than it makes absent collusion:

$$\underbrace{\frac{1}{n} \sum_{j=1}^n b_{1j} \widehat{y}_j}_{\widehat{\pi}_1} \geq \underbrace{b_{11} y_1(c_2)}_{\pi_1^N}.$$

Together with Assumption 6, this inequality ensures that firm 1 prefers to collude with all  $n - 1$  rivals than collude with only a subset of  $j$  rivals by undercutting  $c_{j+1}$  slightly. Notice that Assumption 6 is stronger than Assumption 3; rewriting the inequality as  $y_1(c_2) > \frac{y_k(c_{j+1})}{j} + \frac{\sum_{k=2}^j \frac{b_{1k}}{b_{ii}} y_k(c_{j+1})}{j}$ , it is easy to see that the right-hand side of the inequality exceeds the right-hand side of the inequality in Assumption 3, due to the fact that whenever  $p > c_2$ , firm 1 receives a share in the profits of rival firms.

Assumption 4 above ensures that  $p_1^m(A) < p_i^m(A)$  for all  $i \neq 1$ . Hence, the collusive price,  $\widehat{p}$ , will be at least as high as  $p_1^m(A)$ . To simplify the analysis we will now impose the analog of Assumption 2:

**Assumption 7:**  $\bar{p}(A) < p_i^m(A)$  for all  $i \neq 1$ , where  $\bar{p}(A)$  is the large root of the equation  $\pi_1(\bar{p}(A); A) = b_{11} y_1(c_2)$ .

Given that  $\pi_1(\bar{p}(A); A)$  is quasi-concave,  $\pi_1(\widehat{p}; A) < b_{11} y_1(c_2)$  for all  $\widehat{p} > \bar{p}(A)$ . Hence,  $\bar{p}(A)$  is the upper bound on the collusive price,  $\widehat{p}$ . Assumption 7 implies that the collusive price  $\widehat{p}$  is lower than the ideal collusive price for each firm  $i \neq 1$ . This implies in turn that when the controller of firm  $i \neq 1$  deviates from the collusive scheme, he slightly undercuts  $\widehat{p}$ . In that case,  $s_i = 1$  and  $s_j = 0$  for all  $j \neq i$ , so the direct profit of firm  $i$  is arbitrarily close to  $\widehat{y}_i$ , while the direct profit of all other firms is 0. When the deviant is the controller of firm 1, then  $s_1 = 1$  and  $s_j = 0$  for all  $j \neq 1$ , so the direct profit of all firms but 1 is 0. Consequently, firm 1's controller simply sets a price of  $p_1^m$  which maximizes  $\widehat{y}_1$ , so the resulting direct profit of firm 1 in the current period is  $y_1^m$ . By (10) then, the vector of current profits,  $\pi^{d_i} = (\pi_1^{d_i}, \pi_2^{d_i}, \dots, \pi_n^{d_i})'$ , is such that

$$\pi_j^{d_i} = \begin{cases} b_{j1} \widehat{y}_1^m & i = 1, \\ b_{ji} \widehat{y}_i & i \neq 1. \end{cases} \quad (13)$$

Once the collusive agreement breaks down, firms play the one shot Nash equilibrium in all subsequent periods; in this equilibrium, firm 1 monopolizes the market by slightly undercutting  $c_2$  and the vector of profits,  $\pi^N = (\pi_1^N, \pi_2^N, \dots, \pi_n^N)'$ , is given by (11).

Given the accounting profits of the  $n$  firms under collusion and following a deviation from the collusive scheme, the condition that ensures that the collusive scheme in which all firms charge a price  $\widehat{p}$  can be sustained as a subgame perfect equilibrium is

$$\frac{\gamma_{ii}\widehat{\pi}_i}{1-\delta} \geq \gamma_{ii} \left( \pi_i^{d_i} + \frac{\delta\pi_i^N}{1-\delta} \right), \quad i = 1, \dots, n. \quad (14)$$

The left-hand side of (14) is the infinite discounted payoff of firm  $i$ 's controller under collusion, consisting of the controller's share in firm  $i$ 's collusive profit. The right-hand side of (14) is the controller's share in firm  $i$ 's profit when it undercuts rivals slightly ( $\pi_i^{d_i}$  in the period in which firm  $i$  deviates and  $\pi_i^N$  in all subsequent periods). If (14) holds, no controller wishes to unilaterally deviate from the fully collusive scheme.

Before proceeding, we introduce the notion of “relative imputed shares” that will play an important role in what follows:

**Definition:** Let  $z_{ij} \equiv \frac{b_{ij}}{b_{ii}}$  be the relative imputed share that the equityholders of firm  $i$  have in firm  $j$  (relative to their imputed share in their “own” firm  $i$ ), and let  $Z$  be the relative imputed shares matrix whose characteristic element is  $z_{ij}$ .

Lemma 2 implies that  $z_{ii} = 1$ ,  $z_{ij} < 1$  for all  $i \neq j$ , and  $\frac{1}{n} \sum_{j=1}^n z_{ij} < 1$  for all  $i = 1, \dots, n$ .

