Blockholder Voting*

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Abstract

By introducing a shareholder with many votes (a blockholder) to a standard model of voting, we uncover several striking results. First, if a blockholder is unbiased, she may not vote with all of her shares. This is efficient, as it prevents her vote from drowning out the information provided by other votes. Second, if this blockholder can announce her vote before the vote takes place, other shareholders may ignore their information and vote with the blockholder to support her superior information. Third, if the blockholder is biased, some shareholders will try to counter the blockholder's vote. The results are robust to allowing for information acquisition and trade. This suggests that regulations discouraging or prohibiting abstention, strategic behavior, and/or coordination may reduce efficiency.

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1 Introduction

Shareholder voting has a stark difference with voting in the political context - a shareholder may have many shares, and thus, many votes. Blockholders, shareholders with a large fraction (often defined as 5%) of the shares of a firm, are ubiquitous - in a sample of representative U.S. public firms, Holderness (2009) finds that 96% of the firms have at least one blockholder, and the average stake of the largest blockholder of a firm is 26%.[1] The identity of blockholders may vary substantially from activist investors to passive index funds to even the managers or directors of the firm.

Blockholders play a large role in the governance of firms. The U.S. Securities and Exchange Commission (SEC) has increasingly focused regulatory scrutiny on blockholders for their voting behavior. Investment advisers (including mutual funds) have been told to ensure that their votes are in the best interests of their clients and must publicly report their votes (SEC, 2003). Large investors must report their stakes and intentions when they reach 5% ownership of a firm. Nevertheless, groups of activist investors ("wolf packs") sometimes act in concert but individually avoid the 5% rule, generating both academic and regulatory debate (Coffee and Palia, 2015).

In this paper, we study blockholder voting in a standard theoretical framework and find several striking results:

- *Blockholders may prefer not to vote with all of their shares, and this is efficient*: Consider the voting strategy of an unbiased blockholder, where unbiased means that the blockholder wants her vote to increase the value of the firm. Our first result is that this blockholder may not vote with all of her shares. Given that she wants to maximize the value of the firm, if she doesn't have very precise information, she would prefer that other shareholders' information not be drowned out by her votes. The blockholder is not wasting her unvoted shares; she is acting

[1] He also finds that this distribution is not very different that in the rest of the world.

optimally to improve the efficiency of the vote.[2] We also demonstrate in an extension that given the opportunity to trade shares, the blockholder may not trade her unvoted shares. Given that evidence suggests that many investment advisers try to satisfy SEC requirements by blindly voting their shares with the recommendation of proxy advisers (Iliev and Lowry, 2015, and Malenko and Shen, 2016), our result suggests that such rules may be inefficient.

- *Other shareholders may ignore their own information to vote with an unbiased blockholder:* Our second result looks at the situation where the unbiased blockholder can observably announce its voting intentions before other shareholders vote. This could represent an activist investor or a large pension fund using public statements to communicate their position. In this case, if this blockholder is well informed, some of the shareholders may ignore their information and vote with the blockholder. It is perfectly rational to ignore their information; if the blockholder has a lot of information but not enough shares to express that information, the shareholders essentially supplement the blockholder's shares with their own. Determining which shareholders will support the blockholder requires coordination. Our result then suggests that shareholder coordination to support blockholder voting can improve efficiency; therefore some concerns about coordination by groups of investors such as "wolf packs" may be overstated.

- *When the blockholder is biased, some shareholders will try to counter the bias:* Now consider the situation where the blockholder is known to be biased. We defined bias here as the blockholder supporting one side of the proposal no matter what information it receives. For example, the blockholder could support management's position because she is a manager or has business dealings with the firm that depend

[2] We define efficiency below to be maximizing the probability that the vote matches the true state of the world.

on good relationships with management.[3] Our third result is that, in this case, some shareholders may ignore their information in order to counter the blockholder's bias. Consider the situation where a shareholder receives a signal that management's position adds value. Since the shareholder knows that management is over-weighted in the vote, we show she might either vote *against* the blockholder (to cancel out the blockholder's vote) *or* abstain (which allows some information to enter the vote). The shareholder acts strategically to counter bias and maximize the informativeness of the vote. Notice that this may require coordination by shareholders; in this scenario, coordination is also efficiency enhancing (as it was in the case of the unbiased blockholder discussed above).

The voting model is standard aside from the fact that there is one voter (the blockholder) with more shares than the others. All other shareholders want to improve the value of the firm and receive a signal about which choice from a binary decision will increase value. The blockholder also receives a signal, which we assume to be more informative than an individual shareholder. A vote wins by a majority, and voters may vote strategically (or abstain).

Strategic voting is a key element of our model and the focus of our analysis. Allowing the blockholder and shareholders to vote strategically leads to more informative outcomes. Strategic abstention enhances efficiency alternatively by (i) allowing the blockholder to express its information precisely, (ii) allowing shareholders to step aside when the blockholder has very precise information, (iii) allowing shareholders to express themselves while still

[3] For example, a mutual fund may support management to preserve business ties (see Davis and Kim (2007), Ashraf, Jayaraman, and Ryan (2012), and Cvijanovic, Dasgupta, and Zachariadis (2016)). Another example of blockholder bias is documented in Agrawal (2012), where labor union pension funds may vote for labor-friendly directors. Such bias may also arise as a result of a blockholder's holdings in other firms, as in Azar, Schmalz and Tecu (2017).

countering the bias of a blockholder, and (iv) forcing the biased blockholder to not express its bias. Beyond strategic abstention, shareholders may also ignore their signal to increase informativeness when (i) they vote with the blockholder to support its precise information and (ii) they vote against the biased blockholder to allow information to flow through the remaining votes.

We offer two extensions of the model to make it more realistic. In the first extension, we allow the blockholder to acquire information and trade shares. This provides a robustness check for our result that the blockholder may not vote all of its shares, as it is natural to ask why the blockholder would not trade shares that it does not vote. We show that the blockholder might face a lemons discount when it sells shares as investors may infer that the blockholder did not acquire information. In the second extension, we provide a rationale for the SEC rules for investment advisers, given that they seem contrary to economic intuition from our main model. In particular, we show that forcing the blockholder to vote all of her shares could potentially enhance efficiency as it might provide her with incentives to acquire information.

Our analysis is kept simple to present the effects clearly and maintain tractability. We recognize several directions in which we have kept the model uncomplicated. First, the bias (or lack thereof) of the blockholder is common knowledge. Second, the precision of all shareholders' and the blockholder's information is common knowledge. Third, we focus on the most informative equilibrium. There are good reasons to do so, as this is the equilibrium all agents in the model prefer, however there do exist many other equilibria as in all strategic voting models.

In the following subsection, we review the related literature. In Section 2, we set up the model. In Section 3, we analyze the case where the blockholder is unbiased and passive in the sense that it makes no announcement. In Section 4, we have the unbiased blockholder make an announcement before the other shareholders vote, which we call the active blockholder case. In Section 5, we extend the model to allow for information acquisition, trading, blockholder bias, and have a further examination of voting regulation. Section 6 concludes. All proofs that are not in the text are in the Appendix.

1.1 Literature Review

Edmans (2014) and Edmans and Holderness (2016) provide thorough surveys of the theoretical and empirical literature on blockholders. None of the theoretical papers cited in these papers are about voting - blockholders make costly interventions in a firm and/or trade the firm's shares in these papers. Yermack (2010) surveys the literature on shareholder voting and also does not cite any such research.

Theoretical Literature

Feddersen and Pesendorfer (1996) analyze the swing voter's curse, where uninformed voters abstain in order to allow more informed voters to sway the vote. This is the intuition for why the unbiased blockholder may not vote all of her shares in our model, i.e. her information has a value, and so does the information of the remaining shareholders. Our result that shareholders may ignore their information to vote with the unbiased blockholder and improve the outcome is related. A few other papers use this type of logic in different contexts.

Eso, Hansen, and White (2014) study empty voting[4] and find that uninformed shareholders and even biased shareholders may sell their votes (at a zero price) to informed shareholders in order to improve the outcome. Note that our analysis demonstrates that activists don't necessarily have to resort to empty voting strategies to gain votes if they have credibility. Maug (1999), Maug and Rydqvist (2009), and Persico (2004) find that informed voters/shareholders may ignore their information in response to different voting rules.[5] This result is related to our findings on biased blockholders; in

[4]Brav and Matthews (2011) also examine empty voting in a model of a hedge fund who may both trade shares and buy votes, while all other voting is random.

[5]Bhattacharya, Duffy, and Kim (2014) have a similar effect when symmetric voters

these papers, the voters ignore their information to correct the bias of the voting rule, while in our paper the shareholders may ignore their information to deal with the bias of the blockholder. Beyond differences in the questions we address and the asymmetries present in our model, our model is different in that we allow for abstention and for communication (both of which affect the results).

Malenko and Malenko (2016) look at a common value model where shareholders can acquire information and/or purchase it from a proxy advisory service. They focus on an environment where shareholders are symmetric, which is critically different to our asymmetric environment. Harris and Raviv (1988) and Gromb (1993) study whether it is optimal to allocate uniform voting rights for all shares (one shareholder, one vote) but do not incorporate the role of information aggregation in voting that is key in our paper. Cvijanović, Groen-Xu, and Zachariadis (2017) examine the participation decision in a corporate setting where voters are partisan.

Empirical Literature

McCahery, Sautner, and Starks (2016) find that among shareholder engagement measures, voting against management was the second most frequently employed (and the top measure where shareholders exerted "voice") by surveyed institutional investors. Furthermore, many of these investors had engaged managers publicly: 30% had aggressively questioned management on a conference call, 18% had criticized management at the annual general meeting (AGM), 18% had publicized a dissenting vote, 16% had submitted shareholder proposals for the proxy statement, 15% had taken legal action against management, and 13% publicly criticized management in the media. These forms of public engagement lend support to the version of the model where the blockholder moves observably before other shareholders.

Since July 2003, the SEC has required mutual funds to disclose how they

have a different precision of receiving a signal about one state of the world versus the other.

vote proxies with respect to portfolio stocks. Duan and Jiao (2014) find that voting is an important source of governance; for conflictual proxy votes (defined by votes where the proxy advisory firm Institutional Shareholder Services (ISS) opposes management) the probability of mutual funds voting against management is 46.42% higher than for other proposals, while their probability of exit is 3.12% higher. Passive mutual funds, by definition, do not trade very often, and therefore can only affect value through voice (as opposed to exit). Appel, Gormley, and Keim (2016a) find that a one standard deviation increase in ownership by passive funds leads to a 0.75 standard deviation decline in support for management proposals and about a 0.5 standard deviation increase in support for governance proposals. Nevertheless, there are several ways in which mutual fund voting might diverge from blockholder voting behavior in our base model. First, mutual funds might have conflicts of interest and vote more for management.[6] We describe such a situation in our paper as a biased blockholder and examine it in an extension. Second, funds may not do their own research and be influenced by proxy advisory services.[7] We do not examine this in the paper. Third, and possibly most relevant for our paper, the 2003 SEC requirements are not just about disclosure; they require investment advisers to act in the best interests of clients. This could be interpreted as prohibiting mutual funds from abstention and strategic voting. Our main results point to the efficiency enhancing effect of allowing abstention and strategic behavior, but such an interpretation would make this impossible to observe in the data.[8]

Brav et al. (2008) point out that activist hedge funds do not usually

[6] See Davis and Kim (2007), Ashraf, Jayaraman, and Ryan (2012), and Cvijanovic, Dasgupta, and Zachariadis (2016).

