Parametric Recoverability of Preferences*

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Abstract

We recover approximate parametric preferences from consistent and inconsistent consumer choices. The procedure seeks to utilize revealed preference information contained in choices by minimizing its ranking inconsistency with the proposed parametric preferences. We provide a novel characterization of the Varian Inefficiency Index, generalize it to a goodness-of-fit measure of recovered preferences, and decompose the latter into inconsistency and misspecification measures. This provides a reasonable way to test restrictions on parametric models. An application of the method to the data set constructed by Choi et al. (2007) to study choice under risk suggests more pronounced non-expected utility preferences than previously suggested.

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1 Introduction

This paper is a contribution to the applicability of revealed preference theory to the domain of recovering stable preferences from individual choices. The need for such an application emerges from the recent availability of relatively large data sets composed of *individual* choices made directly from linear budget sets.[1] These rich data sets allow researchers to recover approximate *individual* stable utility functions and report the magnitude and distribution of behavioral characteristics in the subject population.

Given a data set constructed from a generic consumer choice problem, which satisfies the Generalized Axiom of Revealed Preference (henceforth GARP), Afriat (1967) suggests a nicely behaved piecewise linear utility function that satisfies the restrictions imposed by the revealed preference relation. This method requires recovering twice the number of parameters as there are observations (Diewert, 1973) and therefore the behavioral implications of such functional forms may be difficult to interpret and apply to economic problems. Varian (1982) builds on this work to construct non parametric bounds that partially identify the utility function, assuming that preferences are convex.

In many cases, however, researchers assume simple functional forms with few parameters that lend themselves naturally to behavioral interpretation. The drawback of this approach is that simple functional forms are often too structured to capture every nuance of individual decision making, and thus preferences recovered in this way are almost always misspecified. That is, the ranking implied by the recovered preferences may be inconsistent with the ranking information implied by the decision maker's choices (summarized through the revealed preference relation). Following this line of reasoning, given a parametric utility function, one should seek a measure to quantify the extent of misspecification and use this measure as a criterion for selecting the element of the functional family which minimizes the misspecification. This measure should apply continuously to inconsistent choice data, and inform the

[1] Notable references are Andreoni and Miller (2002); Fisman et al. (2007); Choi et al. (2007); Ahn et al. (2013); Andreoni and Sprenger (2012).

extent of misspecification by all possible utility functions.

Our proposed measure of misspecification used in recovering preferences relies on insights gained from the literature that quantifies internal inconsistencies inherent in a data set. The Varian (1990) Inefficiency Index is a popular measure of the decision maker's inconsistency. It is calculated by aggregating the minimal budget adjustments required to remove cyclic revealed preference relations, that cause the dataset to fail GARP. We provide the following novel theoretical characterization of the Varian Inefficiency Index: for every continuous and locally non-satiated utility function we calculate the Money Metric Index. This index is an aggregation, taken over all observations, of the minimal budget adjustments required to remove inconsistencies between the ranking information induced by the utility function and the revealed preference information contained in the choices.[2] We prove that the Varian Inefficiency Index equals the infimum of the money metric indices, taken over *all* continuous and locally non-satiated utility functions. Hence, the Varian Inefficiency Index lends itself naturally as a benchmark for minimizing misspecification between the data set and all possible utility functions.

Our proposed procedure of recovering approximate preferences within a restricted parametric family generalizes the principle we introduced in characterizing the Varian Inefficiency Index, by calculating the infimum of the Money Metric Index over the *restricted* subset. If a data set satisfies GARP, the measure we propose quantifies the extent of misspecification that arises solely from considering a specific family of utility functions, rather than all utility functions. If the data set does not satisfy GARP, the measure can be decomposed into the Varian Inefficiency Index and a misspecification index, which is the difference between the Money Metric Index and the Varian Inefficiency Index. Since for a given data set the Varian Inefficiency Index is constant (zero if GARP is satisfied), the measure can be used to recover parametric preferences within some parametric family by minimizing the misspecification.

Furthermore, the procedure can be used to evaluate the increase in mis-

[2]The latter corresponds to the relations between observed choice (the observation) and all feasible alternatives.

specification implied by restricting the set of parameters, and to choose among functional forms. For example, consider some parametric form of non-expected utility that includes expected utility as a special case. Given a data set of choices under risk, one can recover the values of the parameters that minimize misspecification, and evaluate the additional misspecification implied by restricting to expected utility.

To illustrate, we apply our method to recover preferences from data on choice under risk collected by Choi et al. (2007). We recover parameters for the disappointment aversion functional form of Gul (1991)[3] using both the widely used Euclidean-distance-based Non Linear Least Squares (NLLS) and the proposed approach. We identify several important qualitative differences in the recovered parameters. In several cases, the recovered parameters are contradictory with respect to whether subjects are elation loving or disappointment averse, and as such the behavioral conclusions of our analysis may depend critically on the chosen recovery method. Moreover, quantitative differences in the distribution of parameter values in the subject population suggest that the preferences recovered by minimizing inconsistency with revealed preference information put higher weight on first-order risk aversion and lower weight on second-order risk aversion (Segal and Spivak, 1990) than previously found using distance-based approaches. We calculate the additional misspecification implied by restricting to expected utility, and find that choices of between one third and one half of the subjects may be reasonably approximated by expected utility.

Our proposed method of recovering preferences using revealed preference information is fundamentally different from the traditional approach that relies on the distance between observed and predicted choices. While the latter compares only two points and utilizes auxiliary assumptions (e.g. "closer is better") to select among utility functions (each providing different predictions), we point out that choosing an alternative from a menu informs the observer that it is revealed preferred to *every* other feasible alternative. Independently

[3]Within the considered setup of two-states of the world it is observationally equivalent to models of Rank Dependent Utility (Quiggin, 1982).

of the misspecification criterion, a desired recovery method should incorporate as much revealed preference information as available in the data. The proposal made in the current work is one possible candidate that follows this methodology. While possessing nice theoretical and computational properties, other methods that follow similar principle may exist. Studying such potential proposals is an important avenue for future research.

In the next section we reintroduce familiar definitions and results from the literature that applied revealed preference theory to non-parametric recovery of preferences given consistent and inconsistent data sets of choices from budget sets. The discussion regarding the shortcomings of the non-parametric approach in the second section, serves as a motivation for the next section. Our main analytical results and the suggested recovery method are presented in the third section. In the fourth section we apply the proposed method to recover preferences from data on choice under risk collected by Choi et al. (2007). We conclude with five brief discussions on related theoretical, practical and interpretational issues.

2 Non-Parametric Recoverability

2.1 Preliminaries

Consider a decision maker (DM) who chooses bundles $x^i \in \Re^K_+$ $(i \in 1, \ldots, n)$ out of budget menus $\left\{x : p^i x \leq 1,\, p^i \in \Re^K_{++}\right\}$. Let $D = \left\{(p^i, x^i)^n_{i=1}\right\}$ be a finite data set, where x^i is the chosen bundle at prices p^i. The preference information incorporated in the observed choices is summarized by the following binary relations.

Definition 1. An observed bundle $x^i \in \Re^K_+$ is

1. *directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R^0_D x$, if $p^i x^i \geq p^i x$.

2. *strictly directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i P^0_D x$, if $p^i x^i > p^i x$.

3. *revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R_D x$, if there exists a sequence of observed bundles $\left(x^j, x^k, \ldots, x^m\right)$ such that $x^i R^0_D x^j, x^j R^0_D x^k, \ldots, x^m R^0_D x$.

4. *strictly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i P_D x$, if there exists a sequence of observed bundles $\left(x^j, x^k, \ldots, x^m\right)$ such that $x^i R^0_D x^j, x^j R^0_D x^k, \ldots, x^m R^0_D x$ at least one of them is strict.

The data is said to be consistent if it satisfies the General Axiom of Revealed Preference.

Definition 2. Data set D satisfies the *General Axiom of Revealed Preference (GARP)* if for every pair of observed bundles, $x^i R_D x^j$ implies not $x^j P^0_D x^i$.

The following definition relates the revealed preference information implied by observed choices to ranking induced by utility maximization.

Definition 3. A utility function $u : \Re^K_+ \rightarrow \Re$ *rationalizes* data set D, if for every observed bundle $x^i \in \Re^K_+$, $u(x^i) \geq u(x)$ for all x such that $x^i R^0_D x$. We say that D is *rationalizable* if such $u(\cdot)$ exists.

Rationalizability does not imply uniqueness. There could be different utility functions (not related by a monotonic transformation) that rationalize the same data set. Afriat's celebrated theorem provides tight conditions for the rationalizability of a data set.

Theorem. (Afriat, 1967) *The following conditions are equivalent:*

1. There exists a non-satiated utility function that rationalizes the data.

2. The data satisfies GARP.

3. There exists a non-satiated, continuous, concave, monotone utility function that rationalizes the data.

Proof. See Afriat (1967); Diewert (1973); Varian (1982). □

2.2 Shortcomings

2.2.1 Simplicity

The traditional problem of recoverability is to find a utility function that rationalizes the data. Indeed, Afriat's proof of the Theorem is constructive: he shows that if a data set D of size n satisfies GARP then $U(x) = min_i \{U^i + \lambda^i p^i (x - x^i)\}$, where U^i and $\lambda^i > 0$ are $2n$ real numbers that satisfy a set of n^2 inequalities: $U^i \leq U^j + \lambda^j p^j (x^i - x^j)$, rationalizes D. It is important to note that although Afriat's utility function does not rely on any parametric assumptions, it is difficult to directly learn from it about behavioral characteristics of the decision maker, which are typically summarized by few parameters (e.g. risk aversion, ambiguity aversion). Moreover, this utility function that rationalizes the data is generically non-unique. Hence, if one can find a "simpler" (parametric) utility function that rationalizes the data set - it will have equal standing in representing the ranking information implied by the data set. If one accepts that "simple" may be superior, then one should consider paying a price in terms of misspecification. We pursue this line of reasoning by considering the minimal misspecification implied by certain parametric specifications.

2.2.2 Convexity Assumption

Varian (1982) suggests a non-parametric recovery method that partially identifies the subject's preferences by constructing upper and lower bounds on her indifference curves. However, this method imposes the restriction of convexity on the preferences that may be recovered. In Appendix A we demonstrate that if the data set is generated by a DM who correctly maximizes a non-convex preference relation, the ranking implied by Varian's non-parametric bounds may be inconsistent with the underlying preferences of the DM. The reason is that while Afriat's Theorem states that if a data set is rationalizable, there exists a concave utility function that rationalizes it, these convexified preferences rank unobserved bundles differently than a utility function that represents non-convex preferences and rationalizes the same data set. We find that in view of the evolving literature on the importance of non-convex preferences in domains like risk, ambiguity and other-regarding preferences, this exclusion seem to be unwarranted in many contexts. The parametric approach to recoverability permits the observer to identify non-convex preferences within a given functional family.

2.3 Inconsistent Data Sets

Afriat (1973, 1987) and Houtman (1995) use similar methods to those used in Afriat (1967) to recover non-parametrically an approximate utility function, in the sense that the existence of an underlying preference relation is maintained by allowing the DM to not exactly maximize that relation. This approach suffers from the same shortcomings of Afriat (1967) discussed above. The non-parametric approach of Varian (1982) has been extended and developed in Blundell et al. (2003, 2008) and Cherchye et al. (2009), however, to the best of our knowledge, it had not been expanded to include treatment of inconsistent data sets, and doing so will probably entail some behavioral assumptions regarding the nature of the inconsistencies. The parametric approach developed in the current paper, not only extends naturally to inconsistent data sets, but also permits an insightful decomposition of the goodness of fit into measures of inconsistency and misspecification.

The following definition is a generalization of Definition 1. Similar concepts have been introduced into the literature on consistency (Afriat, 1972, 1987; Varian, 1990, 1993) in order to measure how close is a DM to satisfying GARP.[4]

Definition 4. Let D be a finite data set. Let $\mathbf{v} \in [0,1]^n$.[5] An observed bundle $x^i \in \Re^K_+$ is

1. $\mathbf{v}-$*directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R^0_{D,\mathbf{v}} x$, if $v^i p^i x^i \geq p^i x$.

[4] A different but related concept of inconsistency is presented in Echenique et al. (2011).

[5] Throughout the paper we use bold fonts (as $\mathbf{v}$ or $\mathbf{1}$) to denote vectors of scalars in $\Re^n$. For $\mathbf{v}, \mathbf{v}' \in \Re^n$ $\mathbf{v} = \mathbf{v}'$ if $\forall i: v_i = v'_i$, $\mathbf{v} \geqq \mathbf{v}'$ if $\forall i: v_i \geq v'_i$, $\mathbf{v} \geq \mathbf{v}'$ if $\mathbf{v} \geqq \mathbf{v}'$and $\mathbf{v} \neq \mathbf{v}'$ and $\mathbf{v} > \mathbf{v}'$ if $\forall i: v_i > v'_i$. We continue to use regular fonts to denote vectors of prices and goods.

2. $\mathbf{v}-$*strictly directly revealed preferred* to a bundle $x \in \Re_+^K$, denoted $x^i P_{D,\mathbf{v}}^0 x$, if $v^i p^i x^i > p^i x$.

3. $\mathbf{v}-$*revealed preferred* to a bundle $x \in \Re_+^K$, denoted $x^i R_{D,\mathbf{v}} x$, if there exists a sequence of observed bundles $\left(x^j, x^k, \ldots, x^m\right)$ such that $x^i R_{D,\mathbf{v}}^0 x^j, x^j R_{D,\mathbf{v}}^0 x^k, \ldots, x^m R_{D,\mathbf{v}}^0 x$.

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Similarly, consider the following generalization of GARP (Varian, 1990):

Definition 5. Let $\mathbf{v} \in [0,1]^n$. D satisfies the *General Axiom of Revealed Preference Given* $\mathbf{v}$ *(GARP$_{\mathbf{v}}$)* if for every pair of observed bundles, $x^i R_{D,\mathbf{v}} x^j$ implies not $x^j P_{D,\mathbf{v}}^0 x^i$.

The vector $\mathbf{v}$ is used to generate the adjusted relation $R_{D,\mathbf{v}}$ that is acyclic although R_D may contain cycles. Obviously, usually there are many vectors such that D satisfies $GARP_{\mathbf{v}}$. Following are two useful and trivial properties of $GARP_{\mathbf{v}}$:

Fact 1. *Every D satisfies $GARP_{\mathbf{0}}$.*[6]

Fact 2. *If $\mathbf{v}, \mathbf{v}' \in [0,1]^n$ and $\mathbf{v} \geq \mathbf{v}'$ and D satisfies $GARP_{\mathbf{v}}$ then D satisfies $GARP_{\mathbf{v}'}$.*

Varian (1990) proposed an inefficiency index that measures the minimal adjustments of the budget sets which remove cycles implied by choices.[7] While Varian suggests to aggregate the adjustments using the sum of squares, we define this index with respect to an arbitrary aggregator function.

[6] $P_{D,\mathbf{0}}^0$ is the empty relation.

[7] Afriat (1972, 1973) Critical Cost Efficiency Index employs a uniform adjustment for all budgets.

Definition 6. $f:[0,1]^n \to [0,M]$, where M is finite, is an *Aggregator Function* if $f(\mathbf{1})=0,\ f(\mathbf{0})=M$ and $f(\cdot)$ is continuous and weakly decreasing.[8]

For a given aggregator function, this index is a measure of the decision maker's inconsistency.

Definition 7. Let $f:[0,1]^n \to [0,M]$ be an aggregator function. *Varian's Inefficiency Index* is[9],

$$I_V(D,f) = \inf_{\mathbf{v}\in[0,1]^n : D \text{ satisfies } GARP_{\mathbf{v}}} f(\mathbf{v})$$

Fact 3. *$I_V(D,f)$ always exists.*[10]

3 Parametric Recoverability

This section proposes a loss-function that measures the inconsistency between the ranking information encoded in choices made within a data set and a given utility function. For a data set that satisfies GARP, this will constitute a measure of the misspecification in representing the data set by the utility function.

Consider, for example, a data set of a single observation $D=\{(p^1,x^1)\}$ and two candidate utility functions u and u' (the two utility functions represent the parametric restriction) as depicted in Figure 3.1. The data set includes only a single observation, hence is trivially consistent. However, both utility functions

[8] An aggregator function f is weakly decreasing if for every $\mathbf{v},\mathbf{v}' \in [0,1]^n$: $\begin{array}{lcl} \mathbf{v} \geq \mathbf{v}' & \implies & f(\mathbf{v}) \leq f(\mathbf{v}') \\ \mathbf{v} > \mathbf{v}' & \implies & f(\mathbf{v}) < f(\mathbf{v}') \end{array}$. One may wish to restrict the set of potential aggregator functions to include only separable functions that satisfy the cancellation axiom. All our examples belong to this restricted set (and assume an additive structure). The theoretical result does not require the richness of possible aggregator functions. It remains an interesting theoretical exercise to axiomatically characterize possible aggregator functions.

[9] Consider a data set of two points $D=\left\{\left(p^1,x^1\right);\left(p^2,x^2\right)\right\}$ such that $p^1x^2=p^1x^1$ but $p^2x^1<p^2x^2$. D is inconsistent with GARP or GARP$_{\mathbf{1}}$ (since $x^1R_Dx^2$ and $x^2P_D^0x^1$), but consider the series $\mathbf{v}_l=(1-\frac{1}{l},1)$ where $l\in\mathbb{N}_{>0}$. It is easy to verify that for every $l\in\mathbb{N}_{>0}$, D satisfies $GARP_{\mathbf{v}_l}$.

[10] $f(\cdot)$ is bounded and by Fact 1, the set $\{\mathbf{v}\in[0,1]^n : D\ satisfies\ GARP_{\mathbf{v}}\}$ is non-empty.

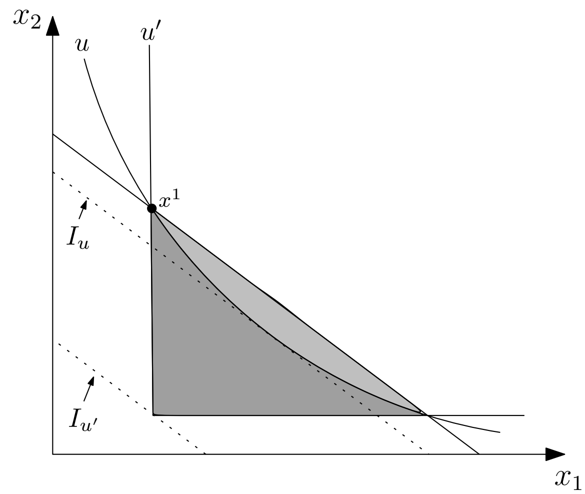


Figure 3.1: Measuring misspecification with budget adjustments

fail to rationalize the data since for both utility function there exist feasible bundles that are preferred to x^1 according to the respective utility function. Consider the unobserved bundles in the lightly shaded region of Figure 3.1. These are bundles to which x^1 is directly revealed preferred and yet are ranked higher than x^1 by the utility function u. In other words, u is misspecified since for these bundles the ranking induced by u is inconsistent with the ranking implied by choices. Yet, if we look at the union of the light and dark shaded regions, it is easy to see that all inconsistencies with the revealed preference information implied by u are also implied by u'. In this sense, we say that the utility function u dominates u' and that u' is more misspecified than u.[11]

Our proposed loss-function seeks the minimal adjustment to the expenditure levels such that all inconsistencies between the revealed preference information and the ranking information are removed. In Figure 3.1, I_u and $I_{u'}$ are the highest expenditure levels (keeping the prices constant) such that there is no affordable bundle that is ranked strictly higher than x^1 by the utility functions u and u' respectively. Since $I_{u'} < I_u < p^1 x^1$ it is evident that

[11]Following this example of a single data point, it might be tempting to conclude that as the preferences become less convex (for the same prediction), the misspecification diminishes. However, this intuition is misleading since in larger data sets the variability in prices may be high enough so that less convex preferences will result in more misspecification than the more convex one.

although both utility functions are misspecified, the misspecification implied by u is smaller than the misspecification implied by u' relative to the data set.[12] In the following subsection we introduce theoretical foundations for this approach.