**Lemma 3:** The collusive scheme whereby all firms charge  $\widehat{p}$  can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that

$$\underbrace{\frac{1}{n} \sum_{j=1}^n z_{ij} \widehat{y}_j}_{\frac{\widehat{\pi}_i}{b_{ii}}} > \underbrace{z_{i1} y_1(c_2)}_{\frac{\pi_i^N}{b_{ii}}}, \quad i = 1, \dots, n. \quad (15)$$

and

$$\delta \geq \widehat{\delta}^{po}(A) \equiv \max \left\{ \widehat{\delta}_1(A), \dots, \widehat{\delta}_n(A) \right\},$$

where

$$\widehat{\delta}_1(A) \equiv \frac{y_1^m - \frac{1}{n} \sum_{j=1}^n z_{1j} \widehat{y}_j}{y_1^m - y_1(c_2)}, \quad (16)$$

and

$$\widehat{\delta}_i(A) \equiv \frac{\widehat{y}_i - \frac{1}{n} \sum_{j=1}^n z_{ij} \widehat{y}_j}{\widehat{y}_i - z_{i1} y_1(c_2)}, \quad i = 2, \dots, n, \quad (17)$$

with  $0 < \widehat{\delta}_i(A) < 1$  for all  $i = 1, \dots, n$ .

**Proof:** Using equations (11), (12), and (13), the definition of  $z_{ij}$ , the necessary condition (14) for collusion at  $\widehat{p}$  can be rewritten as

$$\delta(y_1^m - y_1(c_2)) \geq y_1^m - \frac{1}{n} \sum_{j=1}^n z_{1j} \widehat{y}_j, \quad (18)$$

and

$$\delta(\widehat{y}_i - z_{i1} y_1(c_2)) \geq \widehat{y}_i - \frac{1}{n} \sum_{j=1}^n z_{ij} \widehat{y}_j, \quad i = 2, \dots, n. \quad (19)$$

By definition,  $y_1^m > y_1(c_2)$  and  $y_1^m > \widehat{y}_1 > \frac{1}{n} \sum_{j=1}^n z_{1j} \widehat{y}_j$ , where the last equality follows because  $\widehat{y}_1 > \widehat{y}_2 > \dots > \widehat{y}_n$  and because  $\frac{1}{n} \sum_{j=1}^n z_{1j} < 1$ . Hence, both sides of (18) are positive. By (15),  $\frac{1}{n} \sum_{j=1}^n z_{1j} \widehat{y}_j > y_1(c_2)$  (recall that  $z_{11} = 0$ ), so  $\widehat{\delta}_1(A)$ , which is the value of  $\delta$  at which (18) holds with equality, is between 0 and 1.

As for  $i \neq 1$ , then by Assumption 4,  $\widehat{y}_i > \frac{1}{n-1} \sum_{j \neq i} z_{ij} \widehat{y}_j$ . Adding  $\frac{\widehat{y}_i}{n}$  to both sides and rearranging, yields  $\widehat{y}_i > \frac{1}{n} \sum_j z_{ij} \widehat{y}_j$ . Together with (15),

$$\widehat{y}_i > \frac{1}{n} \sum_j z_{ij} \widehat{y}_j > z_{i1} y_1(c_2).$$

These inequalities in turn ensure that both sides of (19) are strictly positive and that each  $\widehat{\delta}_i(A)$ , which is the value of  $\delta$  at which (19) holds with equality, is between 0 and 1. ■

Absent Assumption 7, the collusive price may be set above  $p_i^m(A)$  for some  $i$ , in which case, if firm  $i$ 's controller deviates from the collusive scheme he will deviate to  $p_i^m(A)$ . In that case,  $\widehat{y}_i$  in (12) will have to be replaced by  $y_i(p_i^m(A))$ .

It is easy to see from Lemma 3 that the incentives of firms to collude depend on cross ownership only through the matrix  $Z$  whose characteristic element is  $z_{ij}$ . In what follows we shall therefore examine how changes in cross ownership affect the matrix  $Z$  and consequently the critical discount factors above which firms wish to collude.

### 4.3 A firm increases its stake in a rival firm by buying shares from an outsider or from the rival's controller

Now, suppose that firm  $r$  increases its stake in firm  $s$ ,  $\alpha_{rs}$  by  $\omega$ . The resulting new PCO matrix is  $A^\omega$ ; it differs from the original PCO matrix only in that its  $rs$ -th entry is  $\alpha_{rs} + \omega$

rather than  $\alpha_{rs}$ . Our main question is whether  $\widehat{\delta}_i(A^\omega)$  is higher or lower than  $\widehat{\delta}_i(A)$ .

