Information acquisition and disclosure in forecasting contests

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Abstract

We study price efficiency of analyst forecasts when competing analysts strategically acquire and disclose information in forecasting contests. In acquiring information, analysts face a tradeoff—while more precise information improves their forecast accuracy and the likelihood of winning the contest, it also increases their conditional signal correlation making differentiation to win the contest harder. In equilibrium, analysts cannot make truthful forecasts if contest rewards are sufficiently high. Increasing contest competitiveness, by scaling up contest rewards: generally, encourages information production, but can discourage information acquisition by the weaker analyst to the point of not acquiring information at all; and reduces price efficiency at higher reward levels. Two ex-ante identical analysts can become differently informed and use opposing forecasting strategies.

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1 Introduction

Analysts play a key role in promoting price efficiency in financial markets and are known to move market prices with their recommendation revisions and earnings forecasts (Givoly and Lakonishok, 1979; Imhoff and Lobo, 1984; Womack, 1996; Kelly and Ljungqvist, 2012). As information intermediaries, financial analysts privately learn about the firms they cover and then communicate this information to markets. Analysts' incentives to privately gather information are in part driven by their relative forecasting performance, e.g., the annual ranking of sell-side analysts for their relative value of research by portfolio managers organized by the well-known *Institutional Investor* magazine (Brown et al., 2015). There is a small but growing literature on analysts' forecasting behaviors when analysts' payoff are primarily based on their relative forecasting accuracy (Aharoni et al., 2017; Banerjee, 2021; Lichtendahl Jr et al., 2013; Ottaviani and Sørensen, 2006). This literature assumes that analysts' information endowment is exogenous and all analysts have the same quality of information.[1] But financial analysts spend a large amount of resources on gathering information about the companies they cover, and importantly, their information gathering exercise also impacts their strategies to communicate this information to the market.

In this paper, we analyze the price efficiency of analyst forecasts when competing analysts strategically gather and disclose information about the firms they cover. Price efficiency is the extent to which analyst forecasts reduce the uncertainty about the future cash flows (fundamental) of the firm they cover. We allow analysts to acquire information by paying a cost, which can vary across analysts. Analysts can thus be heterogenous in their information quality. Unlike previous models, analysts' signal precisions and the conditional correlation among their private signals are endogenous in our model.

We develop a model in which two strategic analysts cover a single firm and issue earnings

[1] Aharoni et al. (2017) is a notable exception for considering analysts with heterogenous information quality.

forecasts based on their noisy private signals about the firm performance. At the beginning, analysts simultaneously, and privately, decide the quality of their signals which they receive in the next period. Analysts have different information acquisition costs and thus their signal qualities can be different in equilibrium. After receiving their signals with the quality chosen at the earlier period, analysts simultaneously issue forecasts about the firm's earnings. Eventually, when the earnings is publicly disclosed, analysts are compensated based on their absolute and relative forecast accuracy (e.g., Aharoni et al., 2017; Banerjee, 2021). For each analyst, his private signal informs him about the distribution of the firm's future earnings and the competing analyst's private signal, because in equilibrium, their signals are correlated conditional on earnings. The extent to which the correlation is endogenously determined in equilibrium via each analyst's signal quality shapes their forecasting strategies and price efficiency.

Prior literature suggests that steeper contest rewards lead to stronger biases or less precise forecasts, since analysts compromise accuracy in order to increase the likelihood of winning a forecasting contest (e.g., Banerjee, 2021; Lichtendahl Jr et al., 2013). Importantly, this result has been shown to hold for exogenous precision levels of analysts' signals. However, when competing analysts can choose the precision of their signals—the case we consider here—steeper contest rewards also affect analysts' incentives to produce information.

To decide equilibrium information acquisition strategies, an analyst faces a tradeoff: on the one hand, greater contest rewards induces an analyst to acquire more precise signal so that he can accurately forecast the earnings and win the contest; on the other hand, more precise signals make analysts' conditional signal correlation higher making differentiation to win the contest harder. We find that, in equilibrium, analysts respond to steeper rewards by producing more information and issuing truthful forecasts only when contest rewards are relatively low. When contest rewards are sufficiently high, the stronger analyst (with a low information cost) acquires more precise information and issues a truthful forecast; the weaker analyst (with a high information cost) acquires less precise information and issues a forecast counter to his signal—he 'flips', resulting in a *flipping* forecasting equilibrium. At sufficiently high reward levels, we find equilibria in which analysts that are ex-ante identical—both have the same information cost—acquire information of different precision levels, and the analyst with more precise information issues a truthful forecast and the other analyst with less precise information flips.