3.1 v-Rationalizability and the Money Metric Index

Afriat (1967) showed that D satisfies GARP if and only if there exists a non-satiated utility function that rationalizes the data. However, if we consider an arbitrary utility function, it will very rarely rationalize the data (even if choices are consistent). Next we define the following generalization of rationalizability:

Definition 8. Let $\mathbf{v} \in [0,1]^n$. A utility function $u(x)$ $\mathbf{v}$-*rationalizes* D, if for every observed bundle $x^i \in \Re^K_+$, $u(x^i) \geq u(x)$ for all x such that $x^i R^0_{D,\mathbf{v}} x$.

That is, the intersection between the set of bundles strictly preferred to an observed bundle x^i according to u, and the set of bundles to which x^i is revealed preferred when the budget constraint is adjusted by v^i, is empty. Notice that $\mathbf{1}-$rationalizability reduces to Definition 3.

To illustrate, consider Figure 3.1, where x^1 is chosen but is not optimal according to utility function u. For every v^1 such that $0 \leq v^1 p^1 x^1 \leq I_u$ there is no x that satisfies $v^1 p^1 x^1 \geq p^1 x$ and is strictly preferred to x^1 according to u. In this case we say that u $v-$rationalizes x^1 (v is a one-dimensional vector that equals v^1). We define the minimum adjustment (supremum v in this case) as the basis for our measure of misspecification. In Figure 3.1 the minimal adjustment required to $v-$rationalize x^1 by utility function u is given by $\frac{I_u}{p^1 x^1}$. Naturally, we would expect utility functions that represent the decision maker's preferences using less misspecification, to require smaller budget adjustments in order to $v-$rationalize the observed choices. This is evident in Figure 3.1 where $I_{u'} < I_u$ captures the intuition that u is less misspecified than u'.

[12] Obviously, this measure is not unique. For example, an alternative measure could use the area contained in the intersection of the upper counter set that goes through x^1 and the budget line. We defer the discussion of this alternative measure to section 5.1.1.

Below we show that the minimal adjustment to the budget set for every observation is given by the value of the Money Metric Utility Function (Samuelson, 1974) at the observation:

Definition 9. The *normalized money metric vector for a utility function* $u(\cdot)$, $\mathbf{v}^{\star}(D,u)$, is such that $v^{\star i}(D,u) = \frac{m(x^i,p^i,u)}{p^i x^i}$ where
$m(x^i,p^i,u) = min_{\left\{y\in\Re_+^K : u(y)\geq u(x^i)\right\}} p^i y$. The *Money Metric Index for a utility function* $u(\cdot)$ is $f\left(\mathbf{v}^{\star}\left(D,u\right)\right)$.

The money metric vector and the money metric utility function upon which it is based, measure, for a given utility function, the minimal expenditure required to achieve at least the same level of utility as the observed choices.[13]

Proposition 1. *Let* $D = \left\{\left(p^i,x^i\right)_{i=1}^n\right\}$ *and let* $u\left(\cdot\right)$ *be a continuous and locally non-satiated utility function.*

1. $u(\cdot)$ $\mathbf{v}^{\star}(D,u)$*-rationalizes* D.
2. $\mathbf{v}^{\star}\left(D,u\right) = \mathbf{1}$ if and only if $u\left(\cdot\right)$ rationalizes D.
3. Let $\mathbf{v}\in[0,1]^n$. $u\left(\cdot\right)$ $\mathbf{v}$*-rationalizes* D if and only if $\mathbf{v}\leqq\mathbf{v}^{\star}(D,u)$.

Proof. Is immediate and is provided in Appendix B for completeness. $\square$

Proposition 1 establishes that $f\left(\mathbf{v}^{\star}\left(D,u\right)\right)$ may be viewed as a function that measures the loss incurred by using a specific utility function to describe a data set. Part 3 shows that $\mathbf{v}^{\star}\left(D,u\right)$ measures the minimal adjustments to the budget sets required to $\mathbf{v}-$rationalize D by u, that is - to remove inconsistencies between the revealed preference information contained in D and the ranking information induced by u.

Part 3 also implies that each coordinate of $\mathbf{v}^{\star}\left(D,u\right)$ is calculated independently of the other observations in the data set. This is a crucial feature of this procedure which deserves some discussion. One may intuitively believe that such independent calculation uses only the directly revealed preference

[13] We include (D,u) in the definition to emphasize that the optimal budget set adjustments depend on both the observed choices and on the specific utility function.

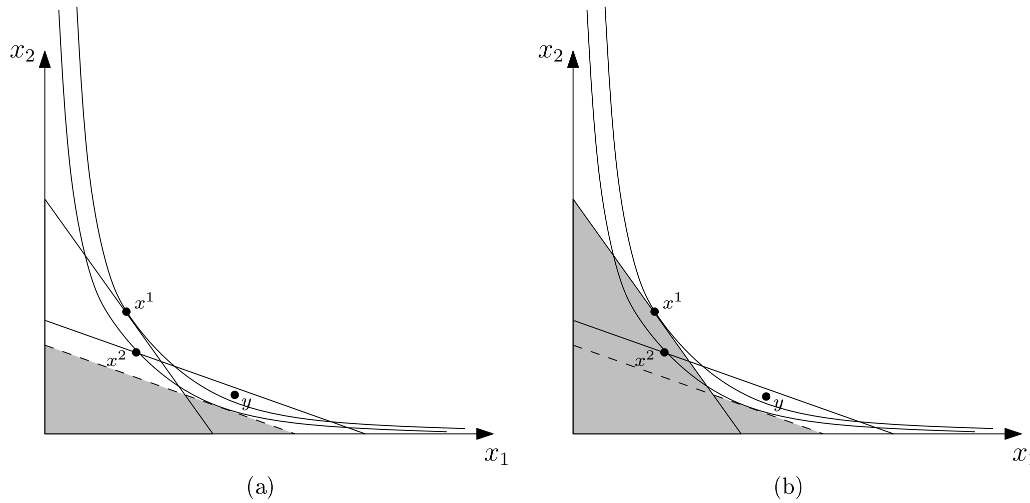


Figure 3.2: The removal of direct inconsistencies removes all indirect inconsistencies

information and may fail to rationalize the data based on the indirect revealed preference information. However, since R_D is the transitive closure of R_D^0, it follows that a utility function is consistent with the directly revealed preference information if and only if it is consistent with all the indirectly revealed preference information. In other words, if the utility function is inconsistent with some indirect revealed preference information, it must be inconsistent with some directly revealed preference information as well.

Figure 3.2 demonstrates this point. The data set includes two observations, where x^1 is directly revealed preferred to x^2. The utility function $u(\cdot)$ ranks x^1 above x^2 but fails to rationalize the data since $u(y) > u(x^1)$ although x^1 is strictly indirectly revealed preferred to y (which is feasible when x^2 is chosen). First, note that if this is the case, it must be that $u(y) > u(x^2)$. That is, $u(\cdot)$ does not rationalize the direct revealed preference information. Second, as is evident from Figure 3.2a, D will be $\mathbf{v}^{\star}-$rationalized by adjusting *only* observation 2's budget set to remove inconsistencies between the utility ranking and the *directly* revealed preference information. More generally, the $\mathbf{v}^{\star}-$ adjustments can be calculated observation-by-observation: for each observation the minimal adjustment is independent of the required adjustments for other observations.[14] Moreover, Figure 3.2b[15] demonstrates that $\mathbf{v}^\star(D,u)$ retains most of the indirect revealed preference information that is consistent with the ranking encoded in the utility function under consideration since $R_{D,\mathbf{v}^\star(D,u)}$ is just the transitive closure of $R^0_{D,\mathbf{v}^\star(D,u)}$.

Part 2 of Proposition 1 is merely a restatement of the familiar definition of rationalizability using the money metric as a criterion. It shows that a non-satiated and continuous utility function $u(\cdot)$ rationalizes the observed choices if and only if it is the case that for all observations there exist no affordable bundles that achieve strictly higher level of utility than the observed choices themselves. In this case we would say that the utility function is correctly specified.

Recall that given an aggregator function $f(\cdot)$, $f(\mathbf{v}^\star(D,u))$ measures the inconsistency between a data set D and a specific preference relation represented by the utility function u. Let $\mathcal{U}^c$ be the set of all continuous and locally non-satiated utility functions. Given a set of utility functions $\mathcal{U} \subseteq \mathcal{U}^c$, the Money Metric Index measures the inconsistency between $\mathcal{U}$ and the data set D.

Definition 10. For a data set D and an aggregator function $f(\cdot)$, let $\mathcal{U} \subseteq \mathcal{U}^c$. The *Money Metric Index of* $\mathcal{U}$ is

$$I_M(D,f,\mathcal{U}) = \inf_{u\in\mathcal{U}} f(\mathbf{v}^\star(D,u))$$

The following observation follows directly from the definition of $I_M(D,f,\mathcal{U})$.

Fact 4. *For every* $\mathcal{U}' \subseteq \mathcal{U} : I_M(D,f,\mathcal{U}) \leq I_M(D,f,\mathcal{U}')$.

In particular, it implies that for every $\mathcal{U} \subseteq \mathcal{U}^c$: $I_M(D,f,\mathcal{U}^c) \leq I_M(D,f,\mathcal{U})$. That is, the value of the Money Metric Index calculated for *all* continuous

[14] An additional implication of this property is that given m data sets D_i of n_i observations, and utility function $u(\cdot)$, if u $\mathbf{v}^\star(D_i,u)-$ rationalizes D_i for every i, then u $\mathbf{v}^\star\left(\bigcup_{i=1}^m D_i,u\right)-$ rationalizes $\bigcup_{i=1}^m D_i$ where $\mathbf{v}^\star\left(\bigcup_{i=1}^m D_i,u\right) = \left(\mathbf{v}^\star(D_1,u)^T,\ldots,\mathbf{v}^\star(D_m,u)^T\right)^T$. Moreover, if $f(\cdot)$ is additive separable (as are all the aggregators mentioned in this paper) then $f\left(\mathbf{v}^\star\left(\bigcup_{i=1}^m D_i,u\right)\right) = \sum_{i=1}^m f(\mathbf{v}^\star(D_i,u))$.

[15] The shaded area represents those bundles that are directly and indirectly $\mathbf{v}^\star(D,u)-$ dominated by x^1.

and locally non-satiated utility functions is a lower bound on $I_M(D, f, \mathcal{U})$ for every subset of utility functions.

3.2 Decomposing the Money Metric Index

Thus far we have been primarily concerned with GARP-consistent data sets that can be rationalized by some utility function. Given such data sets we argued that $I_M(D, f, \mathcal{U})$ is a natural measure of the misspecification induced by the choice to recover the utility function of the DM using the parametric family $\mathcal{U}$. By Afriat's Theorem, data sets that do not satisfy GARP cannot be rationalized by any utility function. Were we to restrict our analysis to only consistent data sets, the scope of our analysis would be somewhat limited.[16]

The method we propose to construct $\mathbf{v}^{\star}(D, u)$ does not depend on the consistency of the data set D. Therefore, even if a decision maker does not satisfy GARP, we can recover preferences (within the parametric family $\mathcal{U}$) that *approximate* the consistent revealed preference information encoded in the choices. The difficulty with this arises from the fact that $I_M(D, f, \mathcal{U})$ includes both the inconsistency with respect to GARP and the misspecification implied by the chosen parametric family. In this section we study how we can decompose our measure into these two components.

Our strategy in developing the decomposition is to employ Varian (1990) Inefficiency Index as a measure of inconsistency, which is independent of the parametric family under consideration. We prove that the money metric index calculated for *all* locally non-satiated and continuous utility function - $I_M(D, f, \mathcal{U}^c)$ coincides with Varian's Inefficiency index. It follows that $I_M(D, f, \mathcal{U}) - I_M(D, f, \mathcal{U}^c)$ is a measure of misspecification. Note that Varian's Inefficiency Index is independent of any preference ranking, and, as defined, is

[16] Andreoni and Miller (2002), one of the first experimental papers that utilizes revealed preference approach with moderate price variation, finds that a great majority of the subjects satisfy GARP. However, in several recent experimental studies that employ considerable price variation (Choi et al., 2007; Ahn et al., 2013; Choi et al., 2013), about 75 percent of subjects did not satisfy GARP. Most of them can be shown to be very nearly consistent with GARP according to various measures of consistency as Afriat (1972) Critical Cost Efficiency Index, Varian (1990) Inefficiency Index, and the Houtman and Maks (1985) Index.

just a measure of the inconsistency incorporated in the data set. On the other hand, recall that for a family of utility functions $\mathcal{U}$, the Money Metric Index measures the inconsistency between $\mathcal{U}$ and the data set. The following Theorem establishes that Varian's Inefficiency Index can be viewed as a measure of the inconsistency between the set of *all* continuous and locally non-satiated utility functions and the data set.

Theorem 1. *For every finite data set $D = \left\{(p^i, x^i)_{i=1}^n\right\}$ and aggregator function $f : [0,1]^n \to [0, M]$:*

$$I_V(D, f) = I_M(D, f, \mathcal{U}^c)$$

where $\mathcal{U}^c$ is the set of continuous and locally non-satiated utility functions.

Proof. See Appendix C. □

The proof proceeds to show that $I_V(D, f) \leq I_M(D, f, \mathcal{U}^c)$ since if $I_V(D, f) > I_M(D, f, \mathcal{U}^c)$ there exists a utility function $u(\cdot)$ such that $I_M(D, f, \mathcal{U}^c) \leq f(\mathbf{v}^\star(D, u)) < I_V(D, f)$ and D satisfies $GARP_{\mathbf{v}^\star(D,u)}$ in contradiction to the definition of $I_V(D, f)$. On the other hand, we show that if D satisfies $GARP_{\mathbf{v}}$ then $I_M(D, f, \mathcal{U}^c) \leq f(\mathbf{v})$. Moreover, we show that there exists a vector of adjustments $\mathbf{v}$ such that $f(\mathbf{v}) = I_V(D, f)$ and for every $0 \leq \lambda < 1$ D satisfies $GARP_{\lambda\mathbf{v}}$, and therefore we conclude that $I_M(D, f, \mathcal{U}^c) \leq I_V(D, f)$.

Theorem 1 enables us to decompose the Money Metric Index into a familiar measure of inconsistency (Varian's Inefficiency Index) and a natural measure of misspecification that quantifies the cost of restricting preferences to a subset of utility functions (possibly through a parametric form). By monotonicity of I_M (Fact 4), for every $\mathcal{U} \subseteq \mathcal{U}^c$:

$$I_V(D, f) = I_M(D, f, \mathcal{U}^c) \leq I_M(D, f, \mathcal{U})$$

Therefore, we can write $I_M(D, f, \mathcal{U})$ as the sum of $I_V(D, f)$ and $I_M(D, f, \mathcal{U}) - I_M(D, f, \mathcal{U}^c)$. The former is a measure of the cost associated with inconsistent choices that is *independent* of any parametric restriction and depends only on the DM, while the latter measures the cost of restricting the preferences to a specific parametric form by the researcher who tries to recover the DM's preferences. This decomposition has the advantage that the two measures are comparable (same units) and are constructed to maintain revealed preference information encoded in the choices. As such, $I_M(D, f, \mathcal{U}) - I_M(D, f, \mathcal{U}^c)$ serves as a natural measure of misspecification that is rooted in economic theory. Two reasons lead us to believe that such a decomposition is essential for any method of recovering preferences of a DM who is inconsistent, although we are not aware of its existence elsewhere in the literature. First, since for a given data set, the inconsistency index is constant (zero if GARP is satisfied) we can be certain that $I_M(D, f, \mathcal{U})$ can be used to recover parametric preferences within some parametric family $\mathcal{U}$ by minimizing the misspecification. Second, only when the decomposition exists, one can truly evaluate the cost of restricting preferences to some parametric family compared to the cost incurred by the inconsistency in the choices.

Figure 3.3 demonstrates the decomposition graphically. Consider a data set of size 2: $D = \{(p^1, x^1), (p^2, x^2)\}$ where $p^i x^i = 1$. The dataset is inconsistent with GARP since $x^i R_D x^j$ and $x^j P_D^0 x^i$ for $i, j \in \{1, 2\}$ $i \neq j$. It is easy to see that for any anonymous aggregator, the Varian Index will be $I_V(D, f) = f(1, v^2)$. Hence, the dashed line (together with the original budget line from which x^1 was chosen) represents graphically the minimal adjustments required for D to satisfy $GARP_{\mathbf{v}}$. Now consider, for example, the singleton set of utility functions that includes the monotonic and continuous function u. We would like to find $\mathbf{v}^\star(D, u)$. Since x^1 is rationalizable by this utility function, then $v^{\star 1}(D, u) = 1$. $v^{\star 2}(D, u)$ is the minimal expenditure required to achieve utility level of $u(x^2)$ under prices p^2, which is represented graphically by the dotted line. $I_M(D, f, \{u\}) = f(1, v^{\star 2}(D, u))$ and since $v^{\star 2}(D, u)$ is smaller than v^2, it implies that $I_M(D, f, \{u\})$ is weakly greater than $I_V(D, f)$. The difference between the original budget line from which x^2 was chosen and the dashed line - $v^2 p^2 x^2$, represents graphically the inconsistency implied by D, while the difference between the dashed line and the dotted line - $v^{\star 2} p^2 x^2$, represents the misspecification implied by u. Their sum is the goodness of fit measured

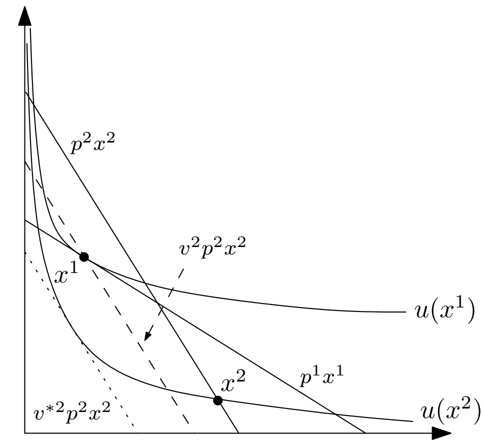


Figure 3.3: Decomposition

by the money metric index. If one considers an alternative utility function u' such that x^1 is not rationalizable by u' (but suppose $v^{\star 2}(D, u') = v^{\star 2}(D, u)$), this would not affect the Varian Index but would imply higher money metric index than u and therefore u' would be more misspecified than u.