To address this question, note from equations (16) and (17) that a change in cross ownership affects the critical discount factors above which firms would like to collude only through its affect on the matrix  $Z$ , which specifies the relative imputed shares of firms in their rivals. Let  $Z^\omega$  be the matrix of relative imputed shares following an increase in firm  $r$ 's stake in firm  $s$  by  $\omega$ . Using Lemma A1 in Gilo, Moshe, and Spiegel (2006), it follows that the  $ij$ 'th entry in the matrix  $Z^\omega$  is given by

$$z_{ij}^\omega = \frac{b_{ij}^\omega}{b_{ii}^\omega} = \frac{b_{ij} + \varepsilon_i b_{sj}}{b_{ii} + \varepsilon_i b_{si}}, \quad \varepsilon_i = \frac{\omega b_{ir}}{1 - \omega b_{sr}} \geq 0. \quad (20)$$

Now, it follows from equations (16) and (17) that

$$\frac{\partial \widehat{\delta}_i(A)}{\partial \omega} = \sum_{j=1}^n \left( \frac{\partial \widehat{\delta}_i(A)}{\partial z_{ij}} \frac{\partial z_{ij}^\omega}{\partial \omega} \right). \quad (21)$$

Equation (16) implies immediately that  $\frac{\partial \widehat{\delta}_1(A)}{\partial z_{1j}} < 0$  for all  $j$  while equation (17) implies that  $\frac{\partial \widehat{\delta}_i(A)}{\partial z_{ij}} < 0$  for all  $i \neq 1$  and all  $j \neq 1$ . Moreover, equation (17) implies that

$$\begin{aligned} \frac{\partial \widehat{\delta}_i(A)}{\partial z_{i1}} &= \frac{-\frac{\widehat{y}_1}{n} (\widehat{y}_i - z_{i1} y_1(c_2)) + y_1(c_2) \left( \widehat{y}_i - \frac{1}{n} \sum_{j=1}^n z_{ij} \widehat{y}_j \right)}{(\widehat{y}_i - z_{i1} y_1(c_2))^2} \\ &= \frac{-\widehat{y}_i \left( \frac{\widehat{y}_1}{n} - y_1(c_2) \right) + \frac{y_1(c_2)}{n} \left( z_{i1} \widehat{y}_1 - \sum_{j=1}^n z_{ij} \widehat{y}_j \right)}{(\widehat{y}_i - z_{i1} y_1(c_2))^2} \\ &= -\frac{\widehat{y}_i \left( \frac{\widehat{y}_1}{n} - y_1(c_2) \right) + \frac{y_1(c_2)}{n} \sum_{j \neq 1}^n z_{ij} \widehat{y}_j}{(\widehat{y}_i - z_{i1} y_1(c_2))^2} < 0, \end{aligned}$$

where the inequality follows because by assumption,  $\frac{\widehat{y}_1}{n} \geq y_1(c_2)$  (otherwise firm 1 has no incentive to collude) and  $\sum_{j \neq 1}^n z_{ij} \widehat{y}_j = z_{ii} \widehat{y}_i + \sum_{j \neq 1, i}^n z_{ij} \widehat{y}_j \geq \widehat{y}_i > 0$  (recall that  $z_{ii} = 1$ ). Hence,

**Lemma 4:**  $\frac{\partial \widehat{\delta}_i(A)}{\partial z_{ij}} < 0$  for all  $i$  and all  $j$ : the critical discount factor above which firm  $i$  wishes to collude is a strictly decreasing function of each of firm  $i$ 's relative imputed shares in rival firms.

Lemma 4 implies that in order to determine the effect of the increase in firm  $r$ 's stake in firm  $s$  by  $\omega$  on firm  $i$ 's incentive to collude, we only need to know how it affects the  $i$ 'th

row in the relative imputed shares matrix  $Z$ . To this end, straightforward differentiation yields

$$\frac{\partial z_{ij}^\omega}{\partial \omega} = \frac{b_{ii} \left( b_{sj} - \frac{b_{si}b_{ij}}{b_{ii}} \right)}{(b_{ii} + \varepsilon_i b_{si})^2} \times \frac{b_{ir}}{(1 - \omega b_{sr})^2}. \quad (22)$$

Theorem 1 in Zeng (2001) ensures that  $b_{sj} \geq \frac{b_{si}b_{ij}}{b_{ii}}$  for all  $j$ . Intuitively,  $b_{sj}$  is the imputed share of firm  $s$  in firm  $j$ , while  $\frac{b_{si}b_{ij}}{b_{ii}}$  is the part of firm  $s$ 's imputed share in firm  $j$  which is due to firm  $i$ . To see why, let  $A^{-i}$  be the modified PCO matrix derived from  $A$  by setting to zero its  $i$ -th row and  $i$ -th column (i.e., “eliminating PCO that involve firm  $i$  from the PCO matrix”), and let  $B^{-i} = (I - A^{-i})^{-1}$  be the associated matrix of imputed shares. Theorem 1 in Zeng (2001) implies that the  $sj$ -th element of  $B^{-i}$ ,  $b_{sj}^{-i}$ , is such that

$$b_{sj}^{-i} = \frac{b_{ii} \left( b_{sj} - \frac{b_{si}b_{ij}}{b_{ii}} \right)}{b_{ii}}, \quad \text{for all } i \neq s. \quad (23)$$