[7] See, for example, Malenko and Shen (2016) and Iliev and Lowry (2015).

[8] Nevertheless, there is some evidence of investment advisers abstaining Iliev et al. (2012) note " that in 6.6% of the director election votes the institutions did not vote." (Footnote 7). In the UK and EU, there is no mandatory disclosure of voting; however, some investors choose to disclose their voting and there is evidence of abstention and voting on both sides of an issue; see for example ShareAction (2015) pp 14-17.

seek control in target firms. The median maximum ownership stake for their sample is about 9.1%. In their sample, activist hedge funds take many forms of public action that is not direct conversation with management or a takeover attempt.[9] Appel, Gormley, and Keim (2016b) points out that the presence of passive mutual funds has facilitated activism by supporting activists with large blocks of shares, and show that this has led to changes in activist presence and tactics and overall outcomes. This links directly to our results on shareholders voting with the blockholder to support its information.

2 Model

The model setup is similar to the strategic voting literature (e.g. Feddersen and Pesendorfer, 1996) with the novel departure that one voter (shareholder) has more votes (shares) than the others—the blockholder. Most of the literature relies on symmetry in order to pin down equilibria. However, in the corporate environment, asymmetry in the number of votes participants have arises naturally. We will allow for asymmetry in numbers of shares and asymmetry in strategies (for shareholders who have equal number of shares).

There are two types of agents who own shares in the firm. There is a blockholder (B) who has $2b$ shares and $2n + 1$ other shareholders (each denoted by S), each of whom owns one share. Henceforth, we use the term shareholder only to refer to one of the $2n + 1$ who each hold only a single share. The assumption that the blockholder has an even and shareholders odd numbers of shares allows us to disregard ties for many of the cases we study. For simplicity, we assume there is no trading of shares. We also assume that the blockholder does not own a majority of the shares:

[9] This includes seeking board representation without a proxy contest or confrontation with management (11.6%), making formal shareholder proposals, or publicly criticizing the company (32.0%), threatening to wage a proxy fight in order to gain board representation or sue (7.6%), launching a proxy contest to replace the board (13.2%), and suing the company (5.4%).

Assumption A1: $n \geq b$

A proposal at the shareholder meeting[10] will be implemented if a majority votes for it. If there is a tie, as is standard in the literature, we will assume that it will be implemented with probability 0.5. There are two states of the world θ, management is correct $(\theta = M)$, and against management $(\theta = A)$, which are both ex-ante equally likely to occur. Let d denote whether management wins $(d = M)$ or loses $(d = A)$. Shareholders and the blockholder have common values; i.e. they both prefer the choice that maximizes the value of the firm. We will relax this assumption in Section 5.3, where we allow the blockholder to be biased and strictly favor one decision. The payoff per share $u(d, \theta)$ from a vote depends on the decision and the state of the world. If the decision matched the state of the world, the payoff is 1 for each share: $u(M, M) = u(A, A) = 1$. If the decision did not match the state of the world, the payoff is 0: $u(M, A) = u(A, M) = 0$.

The blockholder and shareholders get individual imprecise signals $s \in \{m, a\}$ about what the correct state of the world is. This precision is given by $\pi_i(\theta \mid s)$, where $i \in \{B, S\}$.[11]

$$\begin{aligned} \pi_B(M \quad | \quad m) &= \pi_B(A \mid a) = q \\ \pi_S(M \quad | \quad m) &= \pi_S(A \mid a) = p \end{aligned}$$

The probability that the blockholder infers the correct state from the

[10]From Yermack (2010): "In addition to director elections, shareholders may vote on such topics as the appointment of outside auditors, issuances of new shares, creation of equity-based compensation plans, amendments to the corporate charter or bylaws, major mergers and acquisitions, and ballot questions submitted in the form of advisory shareholder proposals. Shareholders may also be asked to ratify certain decisions of the board of directors, such as related-party transactions with members of management. When shareholder approval of an item such as an acquisition becomes time critical, votes may be held at special shareholder meetings called in the middle of a year."

[11]Formally, in the statics literature precision is equal to the reciprocal of the variance; here this is equal to $\pi_i(1 - \pi_i)$ which is monotonic in π_i in the relevant range $\pi_i \in (\frac{1}{2}, 1)$.

signal is q, and that probability for the shareholder is p. We assume that $q \geq p > 0.5$. This indicates that signals are informative, as their precision is above 0.5, and that the blockholder receives a more precise signal than the shareholder. The blockholder presumably receives more precise information because she is a larger investor and possibly has (i) more contact with the firm, (ii) more infrastructure in place to gather information, and (iii) more incentives to gather information. In the analysis, we will discuss what happens when the blockholder's signal varies in the analysis and we will look at information acquisition by the blockholder in the extensions.

We allow the blockholder and shareholders to vote strategically. Given a particular signal, they may vote for M, A, or abstain. Nevertheless, in order to gain some tractability and focus on the key trade-offs of the model, we restrict the voting behavior of the blockholder and shareholders as follows:

Assumption A2: The blockholder is restricted to pure strategies, can vote only an even number of shares, and can not simultaneously vote for both sides of the proposal.

Assuming that the blockholder votes with an even number of shares (like the assumption that the blockholder holds an even number of shares and other shareholders an odd number) allows us to disregard ties for many of the cases we study. This does not affect the results, but simplifies the analysis and presentation. Preventing the blockholder from voting for both sides of the proposal is more a question of presentation, as what will matter in that case would be the net votes the blockholder produces for one side, which could be replicated by voting only a fraction of her votes and abstaining with the rest (which we allow).[12] We also assume:

Assumption A3: Shareholders are restricted to pure strategies.

[12] This choice of presentation might raise the question of whether quorum rules are met. Often these require a majority of shares to be present at a vote. Abstention by those present may be permitted. Alternatively by voting on both sides of an issue, and thus replicating abstention, shareholders in our model can ensure compliance with such a quorum requirement.

This assumption has bite only in case shareholders are indifferent between different voting strategies, and can be understood as restricting the way they respond to indifference. In more general games, allowing mixed strategies may be required to ensure existence of equilibria—we will show that pure strategy equilibria exist in all the environments that we consider. We rely on this assumption to reduce the number of cases that we consider and for tractability. Intuitively, the informationally efficient equilibria that we focus on should require (potentially asymmetric) pure strategies rather than mixed strategies to avoid costly miscoordination. Persico (2004) points out that such a restriction, which he also makes, still allows for rich strategic behavior, which we also demonstrate. Moreover, Esponda and Pouzo (2012) argue that in the voting environment that we consider, pure strategy equilibria are stable, whereas mixed strategy equilibria are not.

The precision of the signals and the structure of the game is common knowledge.

Lastly, the voting setting leads naturally to multiple equilibria, even when the blockholder and shareholders are restricted to pure strategies. It is natural to focus on the equilibria that lead to the best outcome. Note, that since all shareholders have identical preferences and differ only in their information, they all agree on what "best" means here—the equilibria that lead to the highest probability of selecting the decision that matches the state. We will refer to this as the most informative equilibrium. In Section 5.3, we allow the blockholder to be biased, and therefore this may no longer be the blockholder's preferred equilibrium. Nevertheless, this selection criteria will remain the preferred one of the shareholders and we continue to employ it.

In the following sections we consider two variations of the model; first we suppose the blockholder makes no announcements before it votes, which we denote as a passive blockholder. Next we allow the blockholder to observably announce its position before other shareholders, which we denote an active blockholder.

3 Passive blockholder

In this section, we analyze the model outlined above where both the blockholder and shareholders care about maximizing the value of the firm. Looking at how much information the blockholder holds relative to shareholders is critical to understanding what the most informative equilibrium is in this context.

Consider two examples. In the first example, the blockholder has imprecise information, say equal to the precision of an individual shareholder's information. Nevertheless, in this example, the blockholder owns 40% of the shares, while each individual shareholder holds less than 1%. If the blockholder voted with all of its shares in this scenario, the final vote will then mostly reflect the blockholder's information. If instead, the blockholder did not vote all of its shares, the final vote would reflect the information of all shareholders. As the blockholder cares about maximizing the informativeness of the vote, since that will augment the likelihood of increasing firm value, it is natural to posit that the blockholder prefers not to vote all of its shares.

In the second example, the blockholder is perfectly informed, i.e. $q = 1$, but only owns 5% of the shares of a firm. Individual shareholders have imprecise information. In this case, if all individual shareholders vote, the blockholder's vote will have little impact. But if the individual shareholders could delegate the vote to the blockholder, they would prefer to do so, as that would maximize the informativeness of the vote. They can accomplish this by abstaining.

These two examples lead us to two equilibrium outcomes that might maximize informativeness. We prove in Proposition 1 that these are the unique most informative equilibrium outcomes. We first define an optimal vote share for the blockholder, $b^* := \left\lfloor \frac{1}{2} \frac{\ln q - \ln(1-q)}{\ln p - \ln(1-p)} \right\rfloor$, so that $2b^*$ is the even number of shares that most closely corresponds to the optimal vote share (without exceeding it),[13] accounting for the fact that the theoretical optimal weight in decision-making may not correspond to a round number. In particular this definition implies that b^* is the integer that satisfies:

$$\left(\frac{p}{1-p}\right)^{2b^*} \leq \frac{q}{1-q} < \left(\frac{p}{1-p}\right)^{2b^*+1}. \tag{1}$$

We also define a condition that determines which of the two equilibrium outcomes is more informative.

$$\frac{\sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} \left(\frac{p}{1-p}\right)^i}{\sum_{i=0}^{n-b} \binom{2n+1}{i} \left(\frac{p}{1-p}\right)^i} > \frac{q}{1-q} \tag{C1}$$

Finally, we say that a shareholder or blockholder votes sincerely if they vote with their signal.

Proposition 1 *The most informative equilibrium takes the following form:*

(i) When $b \geq b^*$, all shareholders vote sincerely and the blockholder votes sincerely $2b^*$ shares,

(ii) When $b < b^*$ and Condition C1 holds, all shareholders vote sincerely and the blockholder votes sincerely $2b$ shares, and

(iii) When $b < b^*$ and Condition C1 does not hold, the equilibrium is outcome equivalent to one in which only the blockholder votes.

Both these types of equilibria always exist; that is one where shareholders vote sincerely and the blockholder votes sincerely with $\min\{2b, 2b^*\}$ shares; and one where the equilibrium outcome is identical to only the blockholder voting.

We prove the proposition by first observing that if we were to assume that voting was sincere rather than strategic, Nitzan and Paroush (1982) show that, in order to maximize the informativeness of a vote, a planner should

[13] The function $\lfloor x \rfloor$ is called the floor function. It defines the largest integer less than or equal to x.

weight the votes of voters with heterogeneous information according to how precise their signals are. These weights allow us to define b^*. In our model, however, shareholders need not vote sincerely. *Strategic* voting implies they may vote against their signals and/or abstain. We must therefore consider equilibrium behavior (rather than the planner's choice). In the case where the blockholder has more than $2b^*$ shares, this is a simple exercise. McLennan (1998) ensures that in this pure common interest game, everyone voting sincerely with the optimal weights can be implemented as an equilibrium (that is, that shareholders and the blockholder would not want to deviate from voting sincerely according to these vote shares).