It is crucial to note that since, for a given data set, the inconsistency index is constant, the goodness of fit measure can be used to recover parametric preferences within some parametric family. The same idea can be applied to hypotheses testing and model selection. Consider two parametric families $\mathcal{U}$ and $\mathcal{U}'$. A researcher will calculate $I_M(D, f, \mathcal{U}')$ and $I_M(D, f, \mathcal{U})$. As argued before, both incorporate the same inconsistency measure - $I_V(D, f)$, hence the data set D may be better approximated by $\mathcal{U}$ or $\mathcal{U}'$ depending on the relative magnitude of the money metric index. Moreover, an important implication of Fact 4 is that if we impose an additional parametric restriction on preferences (hence reduce the set of possible utility functions we consider), the misspecification will necessarily (weakly) increase. That is, if $\mathcal{U}'$ is a subset of $\mathcal{U}$ that is generated by some parametric restriction, then $\frac{I_M(D,f,\mathcal{U}')-I_M(D,f,\mathcal{U})}{I_M(D,f,\mathcal{U})-I_V(D,f)}$ is a measure of the relative marginal misspecification implied by the restriction of $\mathcal{U}$ to $\mathcal{U}'$. We will tend to accept the restriction if this ratio is low. This methodology resembles statistical hypothesis testing, although the current study does not incorporate any error structure. Inclusion of such structure may provide an interesting avenue for future research, but is not pursued here.[17]

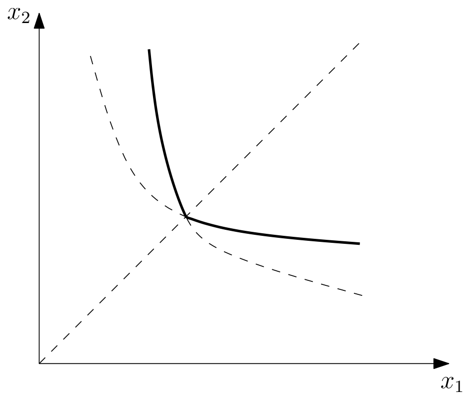
4 Application to Choice under Risk

The goal of this section is to demonstrate the empirical applicability of the Money Metric Index as a criterion for recovering preferences. First, we compare this method and a recovery method that utilizes a loss-function that is based on the Euclidean distance between observed and predicted choices in the commodity space, in particular Non-linear Least Squares (NLLS). Important qualitative differences arise including varied emphasis on first-order versus second-order risk aversion. Additionally, we demonstrate how the suggested method can be used to recover approximate preferences for decision makers who are not strictly rational (in the GARP sense) and assess the degree to which these recovered preferences encode the revealed preference information contained in the choices. Finally, we illustrate how this method can be applied to evaluate nested parametric restrictions, as is the case when we compare models of disappointment aversion with expected utility, as well as non-nested model restrictions, as is the case when we compare various functional forms, e.g. $CRRA$ versus $CARA$.

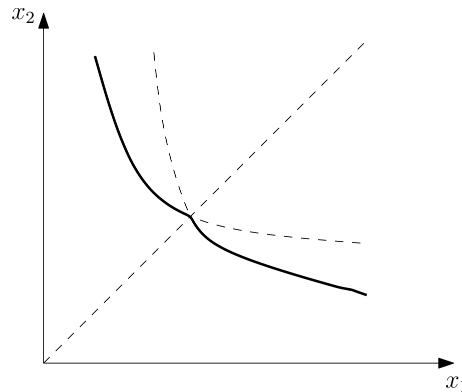
We apply the parametric recoverability method developed in this study and NLLS to a data set of portfolio choice problems collected by Choi et al. (2007). In their experiment, subjects were asked to choose the optimal portfolio using a combination of Arrow securities from linear budget sets with varying prices. We focus our analysis only on the treatment where the two states are equally probable. For each subject, the authors collected 50 observations and proceeded to test these choices for rationality (i.e. GARP) as well as estimate a parametric utility function in order to determine the magnitude and distribution of risk attitudes in the population. Choi et al. (2007) estimate a Disappointment Aversion functional form introduced by Gul (1991) (for equally probable states):[18]

[17]See related discussions in Afriat (1972) and in Varian (1985).

[18]A reader who is not familiar with Gul (1991) model, may find the following footnote



(a) Disappointment aversion, $\beta > 0$



(b) Elation loving, $-1 < \beta < 0$

Figure 4.1: Typical non-expected utility indifference curves induced by Gul (1991) Disappointment Aversion function

$$u(x_1^i, x_2^i) = \gamma w\left(\max\left\{x_1^i, x_2^i\right\}\right) + (1-\gamma)w\left(\min\left\{x_1^i, x_2^i\right\}\right) \tag{4.1}$$

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helpful: Let $p = (p_1, x_1; ...p_n, x_n)$ be a lottery such that $x_1 \leq \cdots \leq x_n$. Assuming (for simplicity) that $ce\,(p) \notin supp\,(p)$, the support of p can be partitioned into elation and disappointment sets: there exists a unique j such that for all $i < j : (x_i, 1) \prec p$ and for all $i \geq j : (x_i, 1) \succ p$. Gul's elation/disappointment decomposition is then given by $r = (x_1, r_1; \cdots; x_{j-1}, r_{j-1})$, $q = (x_j, q_j; \cdots; x_n, q_n)$ and $\alpha = \sum_{i=j}^{n} p_i$ such that $r_i = \frac{p_i}{1-\alpha}$ and $q_i = \frac{p_i}{\alpha}$. Note that $p = \alpha q + (1-\alpha)\,r$. Then:

$$u_{DA}\,(p) = \gamma\,(\alpha)\,E\,(v, q) + (1 - \gamma(\alpha))\,E\,(v, r)$$

and $\exists -1 < \beta < \infty$ such that

$$\gamma\,(\alpha) = \frac{\alpha}{1 + (1-\alpha)\,\beta}$$

where $v\,(\cdot)$ is a utility index and $E\,(v, \mu)$ is the expectation of the functional v with respect to measure μ. If $\beta = 0$ disappointment aversion reduces to expected utility, if $\beta > 0$ the DM is disappointment averse ($\gamma\,(\alpha) < \alpha$ for all $0 < \alpha < 1$), and if $\beta < 0$ the DM is elation seeking ($\gamma\,(\alpha) > \alpha$ for all $0 < \alpha < 1$). Gul (1991) shows that the DM is averse to mean preserving spreads if and only if $\beta \geq 0$ and v is concave. That is, if v is concave then, by Yaari (1969), preferences are convex if and only if the DM is weakly disappointment averse.

For binary lotteries: Let $(x_1, p; x_2, 1-p)$ be a lottery. The elation component is x_2 and the disappointment component is x_1 and $\alpha = 1 - p$ (in our case $\alpha = 0.5$). Therefore:

$$u_{DA}\,(x_1, p; x_2, 1-p) = \gamma\,(1-p)\,v\,(x_2) + (1 - \gamma\,(1-p))\,v\,(x_1)$$

and since $\gamma\,(0) = 0, \gamma\,(1) = 1$ and $\gamma\,(\cdot)$ is increasing, $\gamma\,(\cdot)$ can be viewed as a weighting function, and DA is a special case of Rank Dependent Utility (Quiggin, 1982).

where

$$\gamma = \frac{1}{2+\beta} \qquad \beta > -1 \qquad w(z) \in \left\{ \frac{z^{1-\rho}}{1-\rho}, -e^{-Az} \right\} \quad \rho \geq 0,\ A \geq 0$$

The parameter γ is the weight placed on the better outcome. For $\beta > 0$, the better outcome is under-weighted relative to the objective probability (of 0.5) and the decision maker is *disappointment averse*. For $\beta < 0$, the better outcome is over-weighted relative to the objective probability (of 0.5) and the decision maker is *elation seeking*. In the knife-edge case, when $\beta = 0$, (4.1) reduces to expected utility. β has important economic implication: if $\beta > (=)0$ the decision maker exhibits *first-order (second order) risk aversion* (Segal and Spivak, 1990). That is, the risk premium for small fair gambles is proportional to the standard deviation (variance) of the gamble.[19] First-order risk aversion can account for important empirical regularities that expected utility (with its implied second-order risk aversion) cannot, such as in portfolio choice problems (Segal and Spivak, 1990), calibration of risk aversion in the small and large, and disentangling intertemporal substitution from risk aversion (see Epstein, 1992 for a survey). Figure 4.1 illustrates characteristic indifference curves for disappointment aversion and elation seeking (locally non-convex) subjects, respectively. Additionally, $w(x)$ is a standard utility for wealth function and is represented here by either the $CRRA$ or $CARA$ functional form.

We recover parameters using two different methods: Non-Linear Least Squares (NLLS) based on Euclidean distance and the Money Metric Index developed here. To calculate the Varian Inefficiency Index, $I_V(D, f)$, and the Money Metric Index, $I_M(D, f, \mathcal{U})$,[20] we use both the mean and sum-of-squares

[19] $-1 < \beta < 0$ implies local risk-seeking behavior.

[20] Computing the Varian Index is a hard computational problem (see discussion in Section 5.4), hence we implemented an algorithm that over-estimates the real Varian Index (the details of the implementation are in Appendix D). The implication of this overestimation is that in most of the results that follow, the decomposition of the Money Metric Index overestimates the irrationality component and underestimates the misspecification component. An unavoidable consequence of this computational bias is that in some cases the misspecification component will be negative. That said, while the extent of misspecification with respect to the approximate preferences may be underestimated, the recovered parameters are independent of the calculation of the Varian Index, i.e. minimizing the the Money Metric

		NLLS (SSQ)		Money Metric (SSQ)			Money Metric (MEAN)		
$w(\cdot)$		β	ρ/A	β	ρ/A	I_M	β	ρ/A	I_M
$CRRA$	**GARP (12)**	0.006	1.279	0.413	0.732	0.029	0.249	0.791	0.016
	All (47)	0.171	0.580	0.333	0.356	0.050	0.268	0.387	0.028
$CARA$	**GARP (12)**	-0.07	0.047	0.452	0.022	0.028	0.16	0.024	0.012
	All (47)	0.077	0.028	0.383	0.018	0.060	0.2385	0.019	0.028

Table 1: The median recovered parameters

aggregators:

$$f(\mathbf{v}) \in \left\{ \frac{1}{n} \sum_{i=1}^{n} \left(1 - v^i\right), \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(1 - v^i\right)^2} \right\}$$

For both methods we use an analytical optimization algorithm that allows us to instantaneously recover individual parameters from observed choices for each subject.[21]

4.1 Qualitative Comparison of Methods

In this section we compare differences in recovered parameters according to the choice of recovery methods (NLLS vs Money Metric), specification (CRRA vs CARA), and aggregator (mean vs sum-of-squares). Summary statistics for the recovered parameters are reported in Table 1.[22,23] Additionally, we report the goodness of fit, expressed as the Money Metric Index. The first and third row report the statistics for only those subjects that satisfy GARP (12 out of 47), and the second and fourth row report the statistics for the entire sample.

Index is sufficient and exact. Moreover, in those cases the Money Metric Index is a better approximation to the real Varian Inefficiency Index than the computed value.

[21]For the $CRRA$ functional form we require a restriction of $\rho < 1$ for all subjects that exhibit corner choices. For $\rho \geq 1$ both assets are essential, hence utility is infinitely negative at the corners. This is not a problem for the $CARA$ functional form.

[22]Note that the recovered parameters for NLLS may differ from those reported in Choi et al. (2007) for several reasons: we allow for elation loving ($-1 < \beta < 0$); we permit boundary observations ($x^i = 0$); we use Euclidean norm (instead of the geometric mean); we use multiple initial points (including random) in the optimization routine (instead of a single predetermined point). We were able to replicate the results reported by Choi et al. (2007).

[23]Table 1 reports medians since the recovered parameter values of a handful of subjects are extreme and distort the average statistics.

The summary statistics suggest substantial qualitative and quantitative differences between recovery methods.[24] Since the Money Metric Index does not include any stochastic component, these differences cannot be tested for statistical significance, yet we may still interpret them in terms of economic significance. In other words, numerical differences in the recovered parameters are suggestive of important qualitative differences in behavior. Consider, for example, the higher median value of β reported for all subjects (as well as for the restricted sample of consistent subjects): NLLS results in $\beta \approx 0$ implying that the decision makers are roughly expected utility maximizers on average, i.e. exhibit only second-order risk aversion, whereas the Money Metric Index suggests the opposite, i.e. decision makers are disappointment averse on average and exhibit first-order risk aversion. Note the smaller curvature of the utility function measured by lower median value of ρ for the Money Metric Index, indicates lesser emphasis on second-order risk aversion.[25]

4.2 Recovering Preferences for Inconsistent Subjects

In Section 3.2 we proved the decomposition of the Money Metric Index into the Varian Inefficiency Index - which serves as a measure of inconsistency, and a remainder - which is a measure of misspecification. As such, we recover parameters that are closest to approximating preferences for those subjects who fail GARP .[26] We exclude only those subjects with a value of the Varian Index exceeding 10%.

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[24]The code and disaggregated data is available for download from the online Appendix.

[25]We find important qualitative differences at the individual level as well. For all combinations of loss function and functional form we find some subjects that are disappointment averse according to the Money Metric Index ($\beta > 0$), yet elation loving according to NLLS ($\beta < 0$), or vice versa. For $CRRA$ and the mean aggregator, we find 8 subjects for which the Money Metric Index reports $\beta > 0$ and NLLS reports $\beta < 0$, and none for which the opposite is true. The incidence and subjects affected vary according to functional form and aggregator selected.

[26]Approximate preferences are defined by the set $\tilde{\mathcal{U}} = \{u \in \mathcal{U}^c : I_V(D, f) = I_M(D, f, \{u\})\}$ where D, f, and $\mathcal{U}^c$ are defined as above. In general, this set is not a singleton as the vector of budget adjustments, $\mathbf{v}$, required by the calculation of the Varian Inefficiency Index, is not unique nor is the utility function that rationalizes a given revealed preference relation, $R_{D,\mathbf{v}}$, for a particular vector of

Subject	I_V	β	ρ	I_M
320	0	-0.698	1.025	0.083
206	0.011	0.044	1.793	0.023

Table 2: Comparing consistent and inconsistent subjects

To illustrate, consider Table 2 that compares the recovered parameters using the Money Metric Index for the mean aggregator and the $CRRA$ functional form for two subjects taken from Choi et al. (2007). Subject 320's choices are consistent with GARP while subject 206's are inconsistent. In spite of the fact that 320 is consistent, the parametric preferences considered do not accurately encode the ranking implied by her choices, as it requires 8.3% wasted income on average. On the other hand, the revealed preference information implied by 206's choices are well captured by the parametric family, since it implies inefficiency of only 2.3%, in spite of the fact that her choices are not strictly consistent ($I_V = 0.0105 > 0$, 116 GARP violations between pairs of observations). Additionally, the decomposed misspecification for Subject 206 amounts to only 1.2% ($I_M - I_V$) with respect to her approximate preferences. In other words, although 320 is consistent with GARP, the choices of 206 are better approximated using the specified functional form. As such, the Money Metric Index can be applied uniformly to all data sets, and the appropriateness of a certain functional form can be evaluated ex-post.

Using the decomposition of the Money Metric Index into the Varian Index (measure of consistency) and a residual which measures misspecification, we can calculate the misspecification for each subject (recall that these are underestimations). Figure 4.2 presents the distribution of misspecification in the Choi et al. (2007) sample for various functional forms controlling for the two aggregators we study. Due to the underestimation of the misspecification and the lack of information about the properties of this bias, all that can be learned in certainty is that the percentage of subjects with misspecification exceeding the 5% threshold is considerably higher using the sum-of-squares aggregator function than the mean aggregator. In this case, at least 25% of

adjustments.

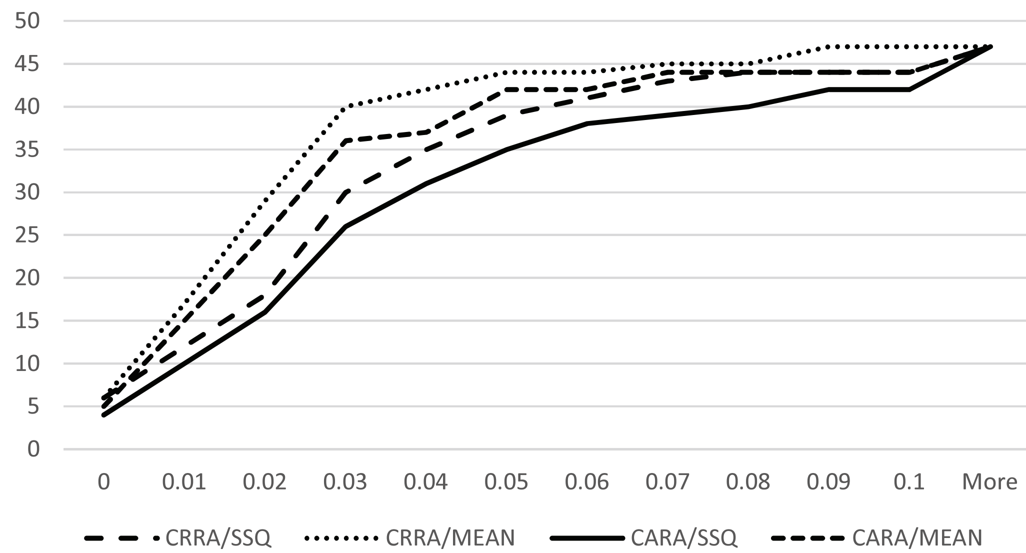


Figure 4.2: Cumulative distribution of misspecification for CRRA/CARA functional forms for mean/sum-of-squares aggregators

subjects exceed the 5% threshold. Results are similar when $CARA$ is used instead of $CRRA$.

4.3 Evaluating a Restriction to Expected Utility

The expected utility model is a nested alternative of the disappointment aversion model, satisfying the restriction that $\beta = 0$. We propose two methods for evaluating whether or not this restriction is justified: one based on the additional misspecification implied by this restriction and the other utilizing the bootstrap method. The former has the advantage of being based on Theorem 1, but suffers from the fact that the computed Varian Inefficiency Index is an upper bound to the real index (and hence the misspecification under the more general model of disappointment aversion is downward biased). The latter can be viewed as appropriate to testing the sensitivity of the recovered parameters to extreme observations and is independent of the overestimation of the inconsistency (through the Varian Inefficiency Index), but is not directly derived from the theoretical considerations explored in the current study.