Therefore, the part of firm  $s$ 's imputed share in firm  $j$  which is due to firm  $i$  is equal to

$$b_{sj} - b_{sj}^{-i} = \frac{b_{si}b_{ij}}{b_{ii}}.$$

Since  $b_{sj}$  is the entire imputed share of firm  $s$  in firm  $j$  while  $\frac{b_{si}b_{ij}}{b_{ii}}$  is only part of it, it is obvious that  $b_{sj} \geq \frac{b_{si}b_{ij}}{b_{ii}}$ . Note also that if firm  $s$  has a stake in firm  $j$  only due to firm  $i$  (i.e., absent firm  $i$  there is no direct and indirect share of firm  $s$  in firm  $j$ ,  $b_{sj}^{-i} = 0$ ), then  $b_{sj} = \frac{b_{si}b_{ij}}{b_{ii}}$  for all  $s \neq j$ .

We now use equation (22) to prove the following result which generalizes Theorem 1 in Gilo, Moshe, and Spiegel (2006) to the case of asymmetric firms.

**Theorem 1:** *Starting with a PCO matrix  $A$ , suppose that firm  $r$  increases its stake in firm  $s$  by some  $\omega > 0$ , so that the new PCO matrix  $A^\omega$  differs from  $A$  only with respect to the  $rs$ -th entry which is increased by  $\omega$ . Then*

- (i)  $\widehat{\delta}_s(A^\omega) = \widehat{\delta}_s(A)$ ,
- (ii)  $\widehat{\delta}_i(A^\omega) = \widehat{\delta}_i(A)$  if  $b_{ir} = 0$  (firm  $i$  has no direct or indirect stake in the acquiring firm  $r$ ), and
- (iii)  $\widehat{\delta}_i(A^\omega) < \widehat{\delta}_i(A)$  otherwise, i.e., for all  $i \neq s$  and  $b_{ir} > 0$ .



**Proof:** (i) Equation (22) implies that if  $i = s$  (firm  $i$  is the target firm  $s$ ), then  $\frac{\partial z_{sj}^\omega}{\partial \omega} = 0$  for all  $j$ . Hence, by Lemma 4,  $\widehat{\delta}_s(A^\omega) = \widehat{\delta}_s(A)$ .

(ii) Equation (22) implies that if  $b_{ir} = 0$  (firm  $i$  has no direct and indirect stake in the investing firm  $r$ ), then  $\frac{\partial z_{ij}^\omega}{\partial \omega} = 0$  for all  $j$ . Again, by Lemma 4,  $\widehat{\delta}_i(A^\omega) = \widehat{\delta}_i(A)$ .

(iii) Now suppose that  $i \neq s$  and  $b_{ir} > 0$ . When  $j = s$ , then  $b_{ii} \left( b_{sj} - \frac{b_{si}b_{ij}}{b_{ii}} \right) = b_{ii} \left( b_{ss} - \frac{b_{si}b_{is}}{b_{ii}} \right) > 0$ , where the inequality follows because part (i) of Lemma 2 establishes that  $b_{ij} < b_{ii}$  for all  $j \neq i$ . Together with the fact that  $b_{ir} \geq 0$ , it follows from equation (22) that  $\frac{\partial z_{ij}^\omega}{\partial \omega} \geq 0$  for all  $i$  and all  $j$ , with a strict inequality for  $j = s$ . Hence, by Lemma 4,  $\widehat{\delta}_i(A^\omega) < \widehat{\delta}_i(A)$  for all  $i \neq s$ . ■

Theorem 1 shows that an increase in the stake of firm  $r$  in firm  $s$  strictly facilitates collusion, except in two special cases in which it has no effect on tacit collusion. The first special case arises when the target firm, firm  $s$ , is the industry maverick. The reason for this is as follows: Lemma A1 in Gilo et al. (2006) implies that following an increase in firm  $r$ 's stake in firm  $s$ , the imputed share of firm  $s$  in any firm  $j$  becomes  $b_{sj}^\omega = b_{sj} + \varepsilon_s b_{sj}$ , where  $\varepsilon_s = \frac{\omega b_{sr}}{1 - \omega b_{sr}}$ . Hence, the relative imputed share of firm  $s$  in firm  $j$  is  $z_{sj}^\omega = \frac{b_{sj}^\omega}{b_{ss}^\omega} = \frac{b_{sj} + \varepsilon_s b_{sj}}{b_{ss} + \varepsilon_s b_{ss}} = \frac{b_{sj}}{b_{ss}}$ , which is independent of  $\omega$ . As a result, firm  $s$ 's incentive to collude is not affected by  $\omega$ . More intuitively, note that if firm  $s$  does not hold either direct or indirect stake in firm  $r$ , then  $b_{sr} = 0$ . In that case,  $\varepsilon_s = 0$ , so  $b_{sj}^\omega = b_{sj}$ . This implies in turn that  $\omega$  affects the imputed shares of firm  $s$  in rival firms only through its imputed share in firm  $r$ . It is not surprising therefore that an increase in  $\omega$  affects  $b_{sj}$  and  $b_{ss}$  by exactly the same factor, which implies in turn that the relative imputed shares of firm  $s$  are not affected by  $\omega$ .