Now consider the case where the blockholder has fewer than $2b^*$ shares. We begin by demonstrating that the equilibria we propose are, in fact, equilibria. We then show that they are the most informative equilibria, where Condition C1 determines which of the two is most informative.

Hence, when the blockholder has too many shares in comparison with her information, she will not vote them all. If she were to vote them all, she would drown out useful information from the other shareholders, impairing the effectiveness of the overall vote. As the blockholder and the shareholders have a common interest in maximizing the value of the firm, the blockholder internalizes this effect and only votes a fraction of her shares. As mentioned in the introduction, for investment advisers (such as mutual funds), the requirement to act in the best interest of investors is often interpreted as prohibiting the blockholder from acting strategically. This result demonstrates the efficiency loss from such a policy.

When the blockholder has too few shares in comparison with her information, there are two equilibria that may be the most informative. One is similar to the above case - the blockholder votes all of her shares sincerely and the other shareholders also vote sincerely. In the other, only the blockholder votes. All other shareholders abstain to allow the blockholder's superior information to determine the vote. Clearly strategic behavior is important to this equilibrium - allowing the shareholders to get out of the blockholder's way by abstaining leads to this result.

For the case where the blockholder has too few shares, we now examine which of the two equilibria is more likely to be more informative. This, of course, depends on Condition (C1). The equilibrium outcome where shareholders abstain is more informative if shareholders' information is poor in comparison to the blockholder and the blockholder has few votes. Perhaps, somewhat less straightforward is the observation that this equilibrium outcome is relatively more informative if there are relatively few shareholders. Equivalently, the equilibrium where shareholders vote sincerely is more informative when they are many shareholders. This is immediate in the limit: the more shareholders there are, the more likely that their sincere vote will accurately match the state; intuitively, when they are few then there is scope for noise and mistakes. We formalize these statements in the following proposition.

Proposition 2 *When $b < b^*$, the most informative equilibrium involves shareholders voting sincerely the (i) higher is p, (ii) the higher is n, (iii) the higher is b and (iv) the lower is q.*

We now study the case where the blockholder can announce its position to the other shareholders before they vote.

4 Active blockholder

Blockholders differ from small shareholders in several ways beyond holding more shares. Up until this point, the only additional difference we have assumed is that the blockholder has better information than shareholders. Here, we suppose that the blockholder can also observably announce a voting strategy before other shareholders vote. Notice that the blockholder need not commit to the announced position as its objectives are aligned with other shareholders.[14] Blockholders may easily be more visible and scrutinized. Any shareholder with over 5% of the shares of a publicly traded firm must publicly disclose this to the SEC. Some blockholders are very public about their activities, such as activist investors. Activist investors make public statements about their intentions and often discuss their intentions with other investors (see Coffee and Palia, 2016).

The analysis of this sequential game proceeds by backwards induction and the equilibrium concept that we apply is Perfect Bayesian Equilibrium: the shareholders observe the blockholder's statement, drawing inferences on the blockholder's signal, and then choose their votes; the blockholder, anticipating equilibrium behavior of the shareholders in response to their signals (and to its own statement), will choose its voting strategy.

First, we argue that in contrast with the case of a passive blockholder, there need not be an equilibrium where all agents vote sincerely. We establish this result first. The intuition for this result also provides some intuition for our more central result on the most informative equilibria. Consider a blockholder with few votes but very precise information. This blockholder can't represent all of its information in its vote. A shareholder with an imperfect signal who has seen the blockholder's vote would then prefer to ignore its information and vote in line with the blockholder (to support the blockholder's information), even if all other shareholders, who have similarly imperfect signals, vote sincerely. This result is contained in the following Lemma.

Lemma 1 *Suppose $2b < 2b^*$, there is no equilibrium where the blockholder and all shareholders vote sincerely.*

Note, however, that there is an equilibrium where all shareholders abstain. This is for the same reason that this equilibrium exists in the passive blockholder case: If no other shareholders are voting, then a single shareholder's

[14] There are, of course, other equilibria, for example involving babbling. Sincere communication will be a feature of the most informative equilibrium.

vote can have no affect on the outcome, and there is no loss to abstaining. However, we argue below that in contrast to the case of the passive blockholder, this cannot be the most informative equilibrium; instead, some voters should mimic the blockholder's vote to make the weight of the blockholder's information correspond to the optimal weight $2b^*$ with the remainder voting sincerely. We proceed by, first, showing such an equilibrium is feasible, and then that it is the most informative.

Proposition 3 *Given an active blockholder, there exist equilibria where the blockholder votes sincerely with* $\min[2b, 2b^*]$ *shares, and*

(i) if $2b \geq 2b^*$*, all shareholders voters vote sincerely, or*

(ii) if $2b < 2b^*$*,* $2b^* - 2b$ *shareholders vote the same way the blockholder does, and the rest of the shareholders vote sincerely.*

We now argue, that the equilibrium characterized in Proposition 3 is the one that is most likely to lead to the action chosen corresponding to the state; i.e., it is the most informative one.

Proposition 4 *There is a unique most informative equilibrium. It is as described in Proposition 3.*

Therefore having some shareholders ignore their information to support the blockholder's information maximizes the probability that the vote matches the state. The most informative equilibrium involves $2b^* - 2b$ shareholders ignoring their information; this clearly leads to a coordination problem. Moreover, the shareholders only want to coordinate because the blockholder has moved first. This suggests that leadership and coordination among shareholders may enhance efficiency.[15]

[15] Allowing for heterogeneity among the shareholders in the precision of their signal creates a natural coordination device: the lower precision shareholders would be the ones who ignore their information and follow the blockholder. The lower precision shareholder only need know their own precision and the distribution of precisions for this to work.

Activist investors are often very public about their positions. For example, Bill Ackman of the Pershing Square Hedge Fund appeared "almost daily on CNBC to take his case directly to investors" (George and Lorsch, 2014). Direct communication with other shareholders was permitted by the SEC's rule 14a-12 in 1999. Communication enters a gray area when investors have sizeable stakes in the firm - an individual OR group stake of 5% or more must be declared publicly to the SEC. The gray area is what constitutes a group - i.e. do communication and parallel actions constitute coordination or not. The purpose of this public declaration to the SEC under section 13(d) is to make other shareholders aware of changes in control of the firm (Lu, 2016). The downside, therefore, of such unobservable coordination is that trading profits are made by the insiders in a "wolf pack" while other shareholders are unaware. However, the trading profits may be needed to incentivize such behavior. Our model demonstrates that there can be benefits from having such a "wolf pack", i.e. having (i) a lead informed activist and (ii) subsequent coordination among some other investors to support the activist's position. As we do not incorporate coordinated trading into the model, we cannot address the costs of such behavior.[16] A different type of coordination might be made feasible by having shareholders invest in a fund, with the fund taking the coordination role. Appel, Gormley, and Keim (2016b) find that passive mutual funds have facilitated activism by supporting activists with large blocks of shares and that this has led to changes in activist presence, tactics, and overall outcomes.

It is natural to compare outcomes and the informational efficiency of the equilibria in the passive blockholder case to that of the active blockholder case. When $2b \geq 2b^*$, the outcome is the same. When $2b < 2b^*$, wel-

[16]Brav, Dasgupta, and Matthews (2016) provide a very different motivation for coordination by a wolf pack in a model that does not include voting. A sufficiently large bloc of shareholders can make a value enhancing change in a firm. These activists have complementarities in their costly decision to take a stake because they get reputation benefits from a successful activism campaign. The complementarities lead to a coordination game.

fare is higher in the active blockholder case. When condition C1 holds (all shareholders vote in the passive blockholder case), there are efficiency gains from the shareholders who ignore their information and vote to support the blockholder's superior information. When condition C1 does not hold (only the blockholder votes in the passive blockholder case), it is clear that there are benefits to having the blockholder's information supported with the appropriate number of votes and to having the other shareholders' information included. This comparison might then suggest that there is a benefit to vocal activist blockholders.

5 Extending the model

We now extend the model to make the setting more realistic. In the first extension, we allow the blockholder to acquire information and trade shares. This provides a robustness check for our result that the blockholder may not vote all of its shares, as it is natural to ask why the blockholder would not trade its excess shares. We demonstrate that there can be a lemons discount if the blockholder trades its shares, i.e., shareholders don't believe the blockholder has acquired information and therefore discount the shares being sold. In the second extension, we examine further the SEC rules for investment advisers. In particular, we show that forcing the blockholder to vote all of its shares could potentially enhance efficiency as it might provide it with incentives to acquire information. This offers a rationale for such a regulation on voting behavior. Lastly, we look at the situation where a blockholder may be biased in the sense that it strictly prefers one outcome and that fact is common knowledge. In the passive blockholder case, this creates opposition among shareholders to the blockholder. When the blockholder is active and can commit to its vote, this can, counterintuitively, lead to the blockholder abstaining.

5.1 Information acquisition and trading

In this subsection, we extend the model to provide an explanation for why, if a blockholder does not vote all of its shares, it does not sell the shares it doesn't vote. The voting game is the same as before (we focus on the active blockholder case), but before the vote takes place we allow for the blockholder to acquire information at a cost and to subsequently buy and sell shares. Information acquisition can provide incentives to have extra shares in the first place - the blockholder only reaps the benefit if it has more shares whose value will increase after the fixed cost of getting the information. The blockholder may not subsequently trade them away as new potential shareholders may be dubious about whether the blockholder acquired information if she is trying to dump shares on the market, and hence have a low willingness to pay for them.

In this extension, the blockholder begins with the same precision of information as a shareholder, i.e., the precision p. The blockholder may then pay a fixed cost c to acquire information, which boosts the precision of its information to $q > p$. We assume that the decision to acquire information is only observable to the blockholder herself.

After the information acquisition stage, we allow for trading. Given the complexity of the model already, we opt for a simplified trading game where we assume the blockholder trades publicly and simultaneously makes take-it-or-leave-it offers (to sell *or* buy). We assume she could sell to new shareholders who can buy one share each and have precision of information p. The blockholder could buy from existing shareholders. The observability of the blockholder's offers simplifies the inference problem of the shareholders, and might reflect that the blockholder is likely to face limits to its ability to disguise large trades. We assume for simplicity that the blockholder can only trade even numbers of shares. Once again, we restrict all agents to pure strategies.

We summarize the details and timing of the game:

1. The blockholder begins with an amount of shares $2b > 2b^*$. There are $2n + 1$ existing shareholders. The blockholder and each of the shareholders have information with precision p.
2. The blockholder can pay c in order to improve the precision of its information to q. This investment decision is not observed.
3. The blockholder can simultaneously and publicly make take-it-or-leave-it offers to sell to potential new shareholders or to buy from existing shareholders. After trading, the number of shares that the blockholder holds is given by $\widehat{2b}$ and the number of shareholders is $2\hat{n} + 1 = 2n + 1 + 2b - \widehat{2b}$.
4. The blockholder announces whether or not she has made the investment in information and her voting intention.
5. Votes are cast.