4.3.1 Misspecification Test

Utilizing a specific functional form, we recover parameters under the restriction that $\beta = 0$ and calculate the additional misspecification implied by this restriction. As proposed in Section 3.2, given the choice of functional form ($CRRA$ or $CARA$) and aggregator (mean or sum-of-squares), we use the ratio $\frac{I_M(D,f,EU)-I_M(D,f,DA)}{I_M(D,f,DA)-I_V(D,f)}$ where DA stands for the disappointment aversion (unrestricted) model , EU stands for expected utility model and f is the chosen aggregator. We allow up to 10% additional misspecification. That is, if the restriction to expected utility implies a proportional increase in the misspecification of more than 10%, then we tend to reject the expected utility specification. Note that since $I_V(D, f)$ is an overestimation of the Varian Inefficiency Index, the calculated ratio is also an overestimation of the real ratio, meaning that the test is actually more strict and will tend to reject expected utility for inconsistent subjects for whom I_V is overestimated.[27] In contrast, the calculation of the Money Metric Index is exact.

The left hand side of Table 3 reports the percentage of subjects with additional misspecification below the 10% threshold. The number of subjects included in each scenario is in brackets. We exclude subjects for three reasons: Subjects with a Varian Inefficiency Index of more than 10% are too inconsistent to consider any reasonable recoverability; those with a Money Metric Index of more than 10% implies that the disappointment aversion specification does not capture their behavior; and those where the Varian Inefficiency Index is overestimated to the extent that it exceeds the value of the Money Metric

[27]For certain subjects it is the case that $I_V(D, f) > I_M(D, f, DA)$ and hence the ratio above is a negative number. In these cases we exclude these subjects from analysis. The incidence of this problem varies depending on the loss function and the choice of functional form.

	misspecification*		**bootstrapping****	
	MEAN	SSQ	MEAN	SSQ
$CRRA$	34.1% (41)	40.0% (35)	26.7% (45)	29.7% (37)
$CARA$	56.4% (39)	40.6% (32)	40.1% (42)	35.3% (34)

*Percentage of subjects for which additional misspecification implied by expected utility restriction is less than 10%.

**Percentage of subjects for which $\beta = 0$ is included in the 95% range of recovered parameters.

(Number of subjects included in each sample in brackets)

Table 3: Evaluating restriction to expected utility using misspecification and bootstrapping

Index. The results between scenarios are qualitatively similar, with a range of subjects consistent with the restriction to expected utility maximization between 13 and 22 (out of 47), depending on the combination of functional form and loss function being used. On the other hand, there does exist some variation in the specific subjects that are excluded due to high values for the Varian Inefficiency or Money Metric Indices as well as which subjects are rejected as expected utility maximizers. While some of this variation is inherent due to the somewhat arbitrary choice of loss function, we will show below how the Money Metric Index can be used to select amongst functional forms.

4.3.2 Bootstrapping

As an alternative to the procedure above, we use a bootstrapping technique in order to determine the sensitivity of recovered parameters to inclusion of all 50 observations. In other words, this procedure provides a sense of how sensitive the recovered parameters are to certain observations by quantifying the variation in recovered parameters that occurs as the composition of the data set varies. The exact procedure is described in detail in Appendix D. This more standard method enables to check the robustness of the conclusions presented in the left hand side of Table 3.

The bootstrapping procedure may be applied to evaluate a parametric restriction. If the interval of recovered β generated by 95% of samples includes

$\beta = 0$ then we may conclude that the expected utility model is a reasonable approximation of individual choices. In the right hand side of Table 3 we present the fraction of subjects for which $\beta = 0$ falls within the 95% range.[28,29] As before, there is some variation in the identity and number of subjects for which this is the case depending on the choice of functional form and loss function. For the combinations tested we find between 11 and 17 subjects may be represented by expected utility according to this procedure.

Although the percentages are somewhat lower, we find there is generally agreement between these procedures for evaluating the expected utility restriction. For example, with respect to the $CRRA$ functional form and mean aggregator, of the 14 subjects with additional misspecification below the 10% threshold, 11 of these subjects satisfy the restriction under the bootstrapping procedure. Additionally, there is one subject that satisfies the bootstrapping criterion for expected utility but is not under the threshold for misspecification. A similar pattern is present for various combinations of functional form and loss function. It is important to note that the lower proportion of expected utility under bootstrapping is in the opposite direction of the bias introduced by the overestimation of I_V.

4.4 Comparison of Non-nested Alternatives

The Money Metric Index also allows one to evaluate non-nested alternatives as is the case if we wish to compare functional forms, for example $CRRA$ versus $CARA$. We can calculate the extent of misspecification implied by each functional form and select the functional form which best represents a decision maker's preferences on a subject by subject basis. For the two loss functions used, we find that most subjects are better represented by the $CRRA$ functional form. The percentage of subjects for which the Money Metric Index is lower using $CRRA$ rather than $CARA$ is reported in Table 4. The number

[28] As above, we exclude subjects for having a Varian Inefficiency Index or Money Metric Index exceeding 10%.

[29] The 95% range is constructed in exactly the same way as with standard statistical bootstrapping procedure by excluding the bottom and top 2.5% percentile of recovered parameters from all samples.

	aggregator	
	MEAN	SSQ
$CRRA$ (vs $CARA$)*	68.9% (45)	80.0% (37)
Expected Utility (allowing $CRRA$ or $CARA$)**	46.3% (41)	48.6% (37)

*Percentage of subjects for which misspecification is lower with $CRRA$ than $CARA$.
**Percentage of subjects for which the additional misspecification implied by expected utility restriction is less than 10% including both $CRRA$ and $CARA$.
(number of subjects included in each sample are in brackets)

Table 4: Choice of utility index and evaluation of expected utility restriction

of subjects included in our calculations is in brackets, again excluding subjects with a Varian Index or Money Metric Index exceeding 10% for both functional forms.

In Section 4.3.1 we evaluated the expected utility model under the restriction of a particular functional form for utility, *$CRRA$ or $CARA$, for all subjects.* In contrast we can evaluate the Money Metric Index for each subject including both functional forms for both the restricted (expected utility) and unrestricted (disappointment aversion) models. Hence, it may be the case that for a single subject the functional form that minimizes misspecification for each model may differ, and of course, the functional form that minimizes misspecification may also differ across subjects. Table 4 reports the results from this more flexible version of the misspecification test above. We conclude that choices of close to half of all subjects may be reasonably approximated by the expected utility model when we allow the utility function to be either $CRRA$ or $CARA$.

5 Short Discussions

5.1 Comments on alternative loss-functions

5.1.1 Area-based Parametric Recoverability

Figure 3.1 suggests an obvious alternative to the money metric as a foundation for measuring misspecification: a measure that is based on the area of

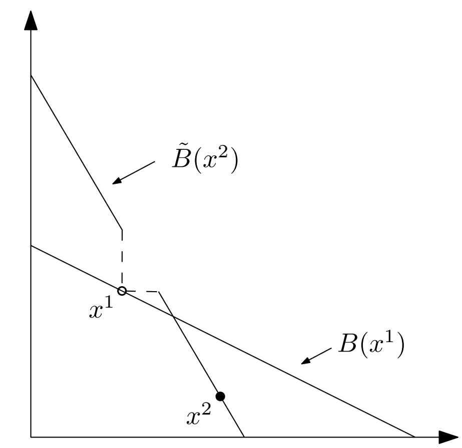


Figure 5.1: Modified budget sets

intersection between the upper contour set corresponding to a specific utility function, and the set of alternatives that are revealed worse than the observed choice. This measure is related to the Minimal Swaps Index, which is a measure of inconsistency proposed recently by Apesteguia and Ballester (2013) for the case of finite number of alternatives. To generalize their method to infinite alternatives set as studied in the current paper, in light of Theorem 1, one needs to calculate an index that is based on the area above for the entire set of continuous and non-satiating utility functions. When the set of utility functions is restricted to a parametric family the Minimal Swaps Index could then measure the inconsistency, while the remainder will represent the misspecification. While the current study demonstrates how to achieve this goal with respect to the Money Metric Index and the corresponding Varian Inefficiency Index, it is not entirely clear how to measure inconsistency directly using areas.

One can define a measure of inconsistency based on the area of intersection between the revealed preferred set and the budget set corresponding to an observed choice. Define the revealed preferred set as *only* those bundles which are either revealed preferred or monotonically dominate a bundle that is revealed preferred to a given bundle. Hence, as illustrated in Figure 5.1, violations of consistency are removed by modifying budget sets so as to eliminate the area of overlap between the budget set and those bundles which are revealed preferred. Hence, we can use this measure to decompose an area index into separate measures of inconsistency and misspecification just as we did with the Money Metric Index.

Nevertheless, an area index is not ideal. First, there does not exist an elegant theoretical analog to Afriat (1987) Theorem with respect to the modified budget sets in Figure 5.1 as there does for the specific type of budget set adjustments utilized in calculating the Money Metric Index (see footnote 36 in Appendix C). Second, computing the inconsistency index suggested above would not be any easier than computing the Varian Inefficiency Index, a problem which is NP-hard. Third, it is a simple exercise to show that choices with modified budget sets as in Figure 5.1 can be easily rationalized by non-convex preferences and, in fact, any recovery procedure based on an area index would be biased towards these types of non-convexities. Put another way, with the area loss function as a criterion, any convex preferences which rationalize the modified data set can be improved upon with similar non-convex preferences. Lastly, the simple area index may lack intuitive interpretation that the Money Metric Index enjoys. All these are surmountable difficulties, that we think are worthwhile pursuing in future work. Ultimately, since the Money Metric Index does not appear to suffer from the same issues we currently believe it dominates the proposed area loss-function both as a measure of misspecification and as a method for recovering preferences.

5.1.2 When Closer is NOT Better

As noted above, a recovery method that employs a loss function that is based only on the distance between observed and predicted choices (as NLLS) fails to account for all the ranking information encoded in the choices, since it compares only the distance between predictions and choices and does not incorporate all other bundles that were feasible but were not chosen. Moreover, if the "true" (unobserved) preferences are not convex, the ranking information induced by a utility function that generates a prediction closer to the observed bundle may be more inconsistent with the "true" ranking of bundles. In other

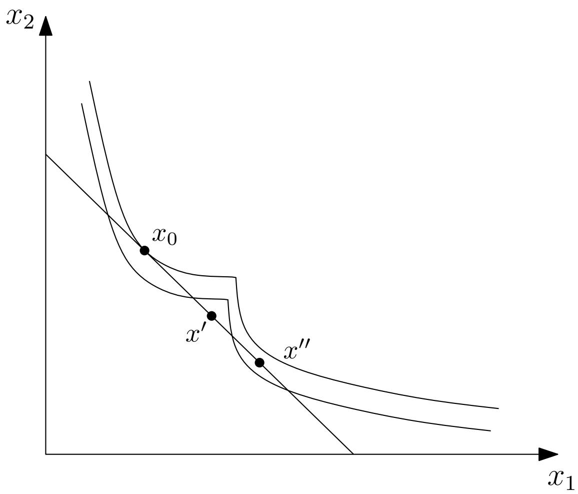


Figure 5.2: Non-convex preferences and a distance-based loss-function

words, the intuition that a closer prediction represents less misspecification relies crucially on the assumption of convex preferences, which is not part of revealed preference theory. Figure 5.2 demonstrates this argument. Consider a choice of x_0 generated by the non-convex preferences depicted in the figure. These preferences would imply that had the DM faced the menu $\{x', x''\}$, she would choose x''. Since x' is closer to x_0 than x'', every recovery method that is based on a distance between observed and predicted choices, would assign a lower loss to preferences with predicted choice at x' than to preferences with predicted choice at x''. This would imply that x' is preferred to x'' - contrary to the "true" preferences that generated the data.

5.2 Relation to Varian (1990)

This work continues the line of thought taken by Varian (1990). There, Varian suggests the money metric as a "natural measure of how close the observed consumer choices come to maximizing a particular utility function" (page 133) and then recommends its usage as a criterion for recovering preferences. He argues that measuring differences in utility space has a more natural economic interpretation than measuring distances between bundles in commodity space. We augment Varian's intuition by providing theoretical and practical substance for the usage of the money metric as a measure of misspecification. First, we demonstrate that the money metric utilizes more preference information encoded in the observed choices to recover preferences than distance-based methods. Second, we relate the budget adjustments implied by the money metric to the Varian Inefficiency Index. Third, we prove that the money metric measure can be constructed observation-by-observation while maintaining most revealed preference information contained in choices.[30] Finally, since we show that the goodness of fit can be decomposed into an inconsistency index and a misspecification index, we introduce several novel applications including evaluating parametric restrictions and model selection.

5.3 Random Utility Maximization

A Random Utility Maximization (henceforth, RUM) model is a probability space over a set of utility functions. A data set is rationalizable if there exists an RUM model such that for every choice problem, the expected frequency of every feasible alternative as generated by the RUM model equals the observed frequency of this alternative.[31]

RUM models have been introduced into economics in the context of population level data, where for each problem only the distribution of choices is observed. In such framework, the population is assumed to be heterogeneous and individuals are assumed to hold deterministic preferences (McFadden, 2005). However, some authors interpret RUM models as describing homogeneous population of individuals with stochastic preferences, or, equivalently, an individual with stochastic preferences that encounters the same choice problem repeatedly (Gul and Pesendorfer, 2006).

The application of RUM to individual level data seems conceptually at-

[30]On the other hand, the computation of the Varian Inefficiency Index is NP-hard since the required budget adjustments are interdependent.

[31]See McFadden (2005), Kitamura and Stoye (2013) and Stoye and Hoderlein (2012) for revealed stochastic preference characterizations and tests for rationalizablity. Gul and Pesendorfer (2006) consider the case where the objects of choice are lotteries and provide necessary and sufficient conditions for a random choice behavior to represent a maximization of some RUM model on lotteries.

tractive. Previously, such application was challenged by experimental results that exhibited monotonicity[32] violations. Recently, extensions were introduced to account for monotonicity violations due to the attraction effect (Gul et al. 2012; Natenzon 2013). However, the finding that subjects exhibit stochastic behavior more frequently when facing "difficult" decision problems (Rubinstein 2002; Agranov and Ortoleva 2014) poses an additional challenge in applying RUM models to individual level data analysis, since it implies deliberate randomization rather than random preferences.

5.4 The Computation of the Varian Inefficiency Index

Afriat (1972, 1973, 1987) and Varian (1990) discuss non-uniform adjustments of the budget lines so that the inconsistencies in the data are removed. Varian (1990) argues that given an aggregator function an optimal vector of adjustments can be found. Moreover, the value of this vector can be interpreted as the inconsistency level of a given data set. The problem of finding this exact value is equivalent to the *minimum cost feedback arc set problem*.[33] Karp (1972) shows that the minimum cost feedback arc set problem is NP-Hard and therefore finding the exact Varian Inefficiency Index is also NP-Hard as suggested in Varian (1990).

Three algorithms to compute a polynomial approximation were suggested in the economics literature. The first algorithm (Tsur (1989) and Algorithm 1 in Alcantud et al. (2010)) suggests to report the vector $\mathbf{v}$ such that v_j is the minimal adjustment required to exclude all x_i such that $x_i R x_j$ from the budget set of observation j. The second algorithm (Algorithm 2 in Alcantud et al. (2010)) is such that v_j is the minimal adjustment required to exclude one x_i such that $x_i R x_j$ from the budget set of observation j. If the data satisfies $GARP_{\mathbf{v}}$, $\mathbf{v}$ is reported, otherwise another point is removed for each observation j and so on until $GARP_{\mathbf{v}}$ is satisfied. The third algorithm (Varian (1993)

[32] A random choice rule is monotonic if the probability of an existing alternative being chosen cannot increase when a new alternative is introduced into the choice set. Monotonicity is a common property to all RUM models.

[33] Given a directed and weighted graph, find the "cheapest" subset of arcs such that its removal turns the graph into an acyclic graph

and Algorithm 3 in Alcantud et al. (2010)) suggests to calculate the minimal adjustment to one of the budget sets, such that one violation of $GARP$ is removed. This minimal value should be substituted into $\mathbf{v}$ and $GARP_{\mathbf{v}}$ should be checked. If the data satisfies $GARP_{\mathbf{v}}$, $\mathbf{v}$ is reported, otherwise another point is removed and the procedure is repeated until the data satisfies $GARP_{\mathbf{v}}$.

Alcantud et al. (2010) show that Algorithms 2 and 3 are better approximations than Algorithm 1 and that they do not dominate each other. Moreover, Alcantud et al. (2010) show that D satisfies $GARP_{\mathbf{v}}$ for the $\mathbf{v}$ found by Algorithms 2 and 3. This implies that these approximations overestimate the actual Varian Inefficiency Index. We do not know of any measure for the quality of this approximation. Also, note that none of these algorithms uses the chosen aggregator function as part of its iterative mechanism. We believe that incorporating the computer science literature on the "minimum cost feedback arc set problem" and using the chosen aggregator may improve considerably the quality of approximation.

5.5 From Inefficiency to Consideration Sets

In the consistency literature, Afriat (1972) and Varian (1990, 1993) view the extent of the adjustment of the budget line as the amount of income wasted by a decision maker relative to a fully consistent one (hence the term "Inefficiency Index"). A related interpretation, mentioned by Houtman (1995), holds that the DM overestimates prices and hence does not consider all feasible alternatives. An alternative interpretation (due to Manzini and Mariotti, 2007, 2012; Apesteguia and Ballester, 2013; Masatlioglu et al., 2012; Cherepanov et al., 2013), views the adjusted budget set as a consideration set which includes only the alternatives from the original budget menu that the DM compares to the chosen alternative. By construction, those bundles not included in the attention set are irrelevant for revealed preference consideration. Another line of interpretation for inconsistent choice data, is measurement error (Varian, 1985; Tsur, 1989). These errors could be the result of various circumstances as (literally) trembling hand, indivisibility, omitted variables etc.

All above interpretations take literally the existence of an underlying "welfare" preferences that generate the data (Bernheim and Rangel, 2009). In addition there exist other plausible data generating processes that result in approximate (and even exact) consistent choices (Simon, 1976; Rubinstein and Salant, 2012). We do not find a clear reason to favor one interpretation over the other, and would rather remain agnostic about the nature of the adjustments required to measure inconsistency.

More importantly, this paper studies the problem of recoverability of preferences and not consistency. That is, we take the data set as the primitive and the utility function as an approximation. As such, the adjustments serve us as a measurement tool ("ruler") for quantifying the extent of misspecification. We view the current work as contributing to the measurement of misspecification and recovery of approximate preferences rather than to the literature that explains how inconsistency arises.

A Non-Parametric Recovery and Non-Convex Preferences (for online publication)

Assume D satisfies GARP. The following definitions follow Varian (1982).

Definition 11. $P_u(x) \equiv \{x' : u(x') > u(x)\}$ is the *strictly upper contour set* of a bundle $x \in \Re_+^K$ given a utility function $u(x)$.