The second special case where collusion is not facilitated arises when the maverick firm has no direct or indirect stake in the acquiring firm (firm  $r$ ). Then, the increase in  $\alpha_{rs}$  does not affect the relative imputed shares of the maverick firm in any way and hence its incentive to collude are not affected either. In all other cases collusion is strictly facilitated. We summarize these conclusions in the next corollary:

**Corollary 1:** *An increase in firm  $r$ 's cross ownership stake in firm  $s$  never hinders tacit collusion and surely facilitates it if and only if (i) each industry maverick has a direct or an indirect stake in firm  $r$ , and (ii) firm  $s$  is not an industry maverick.*

The proof of Theorem 1 provides a simpler proof for Theorem 1 in Gilo, Moshe, and Spiegel (2006): when all firms have the same marginal cost (the case considered in Gilo, Moshe, and Spiegel, 2006),  $\hat{y}_1 = \dots = \hat{y}_n = y_1^m$  and  $y_1(c_2) = 0$ . Hence, equations (16) and (17) imply that  $\frac{\partial \hat{\delta}_i(A)}{\partial z_{ij}} < 0$  for all  $i$  and all  $j$ , so the proof of Theorem 1 implies immediately that  $\hat{\delta}_i(A) \geq \hat{\delta}_i(A^\omega)$  with strict equality if and only if  $i \neq s$  and  $b_{ir} > 0$ .

To illustrate Theorem 1, we will now examine the following example.

**Example:** Consider an industry with three firms where the PCO matrix is

$$A = \begin{pmatrix} 0 & \alpha & 0 \\ \beta & 0 & \beta \\ \eta & 0 & 0 \end{pmatrix}.$$

The associated matrix of imputed shares is given by

$$B = (I - A)^{-1} = \frac{1}{1 - \alpha\beta(1 + \eta)} \begin{pmatrix} 1 & \alpha & \alpha\beta \\ \beta(1 + \eta) & 1 & \beta \\ \eta & \alpha\eta & 1 - \alpha\beta \end{pmatrix}.$$

If firm 1 increases its stake in firm 2 by  $\omega > 0$ , then simple algebra shows that

$$Z^\omega - Z = \begin{pmatrix} 0 & \omega & \beta\omega \\ 0 & 0 & 0 \\ \frac{\alpha\eta\omega}{F} & \frac{\eta\omega(1+\alpha^2-\alpha\beta)}{F} & 0 \end{pmatrix},$$

where  $F \equiv (1 - \alpha\beta)(1 - \alpha(\beta + \omega)) > 0$ . Since the second row in the matrix contains 0's, the relative imputed shares of firm 2 (which plays the role of firm  $s$ ) do not change. Moreover, if firm 3 does not have a stake in firm 1 (which plays the role of firm  $r$ ), then  $\eta = 0$ , and the relative imputed shares of firm 3 do not change as well. Finally, so long as  $\eta > 0$ ,  $z_{12}$ ,  $z_{13}$ ,  $z_{31}$ , and  $z_{32}$  all increase as Theorem 1 shows.

#### 4.4 A firm increases its stake in a rival firm by buying shares from another rival firm

Theorem 1 assumes implicitly that when firm  $r$  increases its stake in firm  $s$ , it buys additional shares from the outside investors or the controller of firm  $s$ . However, cases exist in which

one firm buys shares in a rival firm from a third rival. A case in point is a recent transaction in the global steel industry where Luxemburg-based Arcelor has increased its stake in the Brazilian steelmaker CST from 18.6% to 27.95% by buying shares from Acesita which is also based in Brazil.<sup>15</sup> To examine the effect of such ownership transfers on the incentives to collude, suppose that firm  $r$  increases its stake in firm  $s$  by buying an ownership stake  $\phi$  from firm  $k$ . The resulting PCO matrix  $A^\phi$  is obtained from the original PCO matrix  $A$  by increasing the  $rs$ -th entry in  $A$  by  $\phi$  and lowering the  $ks$ -th entry by  $\phi$ . Equation (2) in Zeng shows that in this case,

$$z_{ij}^\phi \equiv \frac{b_{ij}^\phi}{b_{ii}^\phi} = \frac{b_{ij} + \varepsilon_i^\phi b_{sj}}{b_{ii} + \varepsilon_i^\phi b_{si}}, \quad \varepsilon_i^\phi \equiv \frac{\phi (b_{ir} - b_{ik})}{1 - \phi (b_{sr} - b_{sk})}. \quad (24)$$

Note that if firm  $k$ 's stake remains unchanged, then  $\varepsilon_i^\phi = \varepsilon_i$ . Hence, the expression we used earlier,  $z_{ij}^\omega$ , is a special case of (24). The main difference is that while  $\varepsilon_i \geq 0$ , now  $\varepsilon_i^\phi \gtrless 0$  as  $b_{ir} \gtrless b_{ik}$ .