The following proposition proves that an equilibrium exists where the blockholder acquires information and does not sell its shares in excess of $2b^*$.

Proposition 5 *There are parameters for which an equilibrium exists in the game defined above where (i) the blockholder improves her information to precision q, (ii) the blockholder does not trade any shares ($\widehat{2b} = 2b$) shares, and (iii) she votes $2b^*(q)$ shares.*

The proof specifies the equilibrium strategies and beliefs of the blockholder and the shareholders and the incentive constraints of the blockholder. It demonstrates by numerical example that such an equilibrium exists.

The blockholder may have incentives to trade her excess shares after acquiring information since, (i) if her information acquisition were observable, she could get at least fair value for the shares, and (ii) selling to new potential shareholders could allow for the incorporation of more information.

However, as information acquisition is not observable, there is a lemons discount for traded shares. Shareholders observe shares on the market from the blockholder and assume she did not invest in information. Therefore the blockholder doesn't get full value for the shares, although it still benefits from new information being incorporated (point (ii) above).

There are, in fact, several other reasons that are not modelled here for why a blockholder might have a discount from trading. One is that new potential shareholders might not have useful information to incorporate into the vote, reducing their willingness to pay (e.g. liquidity traders). A second is that we assumed it was observable that the shares sold were coming from the blockholder. If this was not observable, and they were coming from a shareholder instead, there may not be much gains to trade as shareholders will be even more dubious about their value. A third is that the blockholder is assumed to get the full surplus from any trade, which is unlikely to be true. Lastly, there may be practical impediments to trading. This could arise if the blockholder was an index fund who needed to trade to track the index or if the blockholder had to worry about having too much of a price impact from selling shares.

5.2 The beneficial role of regulation

As we point out above, the SEC regulation (SEC, 2003) that requires investment advisers to vote in the best interest of their clients can reduce the efficiency of the information aggregation process in voting. In this subsection, we discuss under what circumstances such a rule could actually *enhance* efficiency. Our argument is that the SEC rule can make the blockholder more likely to acquire information, which in some circumstances can outweigh the inefficiency the rule introduces into the vote.

We interpret the SEC regulation in our model as having two requirements. First, the blockholder has to vote with all her shares. Second, all shareholders and the blockholder must vote sincerely, i.e., vote with their information. Incorporating this regulation into the model implies that the value per share when the blockholder invests in information acquisition is lower, as we have noted previously in the paper. However, this regulation also implies that the value per share when the blockholder does not invest in information acquisition is also lower. This second effect may be stronger as the blockholder might be impounding very poor information into the vote. This can skew the blockholder's decision toward acquiring information. This will be socially beneficial when the blockholder would not have acquired information without the regulation, even though that would have maximized social welfare.

Our approach is to show that such a situation is possible with an example.

Lemma 2 *There are parameters such that the SEC regulation of investment advisers enhances efficiency.*

The SEC rule results in less efficient information aggregation but the difference between private and social incentives to acquire information compensate for this loss.

Notice that the beneficial role of regulation is likely to be eliminated if trading were introduced. If the blockholder preferred to invest in the absence of regulation then the SEC rule clearly has no benefit. Suppose instead that the blockholder preferred not to invest in information in the absence of regulation, then she would gain by trading all of her shares (since she has the same information precision, p, as other shareholders and, so, there are shareholders who would improve the value of the vote by buying the share and voting informatively). The introduction of the SEC rule does not affect how much the blockholder receives from trading all her shares. Instead, if the blockholder chose not to trade then since the SEC rule obliges the blockholder to take actions that it could have chosen to undertake in the absence of the ruling, it can only reduce the blockholder's value. Thus, if the blockholder prefers to trade all her shares in the absence of the SEC rule, she would also prefer to trade them all in the presence of the rule. In particular, this implies that when the blockholder can trade, the SEC ruling could not lead the blockholder to invest in information acquisition that it would not otherwise undertake. Its only possible role would be to aggregate information less efficiently (in the case where the blockholder found it worthwhile to invest and held more than $2b^*$ shares). Thus, in our model, this regulation may be beneficial only if trading is restricted, as might be the case for index funds minimizing tracking error.

5.3 Biased Blockholder

So far we have considered votes where the blockholder and shareholders have had identical objectives. In this section, we suppose that the blockholder is biased and prefers that the action M is chosen regardless of the underlying state. As discussed earlier, this bias may arise because the blockholder is part of management or directly tied to it through business dealings. We maintain that timing, preferences and the quality of signals are known by all.

We begin by considering the case of a passive biased blockholder who makes no announcements. In this case, we demonstrate that in the most informative equilibrium, shareholders will try to oppose the blockholder by either disregarding their signal and vote for A or voting for A when they receive an a signal and abstain when they receive an m signal. The latter strategy allows some information to enter the vote. At the same time, by cancelling out the blockholder, these shareholders can make way for sincere shareholders to have an informative vote. Thus, shareholders react quite differently here in an attempt to minimize the effect of the bias on the vote.

We then consider a biased active blockholder. We allow this blockholder to commit to an observable vote choice up front (rather than just make an announcement). It might seem that the biased blockholder will simply vote all its shares for its preferred position, so that the distinction between the passive and active cases is uninteresting, but this does not happen. With a passive blockholder, since the blockholder's vote is not observed, it cannot have an effect on shareholder behavior. Instead, with an active blockholder, the blockholder's voting choice is observable and *can* change shareholders behavior (both to counteract the votes of this biased party and through the information learned from the vote), and a different outcome can arise. Indeed, we show that the most informative equilibrium has a blockholder who abstains. Such an equilibrium is the most informative because it has all of the shareholders voting sincerely. It exists because if the blockholder were to deviate, shareholders might infer that it has information that the true state is A, in which case they would all vote for A, which is the worst possible outcome for the blockholder.

Both the passive and active cases reflect a couple of central themes of this paper. First, requiring voters to vote in line with their private information may lead to worse outcomes. Allowing shareholders to either vote against the biased blockholder and disregard their signal or abstain increases the informativeness of the vote. This, of course, may require coordination, which also points to an efficiency enhancing role of coordination. Second, requiring voters to vote rather than abstain may lead to worse outcomes. In the active case, the biased blockholder abstains, which allows an informative vote to occur.

5.3.1 Passive blockholder

When the blockholder is passive and votes at the same time as other shareholders, it is immediate that in any equilibrium the biased blockholder prefers to vote all of its shares for its preferred position: In equilibrium, its behavior (or a deviation from equilibrium behavior) is not observed and can have no effect on the choices made by shareholders. Since more votes in favor of its preferred position increase the likelihood that this position is adopted, taking as given the behavior of shareholders, it will vote with all its $2b$ shares for M.

Of course, multiple equilibria can arise. Our focus here is on the equilibria that are optimal from the perspective of shareholders - that is, equilibria that maximize the probability that the proposal adopted matches the underlying state - the most informative equilibria.

First, suppose that no shareholder can abstain. In this situation, the most informative equilibrium involves $2b$ shareholders voting for A, thereby effectively nullifying the biased blockholder's influence, while the remainder vote sincerely.

Proposition 6 *In the case of a biased passive blockholder, the most informative equilibrium with no abstention involves the blockholder voting for M with all of its $2b$ shares, $2b$ of the shareholders voting for A independently of their signals, and the remaining $2n + 1 - 2b$ shareholders voting sincerely.*

Proposition 6 highlights that allowing shareholders to ignore their signals can lead to more efficient outcomes. The efficiency comes from those votes cancelling out the bias of the blockholder and allowing the sincere voting of the remaining shareholders to provide an informative vote. Notice that this involves coordination among shareholders (to choose which shareholders will block the vote) as in the case with the unbiased active blockholder, but here it is to oppose the blockholder, rather than support her. This could still fit into the "wolf pack" scenario, where the blockholder is management and the coordinating shareholders are activists seeking to change management practices.

Next, we point out that equilibria that feature abstention can be even more efficient. We use a simple example that shows how abstaining rather than voting A when observing an m-signal can allow shareholders to reflect their information to some extent, while still mitigating the influence of the biased blockholder. The intuition for this is the following. Take one biased blockholder vote for M and add a vote where the shareholder abstains when observing an m-signal and votes A when observing the a-signal. If the state of the world is M, it is likely that adding the two votes together will produce one vote for M. Otherwise, it is likely that the two votes will cancel each other out. Therefore, this produces some information as compared to the case where a shareholder always votes for A and this vote completely cancels out a vote of the blockholder.

Example 1 *Suppose that $n = 1$ and $b = 1$ so that the blockholder has two shares and there are three shareholders. Then in the absence of abstention, following Proposition 6, the most informative equilibrium involves two of the shareholders voting A independently of their signals and the third voting sincerely. This selects the correct action with probability p.*

Suppose instead that all three shareholders abstain when observing an m-signal and vote A when observing the a-signal. This selects the correct action with probability $\frac{1}{2}(p^3+3p^2(1-p)\frac{1}{2})+\frac{1}{2}(p^3+3p^2(1-p)+\frac{1}{2}3p(1-p^2))$. The first term corresponds to the true state being A. The vote will reflect this when all three shareholders have an accurate signal, or if two have an accurate signal (and the other abstains) the vote will be for A with probability $\frac{1}{2}$. The second term corresponds to the true state being M. If all three shareholders have the correct signal they all abstain and the blockholder's votes will ensure that M is chosen. Similarly if there is only one vote against, and if there are two votes against, M will be implemented with probability $\frac{1}{2}$. The overall expression can be written as $\frac{p\left(3-2p^2+3p\right)}{4}$ and this is strictly greater than p in the range $p > \frac{1}{2}$. Lastly, using McLennan (1998), if this is the most informative set of strategies from a planner's perspective, it must be an equilibrium. We also demonstrate this equilibrium is the unique most informative equilibrium (available upon request).

5.3.2 Active blockholder with commitment

We now turn to consider the case of an active biased blockholder who may commit to a vote observably before the other shareholders vote.[17] If the blockholder abstains (and the shareholders draw no inference about her information) then the most informative equilibrium involves all shareholders voting sincerely. This is a good outcome for a blockholder who is perfectly informed that the state is M, as M is likely to win such a vote.

Of course, a blockholder who knew that the state is A would not be happy with such an outcome. However, voting by this blockholder might reveal that the blockholder knows that the state is A and so would lead shareholders to update their beliefs and to support A even more strongly (indeed when the blockholder is perfectly informed, all shareholders would vote A). To make these arguments clearly, throughout this section we suppose that the blockholder is perfectly informed regarding the underlying state; that is, $q = 1$. Overall, we show that there is an equilibrium where the biased blockholder abstains regardless of her information, and all shareholders vote sincerely.