Next, consider the set of prices at which an unobserved bundle, x, is chosen and the augmented data set continues to be consistent with GARP.

Definition 12. Suppose $x \in \Re_+^K$ is an unobserved bundle, then

$$S(x) = \{p \,|\{(p, x)\} \cup D \,\text{satisfies GARP and}\, px = 1\}$$

For every unobserved bundle x, Varian (1982) employs $S(x)$ to construct lower and upper bounds on the upper and lower contour sets through x.

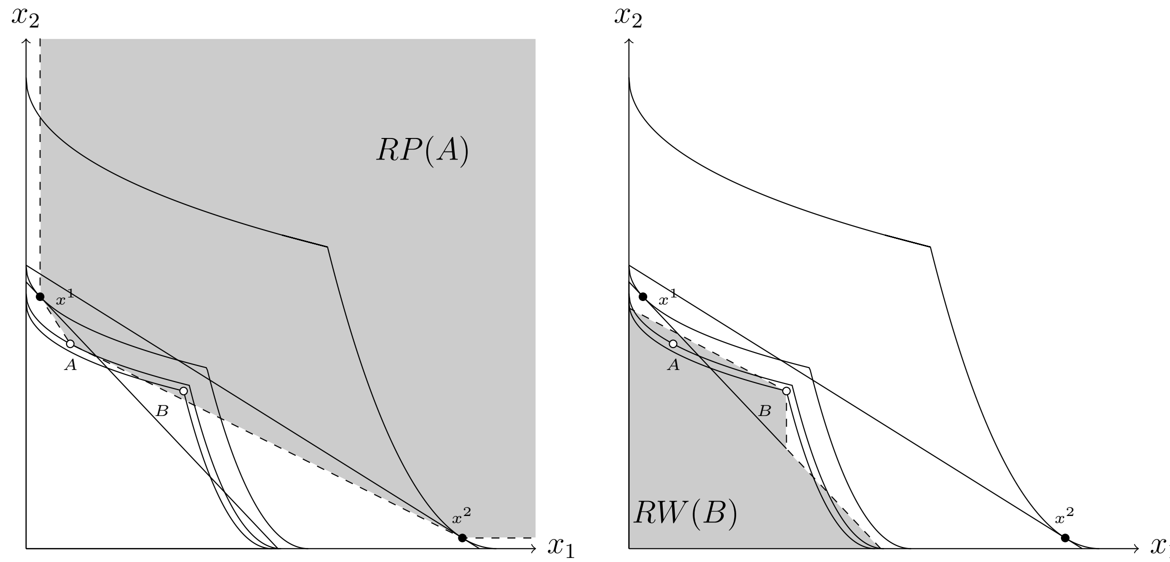
Definition 13. For every unobserved bundle $x \in \Re_+^K$:

1. The *revealed worse set* is $RW(x) \equiv \left\{x' \left| \forall p \in S(x), x P_{D \cup \{p,x\}} x' \right.\right\}$. The *not revealed worse set,* denoted by $NRW(x)$, is the complement of $RW(x)$.

2. The *revealed preferred set* is $RP(x) \equiv \left\{x' \left| \forall p \in S(x'), x' P_{D \cup \{p,x'\}} x \right.\right\}$.

In Fact 5, Varian (1982) (page 953) states: "let $u(x)$ be any utility function that rationalizes the data. Then for all (unobserved bundles - HPZ) x , $RP(x) \subset P_u(x) \subset NRW(x)$". Thus, given a data set that satisfies GARP and a utility function that rationalizes these data, every indifference curve through a given unobserved bundle must be bounded between the revealed worse set and the revealed preferred set of this bundle.

Suppose a DM has to decide how to allocate a wealth of 1 between consumption in two mutually exclusive, exhaustive and equally probable states of the world. The allocation is attained by holding a portfolio of Arrow securities with unit prices $p = (p_1, p_2)$. Figure A.1 presents a data set D of two observations. Portfolio $x^1 = (0.124, 2.222)$ is chosen when prices are $p^1 = (0.450, 0.425)$, and



(a) Violation of the Revealed Preferred Set

(b) Violation of the Revealed Worse Set

Figure A.1: Violations of Fact 5

portfolio $x^2 = (3.850, 0.094)$ is chosen when prices are $p^2 = (0.250, 0.400)$. Notice that since $p^2 < p^1$, every portfolio that is feasible under p^1 is also feasible when prices are p^2, therefore $x^2 R_D^0 x^1$. Now consider two unobserved portfolios $A = (0.390, 1.806)$ and $B = (1.390, 1.390)$. Portfolio A is feasible under both prices, but portfolio B is feasible only under p^2. The revealed preferred set of A and the revealed worse set of B are drawn in panels A.1a and A.1b, respectively. Now consider the following utility function over portfolio $x = (x_1, x_2)$:

$$u(x_1, x_2) = \sqrt{\max\{x_1, x_2\}} + \frac{1}{4}\sqrt{\min\{x_1, x_2\}} \tag{A.1}$$

which represents the preferences of an elation seeking DM (Gul, 1991) with $\beta = -0.75$ and a CRRA utility index with $\rho = 0.5$ over Arrow securities. Therefore, the DM's preferences are not convex and $u(\cdot)$ is not quasi-concave (let alone not concave). The indifference curves drawn in Figure A.1 through x^1 and x^2 demonstrate that this utility function rationalizes the data.

Recall that Fact 5 in Varian (1982) states that for any unobserved bundle

x, if u rationalizes the data then $RP(x) \subset P_u(x) \subset NRW(x)$. However, Figure A.1a clearly demonstrates that while $B \in RP(A)$, it is not true that $B \in P_u(A)$. Similarly, Figure A.1b shows that while $A \in P_u(B)$ it is not true that $A \in NRW(B)$. That is, the ranking of unobserved portfolios implied by the Revealed Preferred and Revealed Worse sets is inconsistent with the ranking of portfolios induced by a utility function that rationalizes the data. In other words, the utility function's indifference curves do not abide by Varian (1982) non-parametric bounds.

Figure A.1 suggests the source of the above inconsistency with Varian's Fact 5: when the DM is elation seeking, her preferences are non-convex and the utility function is not concave. The failure of the nonparametric bounds can be traced back to the construction of the revealed preferred and revealed worse sets. Since by Afriat's Theorem if the data satisfies GARP there exists a concave utility function that rationalizes it, $S(x)$ (Definition 12) is non-empty for every x. However, there may exist a utility function that rationalizes the data for which there is no price vector p that supports x as an optimal choice. Therefore, even if x' is such that $xP_{D\cup\{p,x\}}x'$ for every $p \in S(x)$, it does not imply that the utility function that *never* chooses x will rank x above x'. In Figure A.1a $BP_{D\cup\{p,B\}}A$ for every $p \in S(B)$, however the utility function never chooses B and therefore can rank B below A.[34]

[34] Definitions 12 and 13 can be trivially extended to include observed bundles, and then a similar argument can be constructed for the observed portfolio x^1 in Figure A.1a. Note that the violation of the revealed worse set demonstrated in Figure A.1b cannot occur for an observed bundle since there exists a price vector p that supports the bundle as an optimal choice.

B Proof of Proposition 1 (for online publication)

Notation. Let $x \in \Re^K$ and $\delta > 0$. $B_\delta(x) = \left\{ y \in \Re^K : \|y - x\| < \delta \right\}$.

Definition. A utility function $u : \Re^K \to \Re$ is

1. *locally non-satiated* if $\forall x \in \Re^K$ and $\forall \epsilon > 0$ $\exists y \in B_\delta(x)$ such that $u(y) > u(x)$.
2. *continuous* if $\forall x \in \Re^K$ and $\forall \epsilon > 0$ there exists $\delta > 0$ such that $y \in B_\delta(x)$ implies $u(y) \in B_\epsilon(u(x))$.

Lemma. *If $u(\cdot)$ is a locally non-satiated utility function that rationalizes $D = \left\{ (p^i, x^i)_{i=1}^n \right\}$, then $x^i P_D^0 x$ implies $u(x^i) > u(x)$.*

Proof. Suppose $x^i P_D^0 x$ $(p^i x^i > p^i x)$. Then by the definition of the revealed preference relations (Definition 1), $x^i R_D^0 x$. Since $u(\cdot)$ rationalizes D, $x^i R_D^0 x$ implies $u(x^i) \geq u(x)$. Suppose that $u(x^i) = u(x)$. Since $p^i x^i > p^i x$ $\exists \epsilon > 0$ such that $\forall y \in B_\epsilon(x) : p^i x^i > p^i y$. By local non-satiation $\exists y' \in B_\epsilon(x)$ such that $u(y') > u(x) = u(x^i)$. Thus, y' is a bundle such that $p^i x^i > p^i y'$ and $u(y') > u(x^i)$, in contradiction to $u(\cdot)$ rationalizes D. Therefore, $u(x^i) > u(x)$. $\square$

For what follows, let $D = \left\{ (p^i, x^i)_{i=1}^n \right\}$ and let $u(\cdot)$ be a continuous and locally non-satiated utility function.

Part 1: $u(\cdot)$ $\mathbf{v}^\star(D,u)$*-rationalizes* D

Proof. Suppose that for some observation $(p^i, x^i) \in D$ there exists a bundle x such that $x^i R^0_{D,\mathbf{v}^\star(D,u)} x$ and $u(x^i) < u(x)$. By the definition of the revealed preference relations induced by adjusted data sets (Definition 4.1), $v^{\star i}(D,u) p^i x^i \geq p^i x$. By the normalized money metric definition (Definition 9), $m(x^i, p^i, u) \geq p^i x$. Since $m(x^i, p^i, u)$ is the minimal expenditure required to achieve a utility level of at least $u(x^i)$, the case where the inequality is strict contradicts Definition 9. If $m(x^i, p^i, u) = p^i x$ and $u(x^i) < u(x)$, by continuity of $u(\cdot)$ there exists $\gamma > 0$ such that $u(x^i) < u((1-\gamma)x)$. However, since $p^i(1-\gamma)x < p^i x = m(x^i, p^i, u)$, we reach a contradiction to Definition 9. $\square$

Part 2: $\mathbf{v}^\star(D,u) = \mathbf{1}$ if and only if $u(\cdot)$ rationalizes D.

Proof. First, let us show that if $u(\cdot)$ rationalizes D then $\mathbf{v}^\star(D,u) = \mathbf{1}$. Suppose that for some observation $(p^i, x^i) \in D$, $v^{\star i}(D,u) < 1$, that is: $m(x^i, p^i, u) < p^i x^i$. By Definition 9, there exists a bundle x such that $p^i x < p^i x^i$ and $u(x) \geq u(x^i)$. However, since by Definition 1.2, $x^i P^0_D x$, and since $u(\cdot)$ is a locally non-satiated utility function that rationalizes D, the above proven lemma implies, in contradiction, that $u(x^i) > u(x)$. Thus, $v^{\star i}(D,u) = 1$ for all i. Therefore $\mathbf{v}^\star(D,u) = \mathbf{1}$.

Next, let us show that if $\mathbf{v}^\star(D,u) = \mathbf{1}$ then $u(\cdot)$ rationalizes D. By Definition 9, $\mathbf{v}^\star(D,u) = \mathbf{1}$ implies $m(x^i, p^i, u) = p^i x^i$ for every $(p^i, x^i) \in D$. Suppose that $u(\cdot)$ does not rationalize the data. That is, for some observation (p^i, x^i), there exists a bundle x such that $u(x) > u(x^i)$ and $x^i R^0_D x$. By continuity of $u(\cdot)$ there exist $\gamma > 0$ such that $u((1-\gamma)x) > u(x^i)$. However, since $p^i(1-\gamma)x < p^i x^i = m(x^i, p^i, u)$ we reach a contradiction to Definition 9. $\square$

Part 3: Let $\mathbf{v} \in [\mathbf{0}, \mathbf{1}]^{\mathbf{n}}$. $u(\cdot)$ $\mathbf{v}$-*rationalizes* D if and only if $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$.

Proof. First, let us show that if $u(\cdot)$ $\mathbf{v}$-rationalizes D then $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$. Suppose that $\mathbf{v}$ is such that $u(\cdot)$ $\mathbf{v}$-rationalizes D and for observation i, $v^i > v^{\star i}(D,u)$. By Definition 8, $u(x^i) \geq u(x)$ for all x such that $x^i R^0_{D,\mathbf{v}} x$ or equivalently $v^i p^i x^i \geq p^i x$. By Definition 9 and since $v^i > v^{\star i}(D,u)$ we get that $v^i p^i x^i > m(x^i, p^i, u) = p^i x^{i\star}$ where $x^{i\star} \in argmin_{\left\{y \in \Re^K_+ : u(y) \geq u(x^i)\right\}} p^i y$. It follows that $\exists \epsilon > 0$ such that $\forall y \in B_\epsilon(x^{i\star}) : v^i p^i x^i > p^i y$. By local non-satiation $\exists y' \in B_\epsilon(x^{i\star})$ such that $u(y') > u(x^{i\star}) \geq u(x^i)$. Thus, y' is a bundle such that $v^i p^i x^i > p^i y'$ and $u(y') > u(x^i)$ contradicting that $u(\cdot)$ $\mathbf{v}$-rationalizes D.

Next, let us show that if $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$ then $u(\cdot)$ $\mathbf{v}$-rationalizes D. By Part 1: $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes D. That is, for every observation $(p^i, x^i) \in D$,

$v^{\star i}(D,u)p^ix^i \geq p^ix$ implies $u\left(x^i\right) \geq u\left(x\right)$. Since $\mathbf{v} \leqq \mathbf{v}^{\star}(D,u)$, for every observation $\left(p^i, x^i\right) \in D$, $v^{\star i}\left(D,u\right)p^ix^i \geq v^ip^ix^i$. Therefore, for every observation $\left(p^i, x^i\right) \in D$, $v^ip^ix^i \geq p^ix$ implies $u\left(x^i\right) \geq u\left(x\right)$. Hence, $u\left(\cdot\right)$ $\mathbf{v}$-rationalizes D. $\square$

C Proof of Theorem 1 (for online publication)

Notation. We use the following notations throughout the proof:

- Let $\mathbf{v} \in [0,1]^n$ and $\delta > 0$. $\bar{B}_\delta(\mathbf{v}) = \{\mathbf{v}' \in [0,1]^n : \|\mathbf{v}' - \mathbf{v}\| < \delta\}$.
- $E_v = \{\mathbf{v} \in [0,1]^n : f(\mathbf{v}) = I_V(D,f)\}$
- $\forall \epsilon < M - I_V(D,f) : \quad E_{v+\epsilon} = \{\mathbf{v} \in [0,1]^n : f(\mathbf{v}) = I_V(D,f) + \epsilon\}$.
- $E_G = \left\{\mathbf{v} \in [0,1]^n : \forall r > 0,\ \exists \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D \text{ satisfies } GARP_{\mathbf{v}'}\right\}$.
- $\hat{E} = E_v \cap E_G$.

Lemma 1. *E_v is non-empty, bounded and closed.*

Proof. First, by Fact 3, $I_V(D,f)$ always exists. Second, by Definition 6 $f(\cdot)$ is continuous and bounded. By the Intermediate Value Theorem, for every value of $I_V(D,f)$ there exists a vector $\mathbf{v}$ such that $f(\mathbf{v}) = I_V(D,f)$, concluding that E_v is non-empty. Third, $E_v \subseteq [0,1]^n$ and therefore it is bounded. Finally, since $f(\cdot)$ is continuous it induces a continuous ordering on $[0,1]^n$. Therefore, for every $I_V(D,f)$, the upper contour set and the lower contour set are closed and their intersection, E_v, is closed as well. □

Lemma 2. *$\hat{E}$ is non-empty.*

Proof. Assume $I_V(D,f) < M$. Suppose that $\hat{E}$ is empty, that is $\mathbf{v} \in E_v \Rightarrow \mathbf{v} \notin E_G$ (due to Lemma 1, this condition is not vacuous). Thus, $\forall \mathbf{v} \in E_v,\ \exists r > 0,\ \forall \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D$ violates $GARP_{\mathbf{v}'}$.
Let $r(\mathbf{v}) = \sup_{\{r \in (0,\sqrt{n}] : \forall \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D \text{ violates } GARP_{\mathbf{v}'}\}} r$. $r(\mathbf{v})$ is uniform continuous on E_v since $\forall \mathbf{v}, \mathbf{v}' \in E_v$ if $\| \mathbf{v} - \mathbf{v}' \| < \epsilon$ then by the triangle inequality $|r(\mathbf{v}) - r(\mathbf{v}')| < \epsilon$.[35] Let $\bar{r} = \min_{\mathbf{v} \in E_v} r(\mathbf{v})$. $\bar{r}$ exists since $r(\mathbf{v})$ is continuous on E_v and E_v is bounded and closed (by Lemma 1). In addition, $\bar{r} > 0$ since $\forall \mathbf{v} \in E_v :\ r(\mathbf{v}) > 0$. Then, $\forall \mathbf{v} \in E_v,\ \forall r < \bar{r},\ \forall \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D$ violates $GARP_{\mathbf{v}'}$.

[35] The distance between $\mathbf{v}$ and $\mathbf{v}'$ is at most ϵ, the distance between $\mathbf{v}$ and some $\mathbf{w}$ such that D satisfies $GARP_{\mathbf{w}}$ is $r(\mathbf{v})$ and by the triangle inequality the distance between $\mathbf{v}'$ and $\mathbf{w}$, which serves as a bound on $r(\mathbf{v}')$, is between $r(\mathbf{v}) - \epsilon$ and $r(\mathbf{v}) + \epsilon$.

Thus, we established that for every $I_V(D, f) < M$ if $\hat{E}$ is empty there exists a hypercylinder H of radius $\bar{r} > 0$ around E_v such that if $\mathbf{v}'$ is an interior point in H then D violates $GARP_{\mathbf{v}'}$.

The next step is to show that there exists $0 < \epsilon < M - I_V(D, f)$ such that $E_{v+\epsilon}$ is contained in H (by Lemma 1 $E_{v+\epsilon}$ is non-empty). Suppose that for every $0 < \epsilon < M - I_V(D, f)$ there exists $\mathbf{v}'_\epsilon \in E_{v+\epsilon}$ such that $\mathbf{v}'_\epsilon \notin H$. Then, $\mathbf{v}'_\epsilon$, where $\epsilon \to 0$, is an infinite bounded sequence in $[0, 1]^n$ and therefore it has a convergent subsequence. Denote the limit of this subsequence by $\hat{\mathbf{v}}$. Since $\hat{\mathbf{v}}$ is not an interior point of H it must be that $f(\hat{\mathbf{v}}) \neq I_V(D, f)$. However, by construction, $\lim_{\epsilon \to 0} f(\mathbf{v}'_\epsilon) = I_V(D, f)$, suggesting that $f(\cdot)$ is not continuous. Thus, there exists $\bar{\epsilon}$ such that $E_{v+\bar{\epsilon}} \subset H$. Moreover, since $f(\cdot)$ is continuous $\forall \epsilon \in [0, \bar{\epsilon}) : E_{v+\epsilon} \subset H$.