Using (24) yields

$$\frac{\partial z_{ij}^\phi}{\partial \phi} = \frac{b_{ii}b_{sj} - b_{ij}b_{si}}{(b_{ii} + \varepsilon_i^\phi b_{si})^2} \times \frac{b_{ir} - b_{ik}}{(1 - \phi (b_{sr} - b_{sk}))^2}. \quad (25)$$

Repeating the same steps as in Theorem 1, we obtain the following result:

**Theorem 2:** *Starting with a PCO matrix  $A$ , suppose that firm  $r$  buys a stake  $\phi$  in firm  $s$  from firm  $k$ , so that the new PCO matrix  $A^\phi$  is obtained from  $A$  by increasing the  $rs$ -th entry by  $\phi$  and decreasing the  $ks$ -th by  $\phi$ . Then,*

- (i)  $\widehat{\delta}_s(A^\phi) = \widehat{\delta}_s(A)$ ,
- (ii)  $\widehat{\delta}_i(A^\phi) = \widehat{\delta}_i(A)$  if  $b_{ir} = b_{ik}$  (firm  $i$  has the same imputed share in firms  $r$  and  $k$ ), and
- (iii)  $\widehat{\delta}_i(A^\phi) \leq \widehat{\delta}_i(A)$  for all  $i \neq s$  as  $b_{ir} \geq b_{ik}$ .

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<sup>15</sup>Acesita sold its entire 18.7% stake in CST to Arcelor and to CVRD which is a large Brazilian miner of iron and ore. In addition to its stake in CST, Arcelor also owns stakes in Acesita and in Belgo-Mineira, which is another Brazilian steelmaker (see ‘‘CVRD, Arcelor Team up for CST,’’ *The Daily Deal*, December 28, 2002, M&A; ‘‘Minister: Steel Duties Still Under Study - Brazil,’’ *Business News Americas*, April 8, 2002.)

Theorem 2 implies the following result:

**Corollary 2:** *A transfer of partial cross ownership in firm  $s$  from firm  $k$  to firm  $r$  does not affect tacit collusion if the industry maverick is firm  $s$  or if, at the outset, the industry maverick has the same imputed share in firms  $k$  and  $r$ . Otherwise, the transfer of partial cross ownership facilitates tacit collusion if the industry maverick has a larger imputed share in firm  $r$  (the acquirer) than in firm  $k$  (the seller) but hinders tacit collusion if the reverse holds.*

Proposition 3 in Gilo, Moshe, and Spiegel (2006) also considered the effects of a transfer of partial cross ownership in firm  $s$  from one firm to another but under the special assumption that at the outset all firms hold the exact same ownership stakes in one another. In this case, the matrix  $B$  is symmetric in the sense that its diagonal terms are all the same and its off-diagonal terms are all equal to each other. In particular,  $b_{ir} = b_{ik}$  for all  $i \neq r, k$ , so part (ii) of Theorem 2 shows that  $\widehat{\delta}_i(A^\phi) = \widehat{\delta}_i(A)$  for all  $i \neq r, k$ . Part (i) of Theorem 2 shows in addition that  $\widehat{\delta}_s(A^\phi) = \widehat{\delta}_s(A)$ . As for firms  $r$  and  $k$ , then part (i) of Lemma 2 implies that  $b_{rr} > b_{rk}$  and  $b_{kr} < b_{kk}$ . Hence, equation (25) shows that  $\frac{\partial z_{rj}^\phi}{\partial \phi} \geq 0$  and  $\frac{\partial z_{kj}^\phi}{\partial \phi} \leq 0$  for all  $j$  with strict inequality for  $j = s$ . Hence, by Lemma 4,  $\widehat{\delta}_r(A^\phi) < \widehat{\delta}_r(A)$  and  $\widehat{\delta}_k(A^\phi) > \widehat{\delta}_k(A)$ , implying that the transfer of partial cross ownership in firm  $s$  from firm  $r$  to firm  $k$  strengthen the incentive of firm  $r$  to collude, weaken the incentive of firm  $k$  to collude and has no effect on the incentives of other firms to collude. In the symmetric case considered by Gilo, Moshe, and Spiegel (2006),  $\widehat{\delta}_1(A) = \dots = \widehat{\delta}_n(A)$ , so the incentives of all firms to collude before the transfer of ownership are the same. Hence the transfer of partial ownership turns firm  $k$  (the seller) into a maverick firm and since  $\widehat{\delta}_k(A^\phi) > \widehat{\delta}_k(A)$ , tacit collusion is hindered.

In the present case where firms have asymmetric marginal costs, any firm can potentially be the maverick firm. In particular, Corollary 2 shows that collusion is hindered when the maverick is firm  $k$  and is facilitated if the maverick is firm  $r$ .