Proposition 7 *In the game with a biased active blockholder, there is an equilibrium in which the blockholder always abstains and all shareholders vote sincerely.*

Proof. This is immediate - it can be supported by the off-equilibrium belief that any blockholder who votes knows that the state is A. ■

This result is of interest, since it leads to an efficient outcome - $2n + 1$ shareholders vote sincerely. It is clear, following Condorcet (1785), that the

[17] We allow for commitment here, as the bias of the blockholder has eliminated the common value environment of the model, making communication cheap talk. Such commitment might best be understood as one arising from reputational concerns. In the case of investment advisors, in line with SEC regulations, the blockholder's vote would be observed ex-post. For other kinds of large blockholders, inferences could be drawn from aggregate vote shares.

active blockholder case can therefore use the shareholders' information more efficiently than the passive blockholder case, where some shareholders are not voting sincerely in order to counter the bias of the blockholder.[18] The next result shows that the equilibrium described in Proposition 7 is, in fact, the most informative one in the case of an active blockholder.

Proposition 8 *In the game with a biased active blockholder, the equilibrium outlined in Proposition 7 is the most informative equilibrium.*

This equilibrium incorporates the information of shareholders as efficiently as possible. Although the biased blockholder is present, she has no influence whatsoever in equilibrium. The biased blockholder with precise information that A is the state of the world would prefer to enter and reduce the informativeness of the vote (by having some of the shareholders dedicate themselves to cancelling her vote rather than voting sincerely). However, this would reveal the biased blockholder's information in the signalling game, and she must therefore abstain.

6 Conclusion

Our paper extends a standard voting environment by introducing a voter who has multiple votes: the blockholder. This is natural in a corporate setting and leads to striking results. Contrary to the common wisdom promoted by regulators, we demonstrate that allowing for abstention can increase the informativeness of a vote. An unbiased blockholder with not very precise information but a lot of shares would prefer not to vote all of its shares to allow other shareholder information to be used in the vote. We also demonstrate that allowing shareholders to act strategically and coordinate can also

[18]Note also that (prior to the realization of the state) the blockholder is indifferent about whether it is passive or active since in either case, the vote is equally likely to result in M or A being chosen. Thus, whether or not it incorporates the utility of the blockholder, maximizing ex-ante welfare would favour an active blockholder.

increase the informativeness of a vote. Shareholders may do so to support an unbiased blockholder's information or to counter a biased blockholder's vote.

In our analysis, we have made many simplifications in order to demonstrate the basic driving forces when a blockholder is voting. It would be of interest to extend the model to allow for a more realistic environment. Some avenues that could be pursued are examining situations where there is uncertainty about the bias of the blockholder, allowing for a richer model of share trading, and looking at multiple blockholders.

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7 Appendix

7.1 Proof of Proposition 1 (Passive Blockholder voting)

Proposition 1 The most informative equilibrium when:

(i) $b \geq b^*$ involves all shareholders voting sincerely and the blockholder votes sincerely $2b^*$ shares,

(ii) $b < b^*$ and Condition C1 holds involves all shareholders voting sincerely and the blockholder voting sincerely $2b$ shares, and

(iii) $b < b^*$ and Condition C1 does not hold is outcome equivalent to one in which only the blockholder votes.

We prove our result through a series of Lemmas.

It is intuitive that a voter with better information should have more influence on the outcome—a planner with direct access to the signals would update according to Bayes' rule and more informative signals would have more influence on the posterior belief. In the same way, a voter with better information should be granted more votes. Specifically, Theorem 1 of Nitzan and Paroush (1982) implies the following:

Lemma 3 ***(Application of Nitzan and Paroush, 1982)*** *If voting is sincere and weights (or vote shares) can be allocated among all voters to maximize the informativeness of the vote, then the weight of each voter should depend only on their own information (p or q). In particular, the blockholder's vote share should be $\ln\left(\frac{q}{1-q}\right)$ and each shareholders' vote share should be $\ln\left(\frac{p}{1-p}\right)$. Given each of the shareholders has a single vote, the blockholder should have $\frac{\ln q - \ln(1-q)}{\ln p - \ln(1-p)}$ votes.*

We next examine equilibrium behavior, demonstrating that the equilibria we describe are indeed equilibria.

Lemma 4 *There is always an equilibrium where the blockholder votes sincerely with* $\min\{2b^*, 2b\}$ *votes and all shareholders vote sincerely.*

Proof. First, suppose that $b > b^*$ and consider the blockholder's problem when all informed shareholders are voting sincerely. Then since the blockholder has aligned preferences with all other shareholders, the blockholder's problem, given the behavior of all other voters is analogous to a planner's problem—it is immediate following Proposition 3 that sincere voting according to the vote share b^* is optimal. An identical argument suggests that in this case it is optimal for an informed shareholder to vote sincerely in this situation. Essentially, this is the result of McLennan (1998) applied in this context.

Next suppose that $b^* > b$. First we consider the behavior of the blockholder. Again, Proposition 3 suggests that if it were feasible the blockholder would want to vote sincerely with more votes than he has available. It is intuitive and simple to show that in this case, he would vote sincerely with as many votes as available.

Finally, consider where $b^* > b$ and the blockholder and all other informed shareholders vote sincerely. The intuition here is that the circumstances where an informed shareholder is pivotal are circumstances that primarily reflect the signals of other informed shareholders rather than the more informed signal of the blockholder. In this case, the additional information conveyed by a sincere vote by the informed shareholder leads to a more accurate decision and more efficient outcome.

Formally, note that given the symmetry in the problem, the probability of a shareholder being pivotal when the state is A is the same as the probability of a shareholder being pivotal when the state is M, we write this as $\pi_S(piv) = \pi_S(piv \mid M) = \pi_S(piv \mid A)$. It follows that the expected utility of a shareholder of voting her share for choice M over abstaining (which we denote by $\emptyset$) when observing a signal m is:

$$EU_S(M, \emptyset|m) = p\pi_S(piv \mid M) > \frac{1}{2}p\pi_S(piv \mid M) + \frac{1}{2}(1-p)p\pi_S(piv \mid A) = \frac{1}{2}\pi_S(piv) \tag{2}$$

where the first equality follows on noting that if the shareholder is pivotal and votes for M, the decision matches the state of the world (and the shareholder earns 1 rather than 0) only in case the state is indeed M; given the shareholder observes m this occurs with probability p. The right hand side of the inequality is the expected value of abstaining and noting that in this case there will be a tie and so the action is equally likely to be A or M. The final equality follows on noting $\pi_S(piv) = \pi_S(piv \mid M) = \pi_S(piv \mid A)$.

Since $p > \frac{1}{2}$, the inequality holds; the shareholder prefers to vote for M rather than abstain when all other shareholders and the blockholder are voting sincerely. Similarly, the expected utility of a shareholder who observes an a signal of voting her share for choice A over abstaining is positive.

The benefit of voting against her signal rather than with it when observing a signal m is given by:

$$EU_S(A, M|m) = (1-p)\pi_S(piv \mid A) < \frac{1}{2}\pi_S(piv). \tag{3}$$

It follows that it is strictly optimal for the shareholder to vote with her signal. ■

We must also look at the case where shareholders abstain.

Lemma 5 *There is always an equilibrium where the blockholder votes sincerely with all its votes (or* 2 *or more votes) and all shareholders abstain.*

Proof. It is immediate that if shareholders do not vote then the blockholder votes sincerely (and is indifferent regarding the number of votes with which he does so). If the blockholder votes sincerely with 2 or more votes, then a shareholder can never be pivotal and so abstaining is a best response. ■

Taken together, Lemmas 4 and 5 establish that there is always a multiplicity of equilibria. Given the observation above, it is clear that they cannot be ranked unambiguously. Instead, their relative efficiency depends on parameter values.

The first statement in the Proposition is immediate given arguments in the text and above.

Note that an equilibrium in which two shareholders abstain is outcome equivalent to an equilibrium where one shareholder disregards its information and votes for M while another shareholder disregards its information and votes for A. Therefore any equilibrium with shareholders abstaining implies that there are other equilibria with an identical outcome where shareholders cancel each others' votes out. Therefore, the candidate equilibrium where only the blockholder votes is outcome equivalent to many other equilibria.

When $b < b^*$, we have found two classes of equilibria that are candidates for being the most informative equilibrium: one which is outcome equivalent to only the blockholder voting, and the other where the blockholder votes all $2b$ shares and all shareholders vote sincerely. The first question we must answer is whether there are any more candidate equilibria to consider.

Consider a possible equilibrium where the blockholder votes z_B shares sincerely, z_S shareholders vote sincerely, and the remaining shareholders abstain, where $z_B < z_S < 2n+1$. From our result in Proposition 2, if this equilibrium is more informative than the one where only the blockholder votes, it will be less informative than one where all shareholders vote sincerely. This implies it is less informative than one of the two candidates we have found. If any of the shareholders who abstain in this possible equilibrium would instead vote against their signal or vote for one alternative irrespective of their signal, it would be less informative.

The only possible candidates left are equilibrium take the form of (i) the blockholder voting sincerely z_B shares and less than z_B shareholders vote (sincerely or otherwise); or (ii) shareholders vote regardless of their signal

$z > 2b$ net[19] shares for proposal $J = \{M, A\}$ and less than $z - 2b$ other shares from shareholders are cast sincerely. Equilibria which take the form of (i) are informationally equivalent to the equilibrium where only the blockholder votes, so we will select that equilibrium. Equilibria which take the form of (ii) are informationally inferior to the equilibrium where the blockholder votes $2b$ shares sincerely and all shareholders vote their shares sincerely. Therefore the two candidate equilibria we summarized initially are the only two candidates for most informative equilibrium.

An equilibrium where only the blockholder votes will lead to an outcome that matches the state with probability (and delivers expected utility) of q. An equilibrium where the blockholder votes all of its shares and all shareholders vote sincerely matches the state with probability:

$$q \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + (1-q) \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \tag{4}$$

This expression corresponds to $n+b+1$ or more votes cast in favor of the true underlying state. The first expression corresponds to the blockholder's signal matching the state (with probability q), with the the summation, indicating that $n-b+1$ or more of the $2n+1$ shareholders having signals that match the state. The second expression instead represents the likelihood that the blockholder's signal does not match the state, but that $n+b+1$ or more of the shareholders have signals that do.

We now compare the probability with which each equilibrium matches the state. All shareholders voting leads to a higher probability of doing so,

[19]Net means that if x shareholders are voting for M regardless of their signal and y shareholders are voting A regardless of their signal, $z = x - y > 0$ are voting for M regardless of their signal

and is a more informationally efficient equilibrium when:

$$q\sum_{i=n-b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}+(1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}>q$$

$$\begin{aligned}(1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i} &> q\sum_{i=0}^{n-b}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}\\ \frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}\left(\frac{p}{1-p}\right)^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}\left(\frac{p}{1-p}\right)^i} &> \frac{q}{1-q}.\end{aligned}$$

This is condition C1 in the text.

7.2 Proof of Proposition 2

Proposition 2 *When $b<b^*$, the most informative equilibrium involves shareholders voting sincerely the (i) higher is p, (ii) the higher is n, (iii) the higher is b and (iv) the lower is q.*

Proof. It is convenient to simplify expressions a little by introducing the notation $r:=\frac{p}{1-p}$. Thus, when $b<b^*$, the most informative equilibrium involves informed shareholders voting if:

$$\frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}>\frac{q}{1-q}.$$

(i) Consider

$$\begin{aligned}&\frac{d}{dr}\frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}\\ =\;&\frac{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}ir^{i-1}-\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i\sum_{i=0}^{n-b}\binom{2n+1}{i}ir^{i-1}}{\left(\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i\right)^2}\end{aligned}$$

.