That is, there exists $\bar{\epsilon} > 0$ such that for every $\mathbf{v}' \in [0, 1]^{\mathrm{n}}$ that satisfies $I_V(D, f) \leq f(\mathbf{v}') < I_V(D, f) + \bar{\epsilon} < M$, D violates $GARP_{\mathbf{v}'}$. Since $I_V(D, f)$ is an infimum there is no $\mathbf{v} \in [0, 1]^{\mathrm{n}}$ such that $f(\mathbf{v}) < I_V(D, f)$ and D satisfies $GARP_{\mathbf{v}}$. Thus, there exists $I_V(D, f) < m < M$ such that for every $\mathbf{v} \in [0, 1]^{\mathrm{n}} : f(\mathbf{v}) < m$ and D violates $GARP_{\mathbf{v}}$. That contradicts the maximality of $I_V(D, f)$ as an infimum. Therefore, we have shown that if $I_V(D, f) < M$ then $\hat{E}$ is non-empty.

Finally, suppose $I_V(D, f) = M$. By Definition 6, $\mathbf{0} \in E_v$. By Fact 1, $\mathbf{0} \in E_G$. Thus, also if $I_V(D, f) = M$ then $\hat{E}$ is non-empty. $\square$

Lemma 3. *Let* $\mathbf{v} \in [0, 1]^n$. *If* $\tilde{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ *and* D *satisfies* $GARP_{\tilde{\mathbf{v}}}$, *there exists* $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ *where* $\hat{\mathbf{v}} \leq \mathbf{v}$ *and* D *satisfies* $GARP_{\hat{\mathbf{v}}}$.

Proof. If $\tilde{\mathbf{v}} \leq \mathbf{v}$ then the lemma is trivial. If $\mathbf{v} \leq \tilde{\mathbf{v}}$ then by Fact 2 D satisfies $GARP_{\mathbf{v}}$. By the same fact, D satisfies $GARP_{\hat{\mathbf{v}}}$ for every $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ where $\hat{\mathbf{v}} \leq \mathbf{v}$. Otherwise, define $\hat{\mathbf{v}}$ such that $\forall i \in \{1, \ldots, n\} : \hat{v}^i = \min\{v^i, \tilde{v}^i\}$. By construction, $\hat{\mathbf{v}} \leq \mathbf{v}$ and $\hat{\mathbf{v}} \leq \tilde{\mathbf{v}}$. Since $\forall i \in \{1, \ldots, n\} : |\hat{v}^i - v^i| \leq |\tilde{v}^i - v^i|$ then $\hat{\mathbf{v}} \in B_\delta(\mathbf{v})$. In addition, since $\mathbf{v}, \tilde{\mathbf{v}} \in [0, 1]^n$ then $\hat{\mathbf{v}} \in [0, 1]^n$. Therefore, $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$. Finally, since $\hat{\mathbf{v}} \leq \tilde{\mathbf{v}}$ and D satisfies $GARP_{\tilde{\mathbf{v}}}$ by Fact 2 D satisfies $GARP_{\hat{\mathbf{v}}}$. Thus, we constructed $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ where $\hat{\mathbf{v}} \leq \mathbf{v}$ and such that D satisfies $GARP_{\hat{\mathbf{v}}}$. $\square$

Lemma 4. *Let* $\mathbf{v}' \in \hat{E}$*. D satisfies* $GARP_{\lambda\mathbf{v}'}$ *for all* $\lambda \in [0,1)$*.*

Proof. By Lemma 2 $\mathbf{v}'$ exists. If $\mathbf{v}' = \mathbf{0}$ the Lemma is trivial by Fact 1. Suppose $\mathbf{v}' \geq \mathbf{0}$. Denote $v'_{min} = \min_{\{v'_i>0\}} v'_i$ and let $\delta \in (0, v'_{min})$. Let $\tilde{B}_\delta(\mathbf{v}') = \{\mathbf{v} : \mathbf{v} \leq \mathbf{v}'\} \cap \bar{B}_\delta(\mathbf{v}')$. Let $\tilde{\mathbf{v}} \in \tilde{B}_\delta(\mathbf{v}')$ such that D satisfies $GARP_{\tilde{\mathbf{v}}}$. By Lemma 3 such $\tilde{\mathbf{v}}$ exists and by construction $\tilde{\mathbf{v}} \neq \mathbf{v}'$. Note that choice of δ implies that for every $i \in \{1,\ldots,n\}$, $v'_i > 0 \implies \tilde{v}_i > 0$. Define $\lambda = \{\lambda^i\}_{i=1}^n$ such that if $v'_i = 0$ then $\lambda^i = 0$ and otherwise $\lambda^i = \frac{\tilde{v}_i}{v'_i} > 0$. Then, $\lambda \in [0,1]^n \backslash \{\mathbf{0}\}$. Denote $\bar{\lambda} = \min_{\{\lambda^i>0\}} \lambda^i$. Then, $0 < \bar{\lambda} < 1$. For every $i \in \{1,\ldots,n\}$ define $\hat{v}_i = \bar{\lambda} v'_i$. First, note that $\forall i \in \{1,\ldots,n\}:\ \hat{v}_i \leq \tilde{v}_i$ (if $v'_i = 0$ then $\hat{v}_i \leq \tilde{v}_i = v'_i = 0$, otherwise, $\hat{v}_i = \bar{\lambda} v'_i \leq \frac{\tilde{v}_i}{v'_i} v'_i = \tilde{v}_i$) and by Fact 2 since D satisfies $GARP_{\tilde{\mathbf{v}}}$ then D satisfies $GARP_{\hat{\mathbf{v}}}$. Second, $\hat{\mathbf{v}} = \bar{\lambda}\mathbf{v}'$. Finally, note that $\forall i \in \{1,\ldots,n\}:\ v'_i - \delta \leq \tilde{v}_i \leq v'_i$. Therefore, $\forall i \in \{1,\ldots,n\}:\ 1 - \frac{\delta}{v'_i} \leq \lambda^i \leq 1$ and $1 - \frac{\delta}{v'_{min}} \leq \bar{\lambda} < 1$. Thus, for every $\epsilon > 0$ there exists $\bar{\lambda} > 1 - \epsilon$ such that $\hat{\mathbf{v}} = \bar{\lambda}\mathbf{v}'$ and D satisfies $GARP_{\lambda\mathbf{v}'}$. By Fact 2 for every $0 \leq \lambda \leq \bar{\lambda}$ D satisfies $GARP_{\lambda\mathbf{v}'}$. Hence, D satisfies $GARP_{\lambda\mathbf{v}'}$ for all $\lambda \in [0,1)$. □

Definition. Let $\mathbf{v} \in [0,1]^n$. D satisfies $\boldsymbol{v}$*-Cyclical Consistency* if

$$\begin{aligned} & v^r p^r x^r \geq p^r x^s, v^s p^s x^s \geq p^s x^t, \ldots, v^q p^q x^q \geq p^q x^r \\ \implies\ & v^r p^r x^r = p^r x^s, v^s p^s x^s = p^s x^t, \ldots, v^q p^q x^q = p^q x^r \end{aligned}$$

Lemma 5. *Let* $\mathbf{v} \in [0,1]^n$*. D satisfies* $\mathbf{v}$*-Cyclical Consistency if and only if it satisfies* $GARP_{\mathbf{v}}$*.*

Proof. Suppose D violates $\mathbf{v}$-Cyclical Consistency. Then, there exists a sequence of observations such that $v^r p^r x^r \geq p^r x^s, v^s p^s x^s \geq p^s x^t, \ldots, v^q p^q x^q \geq p^q x^r$ and $v^s p^s x^s > p^s x^t$. By Definition 4, $x^r R^0_{D,\mathbf{v}} x^s, x^s R^0_{D,\mathbf{v}} x^t, \ldots, x^q R^0_{D,\mathbf{v}} x^r$ and therefore $x^t R_{D,\mathbf{v}} x^s$. However, by the same definition $x^s P^0_{D,\mathbf{v}} x^t$. Thus, D violates $GARP_{\mathbf{v}}$. On the other hand, suppose D violates $GARP_{\mathbf{v}}$. There exists a pair of observations (p^t, x^t) and (p^s, x^s) such that $x^t R_{D,\mathbf{v}} x^s$ and $x^s P^0_{D,\mathbf{v}} x^t$. Again, by Definition 4, there exists a subset of observations such that $x^t R^0_{D,\mathbf{v}} x^u, x^u R^0_{D,\mathbf{v}} x^v, \ldots, x^q R^0_{D,\mathbf{v}} x^s$ and since $x^s P^0_{D,\mathbf{v}} x^t$ implies $x^s R^0_{D,\mathbf{v}} x^t$ there is a subset of observations such that $v^t p^t x^t \geq p^t x^u, v^u p^u x^u \geq p^u x^v, \ldots, v^s p^s x^s \geq$

$p^s x^t$. In addition, since $x^s P^0_{D,\mathbf{v}} x^t$ we have $v^s p^s x^s > p^s x^t$. However, this combination violates $\mathbf{v}$-Cyclical Consistency. $\square$

Lemma 6. $I_V(D,f) \leq I_M(D,f,\mathcal{U}^c)$

Proof. If $I_V(D,f)=0$ the lemma follows from definitions 6 and 10. Otherwise, suppose that $I_V(D,f) > I_M(D,f,\mathcal{U}^c)$. Since $I_M(D,f,\mathcal{U}^c) = \inf_{u\in\mathcal{U}^c} f\left(\mathbf{v}^\star(D,u)\right)$ there exists $u \in \mathcal{U}^c$ such that $f\left(\mathbf{v}^\star(D,u)\right) < I_V(D,f)$. By Proposition 1.1 $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes D. By Theorem 6.3.I in Afriat (1987) (p. 179)[36] $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes D if and only if D satisfies $\mathbf{v}^\star(D,u)$-Cyclical Consistency, which is equivalent, by Lemma 5, to D satisfies $GARP_{\mathbf{v}^\star(D,u)}$. However, since D satisfies $GARP_{\mathbf{v}^\star(D,u)}$ and $f\left(\mathbf{v}^\star(D,u)\right) < I_V(D,f)$, $I_V(D,f)$ cannot be the infimum of $f(\cdot)$ on the set of all $\mathbf{v}\in[0,1]^n$ such that D satisfies $GARP_{\mathbf{v}}$. $\square$

Lemma 7. *Let* $\mathbf{v}\in[0,1]^n$ *be such that* D *satisfies* $GARP_{\mathbf{v}}$. *Then* $I_M(D,f,\mathcal{U}^c) \leq f(\mathbf{v})$.

Proof. By Lemma 5, D satisfies $GARP_{\mathbf{v}}$ if and only if D satisfies $\mathbf{v}$-Cyclical Consistency. By Theorem 6.3.I in Afriat (1987) (p. 179) D satisfies $\mathbf{v}$-Cyclical Consistency if and only if there exists a non-satiated continuous utility function $u \in \mathcal{U}^c$ that $\mathbf{v}$-rationalizes D. By Proposition 1.3, $\mathbf{v} \leq \mathbf{v}^\star(D,u)$. Since $f(\cdot)$ is weakly decreasing $f\left(\mathbf{v}^\star(D,u)\right) \leq f(\mathbf{v})$. Therefore, by Definition 10, $I_M(D,f,\mathcal{U}^c) \leq f(\mathbf{v})$. $\square$

Theorem. *For every finite data set* $D=\left\{(p^i,x^i)_{i=1}^n\right\}$ *and aggregator function* $f:[0,1]^n \to [0,M]$ *:*

$$I_V(D,f) = I_M(D,f,\mathcal{U}^c)$$

where $\mathcal{U}^c$ *is the set of contionuous and locally non-satiated utility functions.*

[36] Afriat (1987) does not provide a proof for this theorem. Afriat (1973) provides a proof for the uniform case (same adjustments for all observations) which can be generalized to this theorem. Houtman (1995) studies general cost functions that include the uniform case, the non-uniform case that we use and many other cases. He provides a proof for a general form of Theorem 6.3.I in Afriat (1987) that applies here as well. Note that while Houtman (1995) elaborates on the uniform case, all his statements on this case apply also to the non-uniform linear case that is considered here.

Proof. Let $\mathbf{v}^\star \in \hat{E}$. By Lemma 2 such point exists. For every $\lambda \in [0,1]$ denote $F_\lambda = f(\lambda \mathbf{v}^\star)$. Consider the sequence of intervals $[I_V(D,f), F_\lambda)$. By Lemma 4, D satisfies $GARP_{\lambda \mathbf{v}^\star}$ for all $\lambda \in [0,1)$. Therefore, by Lemma 7, $\forall \lambda \in [0,1)$: $I_M(D,f,\mathcal{U}^c) \leq F_\lambda$. In addition, by Lemma 6, $I_V(D,f) \leq I_M(D,f,\mathcal{U}^c)$. Hence, $\forall \lambda \in [0,1)$: $I_M(D,f,\mathcal{U}^c) \in [I_V(D,f), F_\lambda)$. Since $\lim_{\lambda \to 1} F_\lambda = I_V(D,f)$ we get $I_V(D,f) = I_M(D,f,\mathcal{U}^c)$. $\square$

D The Code (for online publication)

Preliminaries

This appendix describes the code designed to implement the indices and estimations mentioned in the paper. The code accommodates the data gathered by the symmetric treatment in Choi et al. (2007) and supplies recovery procedures using the family of disappointment aversion utility functions introduced in Gul (1991) with CRRA or CARA utility indices. We hope that this appendix will ease the process of adapting this software to other data sets and other families of parametric utility functions.

Consistency Tests (HPZ_Subject_Consistency)

To construct the relations mentioned in Definition 1, we first calculate a matrix (REF) such that the cell in the i^{th} row and the j^{th} column stores $p^i x^i - p^i x^j$. If this difference is non-negative we say that $x^i R_D^0 x^j$ (DRP matrix) while if it is strictly positive we say that $x^i P_D^0 x^j$ (SDRP matrix).[37] Then we use the Floyd–Warshall algorithm (Warshall, 1962)[38] to construct the revealed preferred relation (RP matrix) which is the transitive closure of the directly revealed preferred relation. Finally, we construct the strictly revealed preferred relation (SRP matrix).

Using these relations we implement three consistency tests for a given data set D. $SARP$ ($x^i R_D x^j$ and $x^j R_D x^i$ implies $x^i = x^j$) , $GARP$ ($x^i R_D x^j$ implies not $x^j P_D^0 x^i$) and $WARP$ ($x^i R_D^0 x^j$ implies not $x^j P_D^0 x^i$). For each test we report the number of violations and the number of inconsistent pairs of observations. If $GARP$ is not satisfied we calculate the inconsistency indices described in the next section. If there are no $GARP$ violations, we report that the Afriat index equals 0, the Varian indices equal 0, 0 and 0 (minimum, mean and sum

[37] We introduce a variable named THRESHOLD (initialized to Matlab's epsilon) to allow for some flexibility in these definitions. x^i is directly revealed preferred over x^j if $p^i x^i + THRESHOLD > p^i x^j$ while x^i is strictly directly revealed preferred over x^j if $p^i x^i > p^i x^j + THRESHOLD$.

[38] We use an external graph theory package (matlab_bgl) that implements this algorithm.

of squares, respectively) and that the Houtman Maks index equals 50.

Inconsistency Indices

Afriat's Index (HPZ_Afriat_efficiency_index)

Definition of the *Afriat's Inefficiency Index* is[39]

$$I_A(D) = \inf_{\lambda \in [0,1]: D \text{ satisfies } GARP_{\lambda \mathbf{1}}} 1 - \lambda$$

We use a bisection search to approximate Afriat's index as suggested by Houtman and Maks (1987), Varian (1990) and Houtman (1995). The input is a matrix (expenditure) such that the cell in the i^{th} row and the j^{th} column contains $p^i x^j$. We initialize the index (AFRIAT) to $\frac{1}{2}$ and the bounds to 0 and 1. In each iteration we adjust the matrix by multiplying its main diagonal elements by AFRIAT. The adjusted data is checked for $GARP$. As in any bisection search, if $GARP$ is satisfied the next examined index is the average of the current index and the upper bound while the lower bound is changed to the current index. If $GARP$ is not satisfied the next examined index is the average of the current index and the lower bound while the upper bound is changed to the current index. The number of iterations determines the extent of approximation. We use 30 iterations and therefore we approximate $I_A(D)$ to a level of $2^{-30} \approx 10^{-9}$. To follow the definition we report one minus the result of the algorithm.[40]

Varian's Index (HPZ_varian_efficiency_index)

The definition of *Varian's* Inefficiency *Index* is

$$I_V(D) = \inf_{\mathbf{v} \in [\mathbf{0},\mathbf{1}]^{\mathbf{n}}: D \text{ satisfies } GARP_{\mathbf{v}}} f(\mathbf{v})$$

Calculating Varian's index is an NP-hard problem and we use Algorithm 3 in Alcantud et al. (2010) to approximate it (from above). The input is a matrix

[39] See Afriat (1972, 1973).

[40] The procedure reports λ.

(expenditure) such that the cell in the i^{th} row and the j^{th} column contains $p^i x^j$. The vector of adjustments (denote by $\mathbf{v}$ and called var in the code) is initialize to $\mathbf{1}$. The main part of the procedure is embedded in a loop that ends only when the data satisfies $GARP_{\mathbf{v}}$. If indeed $GARP_{\mathbf{v}}$ is satisfied the procedure is done and the vector of adjustments is the result of the loop. Otherwise, we construct the matrix Pert_mat such that the cell in the i^{th} row and the j^{th} column contains $\frac{p_j x_i}{v_j p_j x_j}$ (v_j is the j^{th} element of $\mathbf{v}$) if $x_i R_{\mathbf{v},D} x_j$ and $x_j p^0_{\mathbf{v},D} x_i$ ($GARP_{\mathbf{v}}$ violation) and zero otherwise. The maximal element of Pert_mat is picked and substituted into the corresponding element in the vector of adjustments (the substitution is by multiplication with the previous value). Finally, when the loop ends, the vector of adjustments is aggregated by three distinct aggregators of wastes $(1 - v_i)$: maximum $(\max_i (1 - v_i))$, mean $(\frac{1}{n}\sum_{i=1}^{n}(1 - v_i))$ and average sum of squares $(\frac{1}{n}\sum_{i=1}^{n}(1 - v_i)^2)$, and these three numbers are reported.[41]

Houtman Maks Index (HPZ_Houtman_Maks_efficiency_index)

The *Houtman Maks efficiency index* is defined as the size of the largest subset of observations that satisfy $GARP$.[42] The input is a matrix in which the cell in the i^{th} row and the j^{th} column contains 1 if $x_i P^0_D x_j$ and 0 otherwise. This matrix is turned into a list of the pairs in the relation P^0_D. This list serves as an input to a program that was used in Dean and Martin (2011) which returns an approximation of the minimum number of removals needed for acyclicality.[43] The procedure returns the approximated size of the largest subset of observations that satisfy $GARP$.