**Example:** Consider a duopoly in which firm 1 holds a stake of  $\alpha > 0$  in firm 2 and firm 2

does not hold a stake in firm 1. Then,

$$B = (I - A)^{-1} = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}.$$

Since the two diagonal terms in  $B$  are equal to 1, the matrix of relative imputed shares is such that  $Z = B$ . Using (16) and (17),

$$\widehat{\delta}_1(A) = \frac{y_1^m - \frac{1}{2}(\widehat{y}_1 + \alpha\widehat{y}_2)}{y_1^m - y_1(c_2)} \quad \text{and} \quad \widehat{\delta}_2(A) = \frac{1}{2}.$$

Hence,

$$\Delta \equiv \widehat{\delta}_1(A) - \widehat{\delta}_2(A) = \frac{y_1^m - (\widehat{y}_1 + \alpha\widehat{y}_2) + y_1(c_2)}{2(y_1^m - y_1(c_2))}.$$

Firm 1 is the industry maverick if  $\Delta > 0$  and firm 2 is the industry maverick if  $\Delta < 0$ .

To see that  $\Delta$  can be either positive or negative, let  $Q(p) = 1 - p$ ,  $c_1 = 1/4$ , and  $c_2 = 1/3$ . Then, the monopoly prices are  $p_1^m = 5/8$  and  $p_2^m = 2/3$ . It can be checked that  $\frac{\widehat{y}_1}{2} \geq y_1(c_2)$  for all  $\widehat{p} \in [5/8, 2/3]$ , so in this range, firm 1 is better-off colluding than undercutting firm 2 slightly. Straightforward calculations establish that  $\Delta$  falls with  $\alpha$  when  $\widehat{p} \in [5/8, 2/3]$  and is positive if  $\alpha < \alpha^*$  and negative if  $\alpha > \alpha^*$ , where

$$\alpha^* = \frac{576\widehat{p}^2 - 720\widehat{p} + 257}{192(1 - \widehat{p})(3\widehat{p} - 1)}.$$

Hence, firm 1 is the industry maverick if  $\alpha < \alpha^*$  and firm 2 is the industry maverick if  $\alpha > \alpha^*$ . For  $\widehat{p} \in [5/8, 2/3]$ , the critical value  $\alpha^*$  is a U-shaped function of  $\widehat{p}$  and its value is bounded from below by 0.505 and from above by 0.516.

The intuition for this is straightforward. Without PCO (i.e.,  $\alpha = 0$ ), firm 1 is always the industry maverick. However, investments of firm 1 in firm 2 boost firm 1's incentive to collude because part of its profit derives from its stake in firm 2. When firm 1's stake in firm 2 is sufficiently large, firm 1 is more reluctant to deviate from the collusive scheme than firm 2.

## 4.5 Conditions for firm 1 to be the maverick

Recall from Section 3 that when only firm 1 invests in rivals, firm 1 is the industry maverick. In this section, we provide sufficient (but not necessary) conditions that ensure that firm 1

continues to be the industry maverick even in the presence of multilateral PCO arrangements.

To this end, note from (16) and (17) that

$$\begin{aligned}
\widehat{\delta}_1(A) - \widehat{\delta}_i(A) &= \frac{\left(y_1^m - \frac{1}{n} \sum_{j=1}^n z_{1j} \widehat{y}_j\right) (\widehat{y}_i - z_{i1} y_1(c_2)) - \left(\widehat{y}_i - \frac{1}{n} \sum_{j=1}^n z_{ij} \widehat{y}_j\right) (y_1^m - y_1(c_2))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))} \\
&= \frac{(\widehat{y}_i - z_{i1} y_1^m) y_1(c_2) - \frac{1}{n} \sum_{j=1}^n z_{1j} \widehat{y}_j (\widehat{y}_i - z_{i1} y_1(c_2)) + \frac{1}{n} \sum_{j=1}^n z_{ij} \widehat{y}_j (y_1^m - y_1(c_2))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))} \\
&= \frac{- (\widehat{y}_i - z_{i1} y_1^m) \left(\frac{\widehat{y}_1}{n} - y_1(c_2)\right) - \frac{1}{n} \sum_{j \neq 1} z_{1j} \widehat{y}_j (\widehat{y}_i - z_{i1} y_1(c_2))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))} \\
&\quad + \frac{\frac{1}{n} \sum_{j \neq 1} z_{ij} \widehat{y}_j (y_1^m - y_1(c_2))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))},
\end{aligned}$$

where the last equality follows because by definition,  $z_{11} = 1$ . Adding and subtracting  $\frac{1}{n} \sum_{j \neq 1} z_{ij} \widehat{y}_j (\widehat{y}_i - z_{i1} y_1(c_2))$  and  $\frac{1}{n} \sum_{j \neq 1} z_{ij} \widehat{y}_j (\widehat{y}_1 - n y_1(c_2))$  to the numerator and rearranging terms,