The denominator is positive, the numerator is positive if and only if:

$$\frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}ir^i}{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i} > \frac{\sum_{i=0}^{n-b}\binom{2n+1}{i}ir^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}.$$

Note that $\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}ir^i > (n+b+1)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i$ and $\sum_{i=0}^{n-b}\binom{2n+1}{i}ir^i < (n-b)\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i$.

Given that $\frac{(n+b+1)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i}{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i} > \frac{(n-b)\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}$ reduces to $n+b+1 > n-b$ which is certainly true, the result follows.

(ii) Define the probability of getting the decision correct when all of the shareholders vote sincerely as.

$$F(n) = q\sum_{i=n-b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}+(1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}.$$

Given that q is the probability of getting the decision correct when only the blockholder votes, we want to show that when $F(n) \geq q$, $F(n+1) > q$ to prove the result. We start by writing out $F(n+1)$.

$$\begin{aligned} F(n+1) &= q\sum_{i=n-b+2}^{2n+3}\binom{2n+3}{i}p^i(1-p)^{2n+3-i} \\ &\quad +(1-q)\sum_{i=n+b+2}^{2n+3}\binom{2n+3}{i}p^i(1-p)^{2n+3-i} \end{aligned}$$

We can write the first term of $F(n+1)$ as:

$$
\begin{aligned}
&\sum_{i=n-b+2}^{2n+3} \binom{2n+3}{i} p^i(1-p)^{2n+3-i} \\
= \quad & \begin{array}{c} p^2 \sum_{i=n-b}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} + 2p(1-p) \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} \\ +(1-p)^2 \sum_{i=n-b+2}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} \end{array} \\
= \quad & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} + p^2 \binom{2n+1}{n-b} p^{n-b}(1-p)^{n+1+b} \\ -(1-p)^2 \binom{2n+1}{n-b+1} p^{n-b+1}(1-p)^{n+b} \end{array} \\
= \quad & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} \\ + \left[\binom{2n+1}{n-b} p - \binom{2n+1}{n-b+1}(1-p)\right] p^{n-b+1}(1-p)^{n+b+1} \end{array} \\
= \quad & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} \\ + \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1}(1-p)^{n+b+1} \left[\frac{p}{n+b+1} - \frac{1-p}{n-b+1}\right] \end{array} \\
= \quad & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} \\ + \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1}(1-p)^{n+b+1} \end{array} .
\end{aligned}
$$

This implies that the second term of $F(n+1)$ can be simplified as follows:

$$
\sum_{i=n+b+2}^{2n+3} \binom{2n+3}{i} p^i(1-p)^{2n+3-i} = \begin{array}{c} \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} + \\ \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1}(1-p)^{n-b+1} \end{array} .
$$

Therefore:

$$
\begin{aligned}
F(n+1) \quad = \quad & \begin{array}{c} q \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} + (1-q) \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i(1-p)^{2n+1-i} \\ +q \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1}(1-p)^{n+b+1} \\ +(1-q) \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1}(1-p)^{n-b+1} \end{array} \\
= \quad & \begin{array}{l} F(n) + q \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1}(1-p)^{n+b+1} \\ +(1-q) \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1}(1-p)^{n-b+1} \end{array} .
\end{aligned}
$$

Note that this is greater than $F(n)$ as long as the following holds.

$$\begin{array}{c} q\frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)}\frac{(2n+1)!}{(n-b)!(n+b)!}p^{n-b+1}(1-p)^{n+b+1} \\ +(1-q)\frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)}\frac{(2n+1)!}{(n-b)!(n+b)!}p^{n+b+1}(1-p)^{n-b+1} \end{array} > 0$$

This can be simplified to:

$$\frac{(2p-1)(n+1)-b}{(2p-1)(n+1)+b}+(\frac{p}{1-p})^{2b}\frac{1-q}{q}>0$$

Note that this expression is increasing in n. Therefore if there exists an n for which $F(n-1)<q$ and $F(n)>q$, this will imply that $F(m)>q$ for all $m>n$, which is what we need for our result. We now demonstrate that there must exist such an n.

The function $F(n)$ is well defined given our assumption that $2b<2n+1$, and the minimum value n can take is b. At $n=b$, $F(n)=q\sum_{i=1}^{2b+1}\binom{2b+1}{i}p^i(1-p)^{2b+1-i}$, which is smaller than q. So it is not possible that $F(n)>q$ for all n. Furthermore, it is not possible that $F(n)<q$ for all n, since as n approaches infinity $F(n)>q$. This is true since Condorcet's Jury Theorem states that in our model, if $q=p$, $F(n)\to 1$ as $n\to\infty$; it can't be smaller for $q>p$. One proof of the Theorem is in Ladha (1992). We have now demonstrated that there must exist an n for which $F(n-1)<q$ and $F(n)>q$.

(iii) The term $\frac{\sum_{i=n-2b+1}^{n+2b}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-2b}\binom{2n+1}{i}r^i}$ is increasing in b iff:

$$\frac{\sum_{i=n-2b-1}^{n+2b+2}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-2b-2}\binom{2n+1}{i}r^i}>\frac{\sum_{i=n-2b+1}^{n+2b}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-2b}\binom{2n+1}{i}r^i}$$

This holds iff:

$$\sum_{i=n-2b-1}^{n+2b+2}\sum_{j=0}^{n-2b}\binom{2n+1}{i}\binom{2n+1}{j}r^{i+j}>\sum_{i=n-2b+1}^{n+2b}\sum_{j=0}^{n-2b-2}\binom{2n+1}{i}\binom{2n+1}{j}r^{i+j}$$

This is immediate - the lower bound of the summation on the left hand side is lower than on the right hand side, while the upper bounds of the summation on the left hand side are greater than on the right hand side. Otherwise, all of the terms are identical and all are positive.

(iv) is immediate. ■

7.3 Proof of results on active blockholders

Lemma 1 *Suppose $2b < 2b^*$, there is no equilibrium where the blockholder and all shareholders vote sincerely.*

Proof. We prove this by contradiction. Consider the behavior of one of the shareholders when all the other shareholders and the blockholder vote sincerely.

An individual shareholder's vote is consequential when the vote is otherwise tied. That is there $n+b$ votes for each proposal.

This may reflect that the blockholder's signal is accurate and $n-b$ of the shareholders hold an accurate signal, with the remaining $n+b$ holding an inaccurate signal; or that the blockholder's signal is inaccurate and $n+b$ of the shareholders hold an accurate signal (with the remainder an inaccurate one). These events occur with probability

$$q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b}(1-p)^{n+b}, \text{ and}$$
$$(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b}(1-p)^{n-b},$$

respectively.

It follows that voting sincerely in opposition to the blockholder yields $p\left[(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b}(1-p)^{n-b}\right]$; instead voting along with the blockholder in opposition with his own signal yields $(1-p)\left[q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b}(1-p)^{n+b}\right]$. Voting sincerely in all circumstances requires that the former is greater than the latter. Equivalently $(\frac{p}{1-p})^{2b+1} > \frac{q}{1-q}$. This is false since $2b < 2b^*$ and by definition of b^*. ■

Proposition 3 *Given an active blockholder, there exist equilibria where the blockholder votes sincerely with* $\min[2b, 2b^*]$ *shares, and*

(i) if $2b \geq 2b^*$*, all shareholders voters vote sincerely, or*

(ii) if $2b < 2b^*$*,* $2b^* - 2b$ *shareholders vote in an identical fashion to the blockholder, and the rest of the shareholders vote sincerely.*

Proof. Part (i): If $2b \geq 2b^*$, we need to check that there is an equilibrium where the blockholder vote sincerely with $2b^*$ shares and all shareholders vote sincerely. As this is sequential, we first take as given that the blockholder voted $2b^*$ shares for choice X.

As in the proof of Lemma 1, an individual shareholder's vote is consequential when the vote is otherwise tied. This may arise when the blockholder's signal is accurate and occur with probability $q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b^*}(1-p)^{n+b^*}$ or when the blockholder's signal is inaccurate and occur with probability $(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b^*}(1-p)^{n-b^*}$.

It follows that the shareholder prefers voting sincerely in opposition to the blockholder rather than voting in line with the blockholder in opposition to its signal when $p\left[(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b^*}(1-p)^{n-b^*}\right] \geq (1-p)\left[q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b^*}(1-p)^{n+b^*}\right]$, or, equivalently $\left(\frac{p}{1-p}\right)^{2b^*+1} > \frac{q}{1-q.}$ which follows from the definition of b^* as in (1).[20]

Lastly, given the subsequent behavior of shareholders, the blockholder prefers to vote sincerely with $2b^*$ shares as this maximizes the common payoff of the game (as in Nitzan and Paroush (1982)).

Part (ii): If $2b < 2b^*$ the proof of Lemma 1 shows that if all other shareholders vote informatively, a shareholder has the incentive to vote the

[20] It is simple to show that this also implies that the shareholder prefers voting sincerely to abstention. Similarly, this will be the case for related arguments in this proof.

Of course, this analysis depends on inferences that shareholders draw from the blockholders voting behavior. There are many off-equilibrium beliefs that would support this behavior. For example, it is sufficient, here and below, to assume that shareholders suppose that the blockholder's signal reflects the its net votes, and beliefs are passive (i.e. the blockholders signal is equally likely to be of either type) in case of a tie (for example, if the blockholder abstains on all votes).

same way as the blockholder irrespective of her signal.

However, in order to establish the result, we must prove two things.

First, we must demonstrate that if $2b^* - 2b$ other shareholders are voting with the blockholder and all other shareholders are voting sincerely, then a shareholder will vote sincerely.

Again, to establish this it is useful to write down the probability that the vote is tied and the blockholder is correct. This involves $n+b$ votes with the blockholder and $n+b$ against. The latter must come from those who vote sincerely and there are $2n-(2b^*-2b)$ of these.[21] Thus this probability is $q\frac{2n+2b-2b^*}{(n+b-2b^*)(n+b)}p^{n+b-2b^*}(1-p)^{n+b}$. Similarly, the probability that the vote is tied and the blockholder is wrong is $(1-q)\frac{2n+2b-2b^*}{(n+b)(n+b-2b^*)}p^{n+b}(1-p)^{n+b-2b^*}$. It follows that the shareholder prefers voting sincerely in opposition to the blockholder to voting in line with the blockholder when $p\left[(1-q)\frac{2n+2b-2b^*}{(n+b)(n+b-2b^*)}p^{n+b}(1-p)^{n+b-2b^*}\right] \geq (1-p)\left[q\frac{2n+2b-2b^*}{(n+b-2b^*)(n+b)}p^{n+b-2b^*}(1-p)^{n+b}\right]$ or, equivalently $\left(\frac{p}{1-p}\right)^{2b^*+1} \geq \frac{q}{1-q}$ which follows from (1).