Nonlinear Least Squares Method (HPZ_NLLS)

The Nonlinear Least Squares estimation procedure finds the parameters that minimize the aggregated distance between bundles predicted by utility maximization and observed bundles.

[41] For the max aggregator, the number reported by the package is $min_i (v_i)$.

[42] See Houtman and Maks (1985).

[43] Downloaded from Daniel Martin's personal website on November 5^{th} 2011.

Input

The input includes the subject ID, the number of observations, the chosen quantities and the given prices.[44] The user defined parameters:

Data parameters (within Choi et al. (2007))

1. The treatment in Choi et al. (2007) (treatment, in the current version the asymmetric treatments are disabled).

2. Correction for corner choices (zeros_flag=1 means no correction while zeros_flag=2 implements the correction suggested in page 1929 in Choi et al. (2007) with $\omega = 0.001$, using the function HPZ_No_Corners).

Functional family parameters (within Disappointment Aversion)

1. The vNM utility function (when function_flag equals 1 the function is CRRA while when it equals 2 it is CARA).

2. Restricted forms (beta_zero_flag=true fixes the disappointment aversion parameter to zero to obtain expected utility).

3. Elation loving (beta_flag=1 allows for negative values for the disappointment aversion parameter while beta_flag=2 restricts disappointment aversion parameter to be non-negative).

4. Correction for the disappointment aversion parameter in asymmetric treatments (asymmetric_flag=1 follows Gul (1991) while asymmetric_flag=2 follows the implementation in Choi et al. (2007), in the current version these treatments are disabled).

5. Restricted risk aversion for cases where there are corner choices and the chosen vNM utility function is CRRA (restricted_rho). This parameter is determined within the code.

[44]In the case of Choi et al. (2007), the prices are the reciprocals of the maximum quantities given in the online data sheet.

Estimation procedure parameters (within NLLS)

1. The distance norm (when metric_flag equals 1 the distance is measured by the Euclidean norm while when it equals 2 it is measured by the geometric mean norm implemented in Choi et al. (2007)).[45]

2. Estimation approach (when numeric_flag equals 1 numeric approximation is used to recover predicted choices while when it equals 2 we use the analytic first order conditions).

3. The minimal number of repetitions with identical minimal aggregate distance to establish convergence (NLLS_min_counter).

4. Maximal number of repetitions (max_starting_points, currently determined within the code).

5. Maximal estimation time (NLLS_max_time_estimation measured in minutes, infinity if time is not a constraint).

6. Parallel computing (when parallel_flag=true the matlabpool command is used, otherwise no parallel computing).

7. Output (determines the additional measures reported for the chosen parameters).

[45] The distance norm used in Choi et al. (2007) (implemented in HPZ_ldr_Criterion):

$$\sum_{i=1}^{n}\left(\ln\frac{x_2^{observed}}{x_1^{observed}}-\ln\frac{x_2^{predicted}}{x_1^{predicted}}\right)^2$$

while the Euclidean distance aggregator (implemented in HPZ_Euclid_Criterion):

$$\sum_{i=1}^{n}\sqrt{\left(x_1^{observed}-x_1^{predicted}\right)^2+\left(x_2^{observed}-x_2^{predicted}\right)^2}$$

Main Procedure

The procedure begins with correcting corner choices (if required by the user) and generating random initial points subject to the restrictions on the functional family (HPZ_Initial_Points).[46]

For each initial point we search for the element of the disappointment aversion functional family (defined by two parameters) that minimizes the distances between the predicted bundles and the observed bundles. For each initial point, the recovered parameters are held in results, while the value is stored in criterion. Since in many cases the procedure encounters local minima, we repeat the estimation procedure as long as the best parameters combination (yield minimal aggregate distance) is recovered less than NLLS_min_counter times, provided that the number of estimations did not reach max_starting_points.[47] If the number of repetitions reaches max_starting_points then we report the best estimations, even if recovered less than
NLLS_min_counter times. We also provide an option for time constraint in which the procedure ends when the time limit expires as long as at least 5 estimations took place.

The Parameters Recovery Routine

Given an initial point the recovery is executed using fminsearchbnd[48] which is a version of fminsearch (the unconstrained non-linear optimization routine of Matlab) that allows for simple bounds on the parameters. When one of the parameters is fixed to zero, the optimization is uni-dimensional and the objective function is implemented in HPZ_Criterion_Extreme_Param. In case

[46]The first initial point is $(e^{e^{-3}} - 1, e^{-2})$ as chosen by Choi et al. (2007), the second is $(0, 0)$ while the rest are chosen randomly (using Matlab's rand function which simulates a standard uniform distribution on the open interval $(0, 1)$).

[47]Considerable part of HPZ_NLLS is dedicated to the implementation of this ad-hoc mechanism, specifically to the adjustments needed when a new best is recovered. equal_fval_counter counts the number of estimations that recovered the minimal value and optimal_parameter_matrix stores the results of those estimations.

[48]Released by John D'Errico in July 2006.

no parameter is fixed, the optimization is bi-dimensional and the objective function is implemented in HPZ_Criterion. Given parameter(s), these functions calculate the aggregate difference between the predicted bundles and the observed bundles. The optimal choices can be calculated numerically or analytically. The analytical procedure relies on first order condition and is very efficient, while the numerical procedure can be easily adapted to various families of functions.

Numeric Approach (HPZ_Choices) The numerical calculation is carried out by fmincon (the constrained non-linear optimization routine of Matlab). In every call this function calculates the optimal choice for all observations. Since these calculations are independent, our code implements them using the parallel computing toolbox of Matlab. This function minimizes the objective function implemented in HPZ_Utility_Helper subject to a linear budget constraint while keeping the quantities non negative. HPZ_Utility_Helper uses HPZ_Utility, CRRA and CARA to calculate the utility level given the parameters of the disappointment aversion utility function, the treatment and the chosen vNM utility function.[49]

Analytic Approach (HPZ_Choices_Analytical) The utility function is

$$u(x,y) = \gamma w(\max\{x,y\}) + (1-\gamma) w(\min\{x,y\})$$

where $\gamma = \frac{1}{2+\beta}$ for $-1 < \beta < \infty$ and $w(x) = \frac{x^{1-\rho}}{1-\rho}$ for CRRA or $w(x) = -e^{-ax}$ for CARA. Denote $p = \frac{p_x}{p_y} = \frac{m_y}{m_x}; m_y = \frac{M}{p_y}; m_x = \frac{M}{p_x}$ (if $m_x = m_y$ we denote both by m).

We first elaborate on the CRRA analysis. The marginal rate of substitu-

[49] As fmincon is an iterative process, a starting point is required. We perform fmincon twice using two different starting points which are chosen on two different sides of the intersection between the budget line and the 45 degrees line, close to the corners (not including the corners). Then the optimal choice among those two is the one with the higher utility level.

tion:

$$MRS_{xy} = \begin{cases} \frac{1}{1+\beta}\left(\frac{y}{x}\right)^{\rho} & x > y \\ \left[\frac{1}{1+\beta}, 1+\beta\right] & x = y \\ (1+\beta)\left(\frac{y}{x}\right)^{\rho} & x < y \end{cases}$$

The utility maximization problem can be broken to the following cases based on values of β and ρ (we refer only to identifiable cases):

1. $\rho > 0, \beta \geq 0:$

$$(x,y)^d = \begin{cases} \left(\frac{m_x}{1+\frac{[p(1+\beta)]^{1/\rho}}{p}}, \frac{m_y}{1+\frac{p}{[p(1+\beta)]^{1/\rho}}}\right) & p < \frac{1}{1+\beta} \\ \left(\frac{m_y}{p+1}, \frac{m_y}{p+1}\right) & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ \left(\frac{m_x}{1+\frac{1}{p}\left(\frac{p}{1+\beta}\right)^{1/\rho}}, \frac{m_y}{1+\frac{p}{\left(\frac{p}{1+\beta}\right)^{1/\rho}}}\right) & 1+\beta < p \end{cases}$$

2. $\rho > 0, -1 < \beta < 0:$

$$(x,y)^d = \begin{cases} \left(\frac{m_x}{1+\frac{[p(1+\beta)]^{1/\rho}}{p}}, \frac{m_y}{1+\frac{p}{[p(1+\beta)]^{1/\rho}}}\right) & p < 1 \\ \left\{\left(\frac{m}{1+(1+\beta)^{1/\rho}}, \frac{m}{1+(1+\beta)^{-1/\rho}}\right), \left(\frac{m}{1+(1+\beta)^{-1/\rho}}, \frac{m}{1+(1+\beta)^{1/\rho}}\right)\right\} & p = 1 \\ \left(\frac{m_x}{1+\frac{1}{p}\left(\frac{p}{1+\beta}\right)^{1/\rho}}, \frac{m_y}{1+\frac{p}{\left(\frac{p}{1+\beta}\right)^{1/\rho}}}\right) & 1 < p \end{cases}$$

3. $\rho > 0, \beta = -1:$

$$(x,y)^d = \begin{cases} (m_x, 0) & p < 1 \\ \{(m,0), (0,m)\} & p = 1 \\ (0, m_y) & 1 < p \end{cases}$$

4. $\rho = 0, \beta \geq 0:$

$$(x,y)^d = \begin{cases} (m_x, 0) & p < \frac{1}{1+\beta} \\ \{x \geq y, px + y = m_y\} & p = \frac{1}{1+\beta} \\ \left(\frac{m_y}{p+1}, \frac{m_y}{p+1}\right) & \frac{1}{1+\beta} < p < 1+\beta \\ \{\ x \leq y, px + y = m_y\} & 1+\beta = p \\ (0, m_y) & 1+\beta < p \end{cases}$$

Next we consider the CARA analysis. The marginal rate of substitution:

$$MRS_{xy} = \begin{cases} \frac{1}{1+\beta} e^{-a(x-y)} & x > y \\ \left[\frac{1}{1+\beta}, 1+\beta\right] & x = y \\ (1+\beta)\, e^{-a(x-y)} & x < y \end{cases}$$

The utility maximization problem can be broken down to the following cases based on values of β and a (we refer only to identifiable cases):

1. $a > 0, \beta \geq 0:$

$$(x,y)^d = \begin{cases} (m_x, 0) & p < \frac{1}{1+\beta} e^{-am_x} \\ \left(\begin{array}{c} \frac{1}{p+1} \left[m_y - \frac{1}{a} \ln\left(p\left(1+\beta\right)\right)\right], \\ \frac{1}{p+1} \left[m_y + \frac{p}{a} \ln\left(p\left(1+\beta\right)\right)\right] \end{array} \right) & \frac{1}{1+\beta} e^{-am_x} \leq p < \frac{1}{1+\beta} \\ \left(\frac{m_y}{p+1}, \frac{m_y}{p+1}\right) & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ \left(\begin{array}{c} \frac{1}{p+1} \left[m_y - \frac{1}{a} \ln\left(\frac{p}{1+\beta}\right)\right], \\ \frac{1}{p+1} \left[m_y + \frac{p}{a} \ln\left(\frac{p}{1+\beta}\right)\right] \end{array} \right) & 1+\beta < p \leq (1+\beta)\, e^{am_y} \\ (0, m_y) & (1+\beta)\, e^{am_y} < p \end{cases}$$

2. $a > 0, -1 < \beta < 0:$

$$(x,y)^d = \begin{cases} (m_x, 0) & p < \frac{1}{1+\beta} e^{-am_x} \\ \begin{pmatrix} \frac{1}{p+1}\left[m_y - \frac{1}{a}\ln\left(p\left(1+\beta\right)\right)\right], \\ \frac{1}{p+1}\left[m_y + \frac{p}{a}\ln\left(p\left(1+\beta\right)\right)\right] \end{pmatrix} & \frac{1}{1+\beta} e^{-am_x} \le p < 1 \\ \left\{ \begin{array}{c} \left(\frac{1}{2}\left[m_y - \frac{1}{a}\ln\left(1+\beta\right)\right], \frac{1}{2}\left[m_y + \frac{1}{a}\ln\left(1+\beta\right)\right]\right), \\ \left(\frac{1}{2}\left[m_y + \frac{1}{a}\ln\left(1+\beta\right)\right], \frac{1}{2}\left[m_y - \frac{1}{a}\ln\left(1+\beta\right)\right]\right) \end{array} \right\} & p = 1 \\ \begin{pmatrix} \frac{1}{p+1}\left[m_y - \frac{1}{a}\ln\left(\frac{p}{1+\beta}\right)\right], \\ \frac{1}{p+1}\left[m_y + \frac{p}{a}\ln\left(\frac{p}{1+\beta}\right)\right] \end{pmatrix} & 1 < p \le \left(1+\beta\right) e^{am_y} \\ (0, m_y) & \left(1+\beta\right) e^{am_y} < p \end{cases}$$

3. $a > 0, \beta = -1:$

$$(x,y)^d = \begin{cases} (m_x, 0) & p < 1 \\ \left\{(m,0), (0,m)\right\} & p = 1 \\ (0, m_y) & 1 < p \end{cases}$$

Money Metric Method (HPZ_MME_Estimation)

The Money Metric recovery procedure finds the parameters that minimize the aggregated adjustments needed to remove all inconsistencies between the utility function ranking and the revealed preference information.

Input

The input includes the subject ID, the number of observations, the chosen quantities and the given prices. The user defined parameters are

Data parameters (within Choi et al. (2007))

1. The treatment in Choi et al. (2007) (treatment, in the current version the asymmetric treatments are disabled).

2. Correction for corner choices (zeros_flag, see above for details).

Functional family parameters (within Disappointment Aversion)

1. The vNM utility function (function_flag, see above for details).
2. Restricted forms (beta_zero_flag, see above for details).
3. Elation loving (beta_flag, see above for details).
4. Restricted risk aversion for cases where there are corner choices and the chosen utility index is CRRA (restricted_rho). This parameter is determined by the code.

Estimation procedure parameters (within MME)

1. The wastes aggregation function (when aggregation equals 1 it is the maximum function, when it equals 2 it is the average and when it is 3 it is the average sum of squares).
2. Estimation approach (numeric_flag, see above for details).
3. The minimal number of repetitions with identical minimal aggregate distance to establish convergence (MME_min_counter).
4. Maximal number of repetitions (max_starting_points, see above for details).
5. Maximal estimation time (MME_max_time_estimation, see above for details).
6. Parallel computing (parallel_flag, see above for details).
7. Output (see above for details).

Main Procedure

The procedure begins with correcting corner choices (if required by the user) and generating random initial points subject to the restrictions on the functional family (HPZ_Initial_Points).

For each initial point we search for the element of the disappointment aversion functional family (defined by two parameters) that minimizes the aggregated adjustments needed to remove all inconsistencies between the utility function ranking and the revealed preference information. For each initial point, the recovered parameters are held in results, while the value is stored in criterion. Here also we repeat the estimation procedure as long as the best parameters combination (yield minimal value function) is recovered less than MME_min_counter times provided that the number of estimations did not reach max_starting_points (Footnote 47 is relevant here as well). If the number of repetitions reaches max_starting_points then we report the best estimations, even if recovered less than MME_min_counter times. We also provide an option for time constraint in which the procedure ends when the time limit expires as long as at least 3 estimations took place.

The Parameters Recovery Routine

Given an initial point the recovery is executed using fminsearchbnd (see details above). When one of the parameters is fixed to zero, the optimization is unidimensional and the objective function is implemented in HPZ_MME_Helper_Extreme_Param. In case no parameter is fixed, the optimization is two-dimensional and the objective function is implemented in HPZ_MME_Helper. Given parameter(s), these functions calculate the aggregate adjustments needed to remove all inconsistencies between ranking induced by the utility function and the revealed preference information. Both functions use HPZ_MME_Criterion that calculates the three aggregates[50]. The optimal parameters can be calculated numerically or analytically.

Numeric Approach (HPZ_MME) The inputs for the numerical calculation are the observations and the relevant family of utility functions (parameters and utility index). For every observation the level of utility is directly calculated (by HPZ_Utility). If it is a corner choice, HPZ_Grid_Search_MME

[50] Maximum $(\max_i (1 - v_i^\star))$, mean $(\frac{\sum_{i=1}^n (1-v_i^\star)}{n})$ and the sum of squares $(\sum_{i=1}^n (1 - v_i^\star)^2)$.

implements a bisection search (parallel movements of the budget line) to recover the waste incurred by this choice (using the numeric HPZ_choices). If the observation is an interior choice, the fmincon procedure with a nonlinear constraint (HPZ_Utility_Constraint) minimizes the expenditure subject to achieving at least the same utility level as calculated for the observation. We perform fmincon twice using two different starting points which are chosen on two different sides of the budget line close to the corners. If both instances of fmincon were satisfactory (exitflag=1) then the lower expenditure of the two is reported. Otherwise, a grid search is invoked. Proposition 1 guarantees that the optimization can be implemented observation-by-observation. Therefore, our code implements parallel computing over observations.