$$\begin{aligned}
\widehat{\delta}_1(A) - \widehat{\delta}_i(A) &= \frac{\left(z_{i1} y_1^m + \sum_{j \neq 1} z_{ij} \widehat{y}_j - \widehat{y}_i\right) \left(\frac{\widehat{y}_1}{n} - y_1(c_2)\right) + \frac{1}{n} \sum_{j \neq 1} (z_{ij} - z_{1j}) \widehat{y}_j (\widehat{y}_i - z_{i1} y_1(c_2))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))} \\
&\quad + \frac{\frac{1}{n} \sum_{j \neq 1} z_{ij} \widehat{y}_j ((y_1^m - y_1(c_2)) - (\widehat{y}_i - z_{i1} y_1(c_2)) - (\widehat{y}_1 - n y_1(c_2)))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))} \\
&= \frac{\left(z_{i1} y_1^m + \sum_{j \neq 1} z_{ij} \widehat{y}_j - \widehat{y}_i\right) \left(\frac{\widehat{y}_1}{n} - y_1(c_2)\right) + \frac{1}{n} \sum_{j \neq 1} (z_{ij} - z_{1j}) \widehat{y}_j (\widehat{y}_i - z_{i1} y_1(c_2))}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))} \\
&\quad + \frac{\sum_{j \neq 1} z_{ij} \widehat{y}_j \left(\frac{y_1^m - \widehat{y}_1}{n} + \frac{(n-1+z_{i1})y_1(c_2)}{n} - \frac{\widehat{y}_i}{n}\right)}{(\widehat{y}_i - z_{i1} y_1(c_2)) (y_1^m - y_1(c_2))},
\end{aligned}$$

Recalling that  $z_{ii} = 1$ , it follows that  $\sum_{j \neq 1} z_{ij} \widehat{y}_j > \widehat{y}_i$ . Moreover,  $\frac{\widehat{y}_1}{n} \geq y_1(c_2)$  (see Figure 1). Hence, the first term is positive. The following proposition provides sufficient (but not necessary) conditions for the other two terms to be nonnegative, which ensures that firm 1 is the maverick firm in the industry in the sense that  $\widehat{\delta}_1(A) > \widehat{\delta}_i(A)$  for all  $i \neq 1$ .

**Proposition 4:** *Sufficient (but not necessary) conditions for firm 1 (the most efficient firm in the industry) to be the industry maverick is that (i)  $z_{1j} \leq z_{ij}$  for all  $i, j \neq 1$  (the relative imputed share of firm 1 in each firm  $j$  is no greater than the relative share of any other firm in firm  $j$ ), and (ii)  $\frac{\widehat{y}_i}{n} \leq \frac{(n-1+z_{i1})y_1(c_2)}{n}$  for all  $i \neq 1$  (the collusive profit of each firm  $i \neq 1$  is small relative to the profit that firm 1 earns once the collusive scheme breaks down).*

## 5 Conclusion

Acquisitions of one firm's stock by a rival firm have been traditionally treated under Section 7 of the Clayton Act which condemns such acquisitions when their effect "may be substantially to lessen competition." However, the third paragraph of this section effectively exempts passive investments made "solely for investment." As argued in Gilo (2000), antitrust agencies and courts, when applying this exemption, did not conduct full-blown examinations as to whether such passive investments may substantially lessen competition.<sup>16</sup>

In this paper we showed that although there are cases in which passive investments in rivals have no effect on the ability of firms to engage in tacit collusion, an across the board lenient approach towards such investments may be misguided. This is because passive investments in rivals may well facilitate tacit collusion, especially when these investments are multilateral and in firms that are not industry mavericks. In addition, we showed that direct investments by firms' controllers in rivals may either substitute investments by the firms themselves or facilitate collusion further, especially when the controllers have small stakes in their own firms. We believe that antitrust courts and agencies should take account of these factors when considering cases involving passive investments among rivals.

Throughout the paper we have focused exclusively on the effect of PCO on the ability of firms to engage in (tacit) price fixing. However, if in addition to price fixing firms can also divide the market among themselves, then they would clearly be able to sustain collusion for a larger set of discount factors since they would have more instruments (the collusive price and the market shares). In particular, it would be possible to relax the incentive constraints of maverick firms by increasing their market shares at the expense of firms with nonbinding incentive constraints. This suggests in turn that in the presence of market sharing schemes, firms may have an incentive to become industry mavericks in order to receive a larger share of the market. As our analysis shows, one way to become an industry maverick is to avoid

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<sup>16</sup>We are aware of only two cases in which the ability of passive investments to lessen competition was acknowledged: the FTC's decision in *Golden Grain Macaroni Co.* (78 F.T.C. 63, 1971), and the consent decree reached with the DOJ regarding US West's acquisition of Continental Cablevision (this decree was approved by the district court in *United States v. US West Inc.*, 1997-1 Trade cases (CCH), ¶71,767, D.C., 1997).

investing in rivals.<sup>17</sup> Interestingly, this implies that beside the fact that market sharing schemes are harder to enforce (firms need to commit to ration their sales) and are more susceptible to antitrust scrutiny, they have another drawback, which is that they provide firms with a disincentive to invest in rivals and thereby facilitate tacit collusion.

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<sup>17</sup>Indeed, in a previous version of the paper, we showed that under market sharing schemes and cost asymmetries, only the most efficient firm in the industry has an incentive to invest in rivals to sustain collusion while all other firms find it optimal to not invest in rivals.



## 6 References

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