Second, we must demonstrate that if $2b^*-2b-1$ other shareholders are voting with the blockholder and all other shareholders are voting sincerely, then a shareholder will vote with the blockholder. Once again, we can write the probability that the vote is tied without this shareholder, and the blockholder is correct. Here, again this requires $n+b$ shareholders who vote sincerely to vote in opposition to the blockholder, but in this case there are $2n-(2b^*-2b)+1$ of these. This allows us to write this probability as $q\frac{2n+2b-2b^*+1}{(n+b-2b^*+1)(n+b)}p^{n+b-2b^*+1}(1-p)^{n+b}$. Similarly, the probability that the vote is tied and the blockholder is wrong is $(1-q)\frac{2n+2b-2b^*+1}{(n+b)(n+b-2b^*+1)}p^{n+b}(1-p)^{n+b-2b^*+1}$. It follows that the shareholder prefers voting in line with the blockholder than voting sincerely $(1-p)\left[q\frac{2n+2b-2b^*+1}{(n+b-2b^*+1)(n+b)}p^{n+b-2b^*+1}(1-p)^{n+b}\right] \geq p\left[(1-q)\frac{2n+2b-2b^*+1}{(n+b)(n+b-2b^*+1)}p^{n+b}(1-p)^{n+b-2b^*+1}\right]$, or, equivalently, $\frac{q}{1-q} \geq \left(\frac{p}{1-p}\right)^{2b^*}$. This again follows from the definition of b^* and the equivalent condition (1).

[21] Here, we suppose that $n > b^* - b$ for convenience, the case where $n \leq b^* - b$ involves all agents voting with the blockholder and can be established by similar arguments.

Lastly, given the subsequent behavior of shareholders, the blockholder prefers to vote sincerely its $2b$ shares as this maximizes the common payoff of the game (as in Nitzan and Paroush (1982)). ■

Proposition 4 *There is a unique most informative equilibrium for given parameters, and it is described in Proposition 3.*

Proof. When $b > b^*$, it is clear from Nitzan and Paroush (1982) that the equilibrium in Proposition 3 is the most informative equilibrium.

When $b < b^*$, the arguments from Proposition 3 part (ii) make it clear that there is no equilibrium where the blockholder votes all of his shares sincerely, fewer than $2b^* - 2b$ shareholders ignore their signal and vote with the blockholder, and the rest of the shareholders vote sincerely (since a shareholder who is voting sincerely would prefer to switch to vote with the blockholder).

There is no need to consider a conjectured equilibrium where the blockholder does not vote sincerely or abstains. Imagine a conjectured equilibrium where the blockholder abstains and all other shareholders vote sincerely. First of all, this can't be an equilibrium, as the blockholder would deviate. Second of all, while this is the most informative possible equilibrium where the blockholder does not vote sincerely or abstains, it is less informative than the equilibrium in Proposition 3 part (ii) because of the strength of the blockholder's information.[22]

The only possible candidates left are equilibrium that take the form of (a) the blockholder voting sincerely z shares and less than z shareholders vote (sincerely or otherwise);or (b) shareholders vote regardless of their signal $z > 2b$ net[23] shares for proposal $J = \{M, A\}$ and less than $z - 2b$ other shares

[22] We have ruled out mixed strategies by Assumption A3; however, allowing them would not change the result. Here, since there is a coordination problem with which shareholders will support the blockholder, there exist equilibria with mixed strategies. Given these can lead to coordination failure, they will be less informative than the pure strategy equilibrium in Proposition 3 part (ii).

[23] Net means that if x shareholders are voting for M regardless of their signal and y shareholders are voting A regardless of their signal, $z = x - y > 0$ are voting for M regardless of their signal

are cast sincerely. Equilibria which take the form of (a) are informationally equivalent to the equilibrium where only the blockholder votes. While in the passive blockholder there were conditions under which this was the most informative equilibrium, when there is an active blockholder this is not the case. This is because here, the blockholder's information will be reflected in $2b^*$ votes despite the fact he only has $2b$ votes; e.g. when q approaches 1, all of the shareholders will be voting with the blockholder.

Equilibria which take the form of (b) are informationally inferior to the equilibrium in Proposition 3 part (ii). Therefore the equilibrium in Proposition 3 part (ii) is the most informative equilibrium. ■

7.4 Proof of Proposition 5 (Result on information acquisition and trading)

Proposition 5 *There are parameters for which an equilibrium exists in the game defined above where (i) the blockholder improves her information to precision q, (ii) the blockholder does not trade any shares ($\widehat{2b} = 2b$) shares, and (iii) she votes $2b^*(q)$ shares.*

We specify the equilibrium strategies for the proposition: in stage 2, the blockholder invests in acquiring information; in stage 3, there is no trade and any attempt to trade will lead shareholders to believe that the blockholder made no investment in stage 2; in stage 4, the blockholder announces sincerely both that she invested and her voting intentions (to vote $2b^*(q)$ shares in line with her information); in stage 5, shareholders vote with the belief that any stage 4 announcement about investment and voting intention is truthful and respond with behavior that would lead to the most informationally efficient equilibrium given these beliefs.

We proceed by working backwards.

Tautologically, there is nothing to characterize in stage 4; since this is a common interest game and given shareholder behavior in stage 5, the blockholder will be sincere about investment and will vote the appropriate number of shares. In particular, if the blockholder made an investment, she would announce that she would vote $2b^*(q)$ shares in line with her information; in this case, shareholders would simply vote sincerely. Instead, if the blockholder made no investment, she would announce it and announce her abstention[24].

Period 3: In period 3, each shareholder i has a belief μ_3^i regarding the probability that the blockholder has made the investment c in precision q (and this belief might depend on what is offered at this round). Given the proposed equilibrium, beliefs are such that all shareholders believe that the blockholder invested in precision q with probability 1 (i.e., $\mu_3^i = 1$ for all i) when the blockholder doesn't try to buy or sell shares. Otherwise, off-the-equilibrium-path, we assume that shareholders believe the blockholder did not invest with probability 1 (i.e., $\mu_3^i = 0$ for all i).

The payoff on-the-equilibrium-path for the informed blockholder is:

$$2b[q \sum_{i=n-b^*(q)+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + (1-q) \sum_{i=n+b^*(q)+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i}] \tag{5}$$

This represents the value the blockholder gets from keeping all of its shares and voting $2b^*(q)$ of them. The payoff to the informed blockholder from trying to sell $2x \leq 2b - 2b^*(q)$ shares to new shareholders off-the-equilibrium-path is:

[24] We restricted voting to even numbers of shares. The blockholder's optimal amount of shares with no restriction is 1 share when it has precision p. It is straightforward to prove that not voting is better than voting two shares in this situation.

$$(2b-2x)[q\sum_{i=n+x-b^*(q)+1}^{2n+2x+1}\binom{2n+2x+1}{i}p^i(1-p)^{2n+2x+1-i} \tag{6}$$
$$+(1-q)\sum_{i=n+x+b^*(q)+1}^{2n+2x+1}\binom{2n+2x+1}{i}p^i(1-p)^{2n+2x+1-i}]$$
$$+2x\sum_{i=n+x+1}^{2n+2x+1}\binom{2n+2x+1}{i}p^i(1-p)^{2n+2x+1-i}$$

The first and second lines represent the probability that the vote matches the state multiplied by the number of shares the blockholder retains $2b-2x$. The probability that the vote matches the state has increased because the information of new shareholders has been incorporated. The third line represents the amount that the blockholder can sell its shares for. This amount represents the probability that new shareholders believe the vote will match the state given that they believe the blockholder has not invested.

Note that the blockholder wouldn't deviate to buying shares from existing shareholders, whatever their beliefs were. Such a purchase reduces value since the blockholder would be effectively reducing the amount of information in the vote. The payoff of the shareholder from retaining her share is weakly larger (strictly if the blockholder invested in information) than the payoff of the share when it is sold to the blockholder, and therefore an existing shareholder would refuse to sell at the blockholder's maximum offer.

Thus, the first requirement for the proposed equilibrium to exist would be to prove that the blockholder can lose out by deviating to equation 6, for all x, for some parameters. Below, we numerically show this is true for some parameters.

Now we must specify what a blockholder who did not invest in information will do and her payoffs. There are gains from trade here as the blockholder can sell shares which won't aggregate information into the vote to new shareholders who will provide useful information for the vote. This blockholder will sell all of its shares. The blockholder makes strictly positive profit by selling all shares except for the last one, on which she is indifferent between selling and not (hence, we assume she sells). Her payoff is:

$$2b \sum_{i=n+b+1}^{2n+2b+1} \binom{2n+2b+1}{i} p^i (1-p)^{2n+2b+1-i} \tag{7}$$

Period 2: Given that there is no trade in period 3 if the blockholder invests in information, the blockholder must decide whether such an investment is worthwhile in period 2. The blockholder invests in information if:

$$\begin{aligned} & 2b \left[\begin{array}{c} q \sum_{i=n-b^*+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ +(1-q) \sum_{i=n+b^*+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \end{array} \right] - c \\ \geq \ & 2b \sum_{i=n+b+1}^{2n+2b+1} \binom{2n+2b+1}{i} p^i (1-p)^{2n+2b+1-i} \end{aligned} \tag{8}$$

Numerical simulation: There are two conditions that must hold in order for the specified equilibrium to actually be an equilibrium. The first is demonstrating that equation 5 is larger than equation 6 for any $2x$ shares less than $2b - 2b^*(q)$. The second is that equation 8 holds. We provide an example where both of these conditions have been satisfied, which we checked with numerical simulation. Setting $p = 0.51$, $q = 0.6$, $b = 6$, and $n = 11$ satisfies both conditions when the cost c is low enough (e.g. any $c \leq 5$ works). With this simulation, $b^* = 5$, so there was only one value for sharetrading to check ($x = 1$) for the first condition to hold.

7.5 Proof of Lemma 2 (Efficiency of SEC regulation)

Lemma 2 There are parameters such that the SEC regulation of investment advisers enhances efficiency.

We assume that the information acquisition technology is the same as in the section on information acquisition and trading.

To simplify the analysis, we will examine the situation where the blockholder has excess shares, i.e., $b > b^*$.

The value per share when the blockholder does not invest in information acquisition is:

$$p \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + (1-p) \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i}$$

Note that the SEC regulation implies that the blockholder must vote all of its $2b$ shares.

The value per share with investment is:

$$q \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + (1-q) \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i}$$

The blockholder invests in information acquisition under the regulation when:

$$(q-p) \left[\sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} - \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \right] > \frac{c}{2b} \tag{9}$$

In the absence of the regulation, the blockholder would *not* invest in information acquisition when:

$$\left[\begin{array}{c} q \sum_{i=n-b^*+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ +(1-q) \sum_{i=n+b^*+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \end{array} \right] - \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} < \frac{c}{2b} \tag{10}$$

The investment under the regulation leads to a socially desirable outcome relative to the scenario where there is no regulation and no investment if:

$$\left[\begin{array}{c} q\sum_{i=n-b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i} \\ +(1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i} \end{array}\right]-\frac{c}{2n+2b+1}>\sum_{i=n+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i} \tag{11}$$

Thus the regulation improves efficiency when (i) there would be no investment in the absence of the SEC rule (equation 10), (ii) the SEC rule leads to investment (equation 9), and (iii) this is socially efficient (equation 11). This is interesting as long as the set of parameters for which this is true is non-empty.

This is the case with $b = 15$, $c = 2$, $n = 20$, $p = 0.51$. and $q = 0.6$. Note that $b^* = 5$ in this case.