Analytical Approach (HPZ_MME_Analytical) Let (x_0, y_0) be the chosen bundle. For CRRA, denote

$$\widetilde{u}_0 = (2+\beta)(1-\rho)\,u(x_0, y_0) = (\max\{x_0, y_0\})^{1-\rho} + (1+\beta)(\min\{x_0, y_0\})^{1-\rho}$$

Then, by equating the MRS of the indifference curve $\widetilde{u}_0$ to the price ratio we find the minimal expenditure required to achieve $\widetilde{u}_0$. The different cases are based on values of β and ρ (we refer only to identifiable cases):

1. $\rho > 0, \beta \geq 0 : e(p_x, p_y, (x_0, y_0)) = \begin{cases} p_x \left[\frac{\widetilde{u}_0}{1+(1+\beta)^{\frac{1}{\rho}} p^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} + p_y \left[\frac{\widetilde{u}_0}{1+(1+\beta)^{\frac{1}{\rho}} p^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} ((1+\beta)\,p)^{\frac{1}{\rho}} & p < \frac{1}{1+\beta} \\ (p_x + p_y) \left[\frac{\widetilde{u}_0}{2+\beta}\right]^{\frac{1}{1-\rho}} & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ p_x \left[\frac{\widetilde{u}_0}{(1+\beta)+\left(\frac{p}{1+\beta}\right)^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} + p_y \left[\frac{\widetilde{u}_0}{(1+\beta)+\left(\frac{p}{1+\beta}\right)^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} \left(\frac{p}{1+\beta}\right)^{\frac{1}{\rho}} & 1+\beta < p \end{cases}$

2. $\rho > 0, -1 < \beta < 0 : e(p_x, p_y, (x_0, y_0)) = \begin{cases} p_x \left[\frac{\widetilde{u}_0}{1+(1+\beta)^{\frac{1}{\rho}} p^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} + p_y \left[\frac{\widetilde{u}_0}{1+(1+\beta)^{\frac{1}{\rho}} p^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} ((1+\beta)\,p)^{\frac{1}{\rho}} & p < 1 \\ p_x \left[\frac{\widetilde{u}_0}{1+(1+\beta)^{\frac{1}{\rho}}}\right]^{\frac{1}{1-\rho}} + p_y \left[\frac{\widetilde{u}_0}{1+(1+\beta)^{\frac{1}{\rho}}}\right]^{\frac{1}{1-\rho}} (1+\beta)^{\frac{1}{\rho}} & p = 1 \\ p_x \left[\frac{\widetilde{u}_0}{(1+\beta)+\left(\frac{p}{1+\beta}\right)^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} + p_y \left[\frac{\widetilde{u}_0}{(1+\beta)+\left(\frac{p}{1+\beta}\right)^{\frac{1-\rho}{\rho}}}\right]^{\frac{1}{1-\rho}} \left(\frac{p}{1+\beta}\right)^{\frac{1}{\rho}} & p > 1 \end{cases}$

3. $\rho = 0, \beta \geq 0:$

$$e\left(p_x, p_y, (x_0, y_0)\right) = \begin{cases} p_x \widetilde{u}_0 & p \leq \frac{1}{1+\beta} \\ \frac{p_x+p_y}{2+\beta} \widetilde{u}_0 & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ p_y \widetilde{u}_0 & 1+\beta \leq p \end{cases}$$

For CARA, denote

$$\widetilde{u}_0 = (2+\beta)\, u\left(x_0, y_0\right) = -e^{-a \max\{x_0, y_0\}} - (1+\beta)\, e^{-a \min\{x_0, y_0\}}$$

The indifference curve $\widetilde{u}_0$ intersects with the axis if there exists a positive x such that $\widetilde{u}_0 = -e^{-ax} - (1+\beta)$. This is possible only if $-\widetilde{u}_0 > 1+\beta$, or alternatively, only if $\frac{e^{-a \max\{x_0, y_0\}}}{1-e^{-a \min\{x_0, y_0\}}} > 1+\beta$. The different cases are based on values of β and ρ (we refer only to identifiable cases):

1. $\beta \geq 0$ and $-\widetilde{u}_0 > 1+\beta : e\left(p_x, p_y, (x_0, y_0)\right) = \begin{cases} -\frac{p_x}{a} \ln\left(-\left(\widetilde{u}_0 + (1+\beta)\right)\right) & p < -\frac{\widetilde{u}_0+(1+\beta)}{1+\beta} \\ p_x \left[\frac{1}{a} \ln\left(-\frac{p+1}{p\widetilde{u}_0}\right)\right] + p_y \left[\frac{1}{a} \ln\left(-\frac{(1+p)(1+\beta)}{\widetilde{u}_0}\right)\right] & -\frac{\widetilde{u}_0+(1+\beta)}{1+\beta} \leq p < \frac{1}{1+\beta} \\ (p_x + p_y) \left[\frac{1}{a} \ln\left(-\frac{2+\beta}{\widetilde{u}_0}\right)\right] & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ p_x \left[\frac{1}{a} \ln\left(-\frac{(1+\beta)(1+p)}{\widetilde{u}_0 p}\right)\right] + p_y \left[\frac{1}{a} \ln\left(-\frac{(1+p)}{\widetilde{u}_0}\right)\right] & 1+\beta < p \leq -\frac{1+\beta}{\widetilde{u}_0+(1+\beta)} \\ -\frac{p_y}{a} \ln\left(-\left(\widetilde{u}_0 + (1+\beta)\right)\right) & -\frac{1+\beta}{\widetilde{u}_0+(1+\beta)} < p \end{cases}$

2. $\beta \geq 0$ and $-\widetilde{u}_0 \leq 1+\beta$: $e\left(p_x, p_y, (x_0, y_0)\right) = \begin{cases} p_x \left[\frac{1}{a} \ln\left(-\frac{p+1}{p\widetilde{u}_0}\right)\right] + p_y \left[\frac{1}{a} \ln\left(-\frac{(1+p)(1+\beta)}{\widetilde{u}_0}\right)\right] & p < \frac{1}{1+\beta} \\ (p_x + p_y) \left[\frac{1}{a} \ln\left(-\frac{2+\beta}{\widetilde{u}_0}\right)\right] & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ p_x \left[\frac{1}{a} \ln\left(-\frac{(1+\beta)(1+p)}{\widetilde{u}_0 p}\right)\right] + p_y \left[\frac{1}{a} \ln\left(-\frac{(1+p)}{\widetilde{u}_0}\right)\right] & 1+\beta < p \end{cases}$

3. $-1 < \beta < 0$ and $-\widetilde{u}_0 > 1+\beta$ and $-\frac{\widetilde{u}_0+(1+\beta)}{1+\beta} < 1$ $e\left(p_x, p_y, (x_0, y_0)\right) = \begin{cases} -\frac{p_x}{a} \ln\left(-\left(\widetilde{u}_0 + (1+\beta)\right)\right) & p < -\frac{\widetilde{u}_0+(1+\beta)}{1+\beta} \\ p_x \left[\frac{1}{a} \ln\left(-\frac{p+1}{p\widetilde{u}_0}\right)\right] + p_y \left[\frac{1}{a} \ln\left(-\frac{(1+p)(1+\beta)}{\widetilde{u}_0}\right)\right] & -\frac{\widetilde{u}_0+(1+\beta)}{1+\beta} \leq p \leq 1 \\ p_x \left[\frac{1}{a} \ln\left(-\frac{(1+\beta)(1+p)}{\widetilde{u}_0 p}\right)\right] + p_y \left[\frac{1}{a} \ln\left(-\frac{(1+p)}{\widetilde{u}_0}\right)\right] & 1 < p \leq -\frac{1+\beta}{\widetilde{u}_0+(1+\beta)} \\ -\frac{p_y}{a} \ln\left(-\left(\widetilde{u}_0 + (1+\beta)\right)\right) & -\frac{1+\beta}{\widetilde{u}_0+(1+\beta)} < p \end{cases}$

4. $-1 < \beta < 0$ and $-\widetilde{u}_0 > 1+\beta$ and $-\frac{\widetilde{u}_0+(1+\beta)}{1+\beta} \geq 1$

$$e\left(p_x, p_y, (x_0, y_0)\right) = \begin{cases} -\frac{p_x}{a} \ln\left(-\left(\widetilde{u}_0 + (1+\beta)\right)\right) & p \leq 1 \\ -\frac{p_y}{a} \ln\left(-\left(\widetilde{u}_0 + (1+\beta)\right)\right) & p > 1 \end{cases}$$

5. $-1<\beta<0$ and $-\widetilde{u}_0 \leq 1+\beta$:

$$e\left(p_x, p_y,\left(x_0, y_0\right)\right)=\begin{cases} p_x\left[\frac{1}{a} \ln \left(-\frac{p+1}{p \widetilde{u}_0}\right)\right]+p_y\left[\frac{1}{a} \ln \left(-\frac{(1+p)(1+\beta)}{\widetilde{u}_0}\right)\right] & p \leq 1 \\ p_x\left[\frac{1}{a} \ln \left(-\frac{(1+\beta)(1+p)}{\widetilde{u}_0 p}\right)\right]+p_y\left[\frac{1}{a} \ln \left(-\frac{(1+p)}{\widetilde{u}_0}\right)\right] & 1<p \end{cases}$$

6. $\beta=-1$

$$e\left(p_x, p_y,\left(x_0, y_0\right)\right)=\begin{cases} p_x \max \left\{x_0, y_0\right\} & p \leq 1 \\ p_y \max \left\{x_0, y_0\right\} & 1<p \end{cases}$$

7. $a=0, \beta \geq 0$ ($\widetilde{u}_0=\max \left\{x_0, y_0\right\}+(1+\beta) \min \left\{x_0, y_0\right\}$):

$$e\left(p_x, p_y,\left(x_0, y_0\right)\right)=\begin{cases} p_x \widetilde{u}_0 & p \leq \frac{1}{1+\beta} \\ \frac{p_x+p_y}{2+\beta} \widetilde{u}_0 & \frac{1}{1+\beta} \leq p \leq 1+\beta \\ p_y \widetilde{u}_0 & 1+\beta \leq p \end{cases}$$

User Interface (HPZ_Interface)

In order to use this Matlab package follow the following steps:

- Set the MATLAB path to the place that stores the HPZ_PRU_Software folder (using the Set Path option with Add Folder with Subfolders).
- After the path is set, to run the code, the user should write HPZ_Interface command on the MATLAB command window. The data set we currently use as the input data is (Choi et al. (2007), Data_CFGK_2007.csv).
- The Action Selection window: The user is required to choose three actions - consistency analysis (Consistency Tests and Inconsistency Indices), the Nonlinear Least Squares recovery method or the Money Metric recovery method.
- The Subjects Selection window: The user is required to select the analyzed subjects (one or multiple subjects can be chosen). If the Consistency Analysis action was chosen in the Action Selection window, the next window would be the result window (described below). Otherwise, the following window would be the Functional Form Settings window.

- The Functional Form Settings window: The user is required to decide on four issue - the functional form of the utility indices (CRRA or CARA), the computational approach (numerical or analytical), the disappointment aversion parameter boundary ($\beta = 0$, $\beta \geq 0$ or $\beta > -1$) and the adjustment for boundary solutions.

- The Optimization Settings window slightly differs between the Nonlinear Least Squares method and the Money Metric method. For both methods the user is first required to specify the number of times the recovered parameters should be recovered before the process is terminated and the parameters are reported. Increasing the number of convergence points improves reliability but reduces efficiency. Note that when the user selects the number of convergence point there is no time limit on the recovery process. Then, the user is required to specify the allocated time (in minutes). Defining a time limit overrides the choice of the number of convergence points. The parameter estimation process (i.e., fminsearchbnd procedure) would stop when the time is over and report the results that have been computed during the allocated time. Third, the user can choose to use parallel processing (more important for the numerical approach). If the parallel processing is selected then the code would kill other processes running on the machine and use all computing power to run the software. For the Money Metric method, the aggregator of the wastes vector is required (either maximum, mean or average sum of squares). For the Nonlinear Least Squares method, the distance metric should be selected to be either the Euclidean metric or the metric used in Choi et al. (2007).

- The Output File Format window: The user is required to customize the information in the output file. The basic format of the output file includes the recovered parameters and the value of the optimized aggregated criterion. There are five different options for evaluating the resulting parameters including the one with which the optimization was carried out. The five are the three possible Money Metric methods and the two possible Nonlinear Least Squares methods.

- The Output File Notification window: When the routine is done, a window is prompted on the screen including the path to the output file.

Bootstrapping package for Money Metric Method

The bootstrapping module has been developed to provide the parameter distribution for the underlying recovered parameters and compute confidence intervals for the recovered Disappointment Aversion parameter (β) using Money Metric method, while β is bounded to -1. For bootstrapping we used 1000 draws with replacement on all observations of each individual subject. In order to report confidence intervals on recovered β, we sort all 1000 results for all draws, and then report the 25^{th} and 975^{th} ones as the lower bound and upper bound of 95% confidence interval for β, and also report 50^{th} and 950^{th} ones as the lower bound and upper bound of 90% confidence interval for β (we also report mean and standard deviation of the parameter).

User Interface (HPZ_Bootstrapping_Module)

In order to make this module work, the user has to follow the following steps:

- Set the MATLAB path to the place that stores the HPZ_PRU_Software folder and the HPZ_Bootstrapping_Package folder (using the Set Path option with Add Folder with Subfolders).

- After the path is set, to run the code, the user should write HPZ_Bootstrapping_Module command on the MATLAB command window.

- The bootstrapping window includes the following choices: The functional form of the vNM numbers (CRRA or CARA), the computational approach (numerical is currently disabled), the MME aggregation functional (either mean or average sum of squares).

- The Subjects Selection window: The user is required to select the analyzed subjects (one or multiple subjects can be chosen).
- The Output File Notification window: When the routine is done, a window is prompted on the screen including the path to the output file.

References

Afriat, Sydney N., "The Construction of Utility Functions from Expenditure Data," *International Economic Review*, February 1967, *8* (1), 67–77.

_ , "Efficiency estimates of production functions," *International Economic Review*, October 1972, *13* (3), 568–598.

_ , "On a System of Inequalities in Demand Analysis: An Extension of the Classical Method," *International Economic Review*, June 1973, *14* (2), 460–472.

_ , *Logic of Choice and Economic Theory*, Oxford: Clarendon Press, 1987.

Agranov, Marina and Pietro Ortoleva, "Stochastic Choice and Hedging," *Working Paper*, 2014.

Ahn, David, Syngjoo Choi, Shachar Kariv, and Douglas Gale, "Estimating Ambiguity Aversion in a Portfolio Choice Experiment," *Quantitative Economics*, 2013, *forthcoming.*

Alcantud, José Carlos R., Daniel L. Matos, and Carlos R. Palmero, "Goodness-of-fit in optimizing a consumer model," *Mathematical and Computer Modelling*, 2010, *52*, 1088–1094.

Andreoni, James and Charles Sprenger, "Estimating Time Preferences from Convex Budgets," *American Economic Review*, December 2012, *102* (7), 3333–3356.

— **and John Miller**, "Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism," *Econometrica*, March 2002, *70* (2), 737–753.

Apesteguia, Jose and Miguel A. Ballester, "A Measure of Rationality and Welfare," *Working Paper*, 2013.

Bernheim, B. Douglas and Antonio Rangel, "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welafare Economics," *Quarterly Journal of Economics*, 2009, pp. 51–104.

Blundell, Richard W., Martin Browning, and Ian A. Crawford, "Nonparametric Engel Curves and Revealed Preference," *Econometrica*, 2003, *71* (1), 205–240.

—, —, and —, "Best Nonparametric Bounds on Demand Responses," *Econometrica*, 2008, *76* (6), 1227–1262.

Cherchye, Laurens, Ian Crawford, Bram De Rock, and Frederic Vermeulen, "The Revealed Preference Approach to Demand," in Daniel J. Slottje, ed., *Quantifying Consumer Preferences (Contributions to Economic Analysis, Volume 288)*, Emerald Group Publishing Limited, 2009, pp. 247–279.

Cherepanov, Vadim, Timothy Feddersen, and Alvaro Sandroni, "Rationalization," *Theoretical Economics*, September 2013, *8* (3), 775–800.

Choi, Syngjoo, Raymond Fisman, Douglas Gale, and Shachar Kariv, "Consistency and Heterogeneity of Individual Behavior under Uncertainty," *American Economic Review*, December 2007, *97* (5), 1921–1938.

—, Shachar Kariv, Wieland Muller, and Dan Silverman, "Who is (More) Rational?," *American Economic Review*, 2013, *Forthcoming.*

Dean, Mark and Daniel Martin, "Testing for Rationality with Consumption Data: Demographics and Heterogeneity," *Working Paper*, 2011.

Diewert, W. E., "Afriat and Revealed Preference Theory," *Review of Economic Studies*, July 1973, *40* (3), 419–425.

Echenique, Federico, Sangmok Lee, and Matthew Shum, "The Money Pump as a Measure of Revealed Preference Violations," *Journal of Political Economy*, 2011, *119* (6), 1201–1223.

Epstein, Larry G., "Behavior Under Risk: Recent Developments in Theory and Applications," in "Econometric Society" 1992.

Fisman, Raymond, Shachar Kariv, and Daniel Markovits, "Individual Preferences for Giving," *American Economic Review*, May 2007, *97* (5), 1858–1876.

Gul, Faruk, "A Theory of Disappointment Aversion," *Econometrica*, May 1991, *59* (3), 667–686.

_ **and Wolfgang Pesendorfer**, "Random Expected Utility," *Econometrica*, 2006, *74*, 121–146.

_ **, Paulo Natenzon, and Wolfgang Pesendorfer**, "Random Choice as Behavioral Optimization," *Working Paper*, 2012.

Houtman, Martijn, "Nonparametric Consumer and Producer Analysis," Dissertation 1995.

_ **and J.A.H. Maks**, "Determining all Maximal Data Subsets Consistent with Revealed Preference," *Kwantitatieve Methoden*, 1985, *19*, 89–104.

_ **and** _ , "The Existence of Homothetic Utility Functions Generating Dutch Consumer Data," Mimeo 1987.

Karp, Richard M., "Reducibility Among Combinatorial Problems," in R. E. Miller and J. W. Thatcher, eds., *Complexity of Computer Computations*, New York: Plenum, 1972, pp. 167–174.

Kitamura, Yuichi and Jörg Stoye, "Nonparametric Analysis and Random Utility Models," *Working Paper*, 2013.

Manzini, Paola and Marco Mariotti, "Sequentially Rationalizable Choice," *American Economic Review*, 2007, *97* (5), 1824–1839.

Manzini, Paula and Marco Mariotti, "Categorize then Choose: Boundedly Rational Choice and Welfare," *Journal of the European Economic Association*, 2012.

Masatlioglu, Yusufcan, Daisuke Nakajima, and Erkut Ozbay, "Revealed Attention," *American Economic Review*, 2012, *102* (5), 2183–2205.

McFadden, Daniel L., "Revealed Stochastic Preference: a Synthesis," *Economic Theory*, 2005, *26*, 245–264.

Natenzon, Paulo, "Random Choice and Learning," *Working Paper*, 2013.

Quiggin, John, "A Theory of Anticipated Utility," *Journal of Economic Behavior and Organization*, 1982, *3*, 323–343.

Rubinstein, Ariel, "Irrational diversification in multiple decision problems," *European Economic Review*, 2002, *46*, 1369–1378.

_ **and Yuval Salant**, "Eliciting Welfare Preferences from Behavioral Datasets," *Review of Economic Studies*, 2012, *79* (1), 375–387.

Samuelson, Paul A., "Complementarity: An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory.," *Journal of Economic Literature*, 1974, *12* (4), 1255–1289.

Segal, Uzi and Avia Spivak, "First Order versus Second Order Risk Aversion," *Journal of Economic Theory*, June 1990, *51* (1), 111–125.

Simon, Herbert A, "From substantive to procedural rationality," in "25 Years of Economic Theory," Springer US, 1976, pp. 65–86.

Stoye, Jörg and Stefan Hoderlein, "Testing Stochastic Rationality and Predicting Stochastic Demand: The Case of Two Goods," *Working Paper*, 2012.

Tsur, Yacov, “On Testing For Revealed Preference Conditions,” *Economics Letters*, 1989, *31*, 359–362.

Varian, Hal R., “The Nonparametric Approach to Demand Analysis,” *Econometrica*, July 1982, *50* (4), 945–973.

_ , “Non-Parametric Analysis of Optimizing Behavior with Measurement Error,” *Journal of Econometrics*, 1985, *30*, 445–458.

_ , “Goodness-of-fit in Optimizing Models,” *Journal of Econometrics*, 1990, *46*, 125–140.

_ , “Goodness-of-fit for Revealed Preference Tests,” *Working Paper*, 1993.

Warshall, S., “A Theorem on Boolean Matrices,” *The Journal of the American Association of Computing Machinery*, 1962, *9*, 11–12.

Yaari, Menahem E., “Some Remarks on Measures of Risk Aversion and on Their Uses,” *Journal of Economic Theory*, 1969, *1*, 315–329.