7.6 Proof of Proposition 6 (Biased passive blockholder)

Proposition 6 *In the case of a biased passive blockholder, the most informative equilibrium with no abstention involves the blockholder voting for M with all of its $2b$ shares, $2b$ of the shareholders voting for A independently of their signals, and the remaining $2n + 1 - 2b$ shareholders voting sincerely.*

We begin with a lemma that describe strategies that are not optimal for shareholders. This will allow us to characterize strategies for the shareholders as weakly monotone, i.e., a shareholder has weakly monotone strategies when she is at least as likely to vote A when receiving the a-signal as when receiving the m-signal (and similarly for voting M).

Lemma 6 *Shareholder strategies are weakly monotone.*

Proof. We begin by proving that a shareholder would never vote anti-sincerely, i.e. voting M when she gets an a signal and voting A when she gets a m signal. Suppose for contradiction, that there is a shareholder who votes anti-sincerely. Taking all other shareholders strategies as given, we define several relevant pivotal probabilities as follows: Let $\pi^N(A)$ denote the probability that without this voter, the vote is split and the true state is A; $\pi^M(A)$ denote the probability that without this voter, the vote is in favor of M by one vote and the true state is A; $\pi^A(A)$ denote the probability that without this voter, the vote is in favor of A by one vote and the true state is A. We can define $\pi^N(M)$, $\pi^A(M)$, $\pi^M(M)$ similarly.

It follows that we can write down the expected probability of getting the decision correct when this last voter is votes anti-sincerely. This is:

$$\begin{aligned} &\Pr(non\_piv) + \frac{1}{2}\left[(1-p)(\pi^N(A) + \pi^A(A) + \frac{1}{2}\pi^M(A)) + p\frac{1}{2}\pi^A(A)\right] \qquad (12) \\ &+\frac{1}{2}\left[(1-p)(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)) + p\frac{1}{2}\pi^M(M)\right], \end{aligned}$$

where $\Pr(non\_piv)$ denotes the probability of getting the decision correct when the voter is not pivotal. The first square bracket corresponds to the true state being A. In this case, the voter may observe the signal m with probability $(1-p)$, which, given that the voter votes anti-sincerely, leads him to vote for A. In this case, the shareholder chooses the right action when he is pivotal in the event of a tie or the vote is in favor of A by one vote. The vote also leads to a tie and choosing the right action with probability $\frac{1}{2}$ in the case where the vote is in favor of M by one vote. The final term in the first set of square brackets corresponds to the agent getting the signal a and voting for M, which leads to a tie and choosing the right action with probability $\frac{1}{2}$ in the case where the vote was otherwise in favor of A by one vote. The second square bracket is the analogous expression for when the true state is M.

Similarly, if instead the shareholder voted sincerely, the probability that the action chosen would match the state would be:

$$\Pr(non\_piv) + \frac{1}{2}p(\pi^N(A) + \pi^A(A) + \frac{1}{2}\pi^M(A)) + \frac{1}{2}(1-p)\frac{1}{2}\pi^A(A) \tag{13}$$
$$+\frac{1}{2}p(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)) + \frac{1}{2}(1-p)\frac{1}{2}\pi^M(M).$$

Thus the optimality of anti-sincere behavior requires that (12)$\geq$(13), or, equivalently:

$$\begin{array}{c} \pi^N(A)(1-2p) + \pi^A(A)(1-2p-\frac{1-p}{2}) + \pi^M(A)\frac{1-2p}{2} \\ +\pi^N(M)(1-2p) + \pi^M(M)(1-2p-\frac{1-p}{2}) + \pi^A(M)\frac{1-2p}{2} \end{array} \geq 0.$$

This is clearly false on noting that $1-p>0$, $1-2p<0$ and all of the pivotal probabilities (the πs) are non-negative. Therefore this proves that a shareholder would not vote anti-sincerely.

Next, we rule out voting M in the case where the shareholder observes an a signal and abstaining when the shareholder observes an m signal. The probability of the vote matching the state correctly in this case is:

$$\Pr(non\_piv) + \frac{p}{4}\pi^A(A) + \frac{1-p}{4}\pi^N(A) + \frac{1-p}{2}\pi^A(A) \tag{14}$$
$$+\frac{1}{2}p(\frac{1}{2}\pi^N(M) + \pi^M(M)) + \frac{1}{2}(1-p)(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)).$$
$$= \frac{2-p}{4}\pi^A(A) + \frac{1-p}{4}\pi^N(A) + \frac{2-p}{4}\pi^N(M) + \frac{1-p}{4}\pi^A(M) + \frac{1}{2}\pi^M(M)$$

We define two other possible strategies to compare this with. The first is voting for M and ignoring the signal. The probability the vote matches the state in this case is:

$$\Pr(non\_piv) + \frac{1}{2}(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)) + \frac{1}{4}\pi^A(A) \tag{15}$$

The second is abstaining for both signals. The probability the vote matches the state in this case is:

$$\Pr(non\_piv) + \frac{1}{2}(\frac{1}{2}\pi^N(A) + \pi^A(A)) + \frac{1}{2}(\frac{1}{2}\pi^N(M) + \pi^M(M)). \tag{16}$$

The strategy we are considering is better than always voting M when equation 14 is larger than equation 15. Simplifying this relation gives us:

$$(1-p)(\pi^A(A) + \pi^N(A)) > p(\pi^N(M) + \pi^A(M)) \tag{17}$$

The strategy we are considering is better than always abstaining when equation 14 is larger than equation 16. Simplifying this relation gives us:

$$(1-p)(\pi^N(M) + \pi^A(M)) > p(\pi^A(A) + \pi^N(A)) \tag{18}$$

Adding these two conditions (equations 17 and 18) yields:

$$(1-p)(\pi^N(M)+\pi^A(M)+\pi^A(A)+\pi^N(A)) > p(\pi^N(M)+\pi^A(M)+\pi^A(A)+\pi^N(A))$$

Which can't be true as the pivotal probabilities are weakly positive and $1-p < p$. This implies that at least one of the strategies considered dominates the strategy of voting M in the case where the shareholder observes an a signal and abstaining when the shareholder observes an m signal.

Given symmetry, the case that the shareholder strategy of voting A when an m signal is observed and abstaining when an a signal is observed can be proved not to be optimal in an identical manner. ■

Lemma 6 states that strategies are weakly monotone: that is a shareholder who has is at least as likely to vote A when receiving the a-signal as when receiving the m-signal (and similarly for voting M). Following Proposition 2 of Persico (2004), the equilibrium then involves some shareholders voting A independently of their signal and the remaining voting sincerely. It remains to characterize, the number who do so. This is done in Lemma 7.

Lemma 7 *Suppose that a shareholders vote A and the rest vote sincerely then: (i) if $a < 2b$ then the probability of the vote matching the state is higher with $a+1$ voting against and the rest sincere than with a voting against and the rest sincere; and (ii) if $a > 2b+1$ then the probability of the vote matching the state is higher with $a-1$ voting against and the rest sincere than with a voting against and the rest sincere. (iii) $a = 2b$ or $2b+1$ give the same probability of the vote matching the state.*

Proof. (i) We write $R(a)$ to denote the probability that the decision matches the state when a shareholders vote A independently of their signals and the remainder vote sincerely, and $R(a, npiv)$ to denote the probability decision matches the state when the last shareholder among sincere voters is not pivotal when a shareholders vote A independently of their signals and the remainder vote sincerely.

Note that since no shareholders abstain, the only possibility of being pivotal requires that without the last shareholder, the votes for M and A are evenly split (that is, $\pi^M(A) = \pi^A(A) = \pi^M(M) = \pi^M(M) = 0$). Thus, we can write

$$R(a) = R(a, npiv) + \frac{1}{2}p\pi^N(A) + \frac{1}{2}p\pi^N(M).$$

Instead, switching the last shareholder to always vote for A independently of his signal implies that

$$R(a) = R(a, npiv) + \frac{1}{2}\pi^N(A).$$

It following that switching the last shareholder from voting sincerely to always voting A is beneficial if:

$$\frac{1-p}{p}\pi^N(A) \geq \pi^N(M). \tag{19}$$

Next we can write down the pivotal probabilities $\pi^N(A)$ and $\pi^N(M)$ for the case where a shareholders vote for A (and the blockholder uses its $2b$

shares to vote for M). Of the $2n-a$ shareholders who vote sincerely, it must be that $n+a-b$ vote for M for the vote to be tied without the last shareholder. This implies that $\pi^N(A) = \begin{pmatrix} 2n-a \\ n-b \end{pmatrix} (1-p)^{n-b} p^{n+b-a}$ and $\pi^N(M) = \begin{pmatrix} 2n-a \\ n-b \end{pmatrix} p^{n-b}(1-p)^{n+b-a}$. Substituting these expressions into (19) yields

$$p^{2b-a-1} \geq (1-p)^{2b-a-1};$$

since $p > 1-p$; this is holds as long as $2b-a-1 \geq 0$.

(ii) can be similarly established.

(iii) The condition that it is better to have $a-1$ voting sincere than a voting sincere iff $p^{2b-a-1} \geq (1-p)^{2b-a-1}$ holds also for the case of $a = 2b+1$ where this condition holds with equality, implying that the cases $a = 2b$ and $a = 2b+1$ are equivalent in terms of their probabilities of getting the decision right. ■

Finally, to complete the proof of Proposition 6, it remains to check that this is an equilibrium—since it is optimal in the relaxed problem (where incentive constraints do not have to be satisfied in Lemma 7.) it is clearly the optimal (most informative) equilibrium. This is immediate on noting that the conditions in the proof of Lemma 7 that ensure optimality, also ensure that no shareholder has a strict incentive to change their voting strategy. Clearly for any strategy of the shareholders, the biased blockholder optimizes by voting all his shares for M.■

7.7 Proof of Proposition 8 (Biased active blockholder)

Proposition 8 *In the game with a biased active blockholder, the equilibrium outlined in Proposition 7 is the most informative equilibrium.*

Proof. We proceed by describing the set of equilibria that may arise. It is clear that an optimal equilibrium aggregates information efficiently in the second stage, along the lines of the equilibrium of Proposition 6.

First, separating among blockholder types (either full or semi-separating) cannot arise as part of an equilibrium: Any action that is taken only by a blockholder who knows the state is A would lead shareholders to vote A independently of their type (or take other behaviors that guaranteed the outcome A) and so both types of blockholder would prefer to take another action; similarly, any action that is taken only by a blockholder who knows the state is M would lead shareholders to vote M independently of their type, and so would be mimicked by both types.[25]

Thus without loss, the most informative equilibrium involves the blockholder types pooling, and again following the Condorcet result, the best outcome among any such equilibria is the one described in Proposition 6. ■

[25] Assumption A2 restricts the blockholder to pure strategies; however, allowing mixed strategies here would not affect the result.

Consider, fully mixed equilibria, where both blockholder types mix between different actions. For this to arise in equilibria, it must be that the ultimate decision being M in state A is independent of the blockholder behavior. In the second period, this implies that the number of shareholders who vote A independently of their signal while the remainder vote sincerely, is identical irrespective of the blockholder behavior. However, it is clear that this is most efficient when the the number of the shareholders vote who A is 0 following the Condorcet result and so the outcome can be no better than in Proposition 6.