We find that equilibrium information production generally increases with contest rewards, which is intuitive; however, higher reward levels decrease information production of the weaker analyst at some interval of reward levels. In a truth telling equilibrium, the optimal precision of the weaker analyst has an inverted U-shaped relationship with the reward level and at sufficiently high level of reward, he acquires no information at all. The intuition is that, for the weaker analyst, acquiring more precise information with the hope of winning the contest is not worth the cost of information after a certain reward level, leading to a decreasing relationship between his optimal precision and rewards.

The price efficiency of analyst forecasts increases with rewards at low reward levels, decreases with rewards at high reward levels, and remains constant with rewards at substantially high reward levels. At low reward levels, as rewards increase, both analysts acquire more precise information improving the price efficiency. At substantially high reward levels, the optimal precisions of the analysts are maximum (i.e., they know the state variable with certainty) and analysts acquire no more information—the price efficiency remains constant with rewards. The price efficiency decreases with rewards at high reward levels when, in a flipping equilibrium, the stronger analyst's optimal precision is maximum and thus the analyst no longer acquires any information, whereas the weaker analyst's optimal precision increases with rewards. When the market does not know analysts' information cost with certainty—as we assume—it cannot perfectly distinguish between a weak and a strong analyst. This makes the interpretation of analyst forecasts—the mapping of forecasts to analysts' private signals—noisy. At high reward levels, as the optimal precision of the weaker analyst increases, the price efficiency decreases because, given the market's noisy interpretation of the forecasts and the fixed precision of the stronger analyst, the higher the precision of the weaker analyst, the lower the difference in weights the market price places on the information of the stronger and the weaker analyst and the lesser the market's learning.

We find both substitutability and complementarity in information acquisition at different reward levels. The substitutability result is intuitive, because in a forecasting contest, analysts' key incentive is to differentiate, and it is well known that differentiation in action requires substitutability in information acquisition (Colombo et al., 2014; Hellwig and Veldkamp, 2009). The complementarity in information acquisition is a somewhat surprising result in a forecasting contest. The intuition is that in a flipping equilibrium, in which the complementarity effect occurs, has an endogenous mechanism of differentiation—the stronger analyst issues a truthful forecast and the weaker analyst flips. What is required for this endogenous differentiation to succeed is the coordination among analysts' forecasts—analysts coordinate their forecasts to differentiate. The complementarity of information acquisition helps in the coordination process.

We also find that with endogenous information, there exists no mixed strategy forecasting equilibrium as with exogenous information (Banerjee, 2021; Lichtendahl Jr et al., 2013). With the exogenous information, an analyst's equilibrium mixing strategy is such that the analyst's forecasting bias—the probability of forecasting counter to his signal—increases in his signal precision. This is intuitive because higher signal precision implies greater conditional correlation and thus stronger incentives to differentiate by forecasting counter to the signal. This forecasting strategy, however, diminishes an analyst's incentive to acquire information. If an analyst does acquire information, by paying a cost, it will be of little use, because the analyst will be induced to forecast counter to his signal voiding the effect of more precise information.

Our paper contributes to the literature on competitive analyst behavior in a forecasting contest setting. To begin with, Ottaviani and Sørensen (2006) and Lichtendahl Jr and Winkler (2007) show that forecasters who compete to be the most accurate have an incentive to report non-truthfully. For example, Lichtendahl Jr and Winkler (2007) shows that competitive forecasters increase their expected contest scores by exaggerating their probabilistic forecasts. In a continuous state setting, with similar contest incentives, Aharoni et al. (2017) show that the opposite might occur. Namely, when two risk averse analysts compete, the informed analyst might tilt his forecast away from his private signal and towards a commonly observed public signal in order to increase the likelihood of leading the contest; knowing that the uninformed analyst will rely more on this public signal. Lichtendahl Jr et al. (2013) and Banerjee (2021) show that contrarian forecasting behavior might be optimal when the correlation between analysts' signals, conditional on the state variable, is sufficiently high. We contribute to this literature by highlighting the important role of information production in shaping analysts' forecasting behavior. Specifically, by explicitly considering information production, we are able to analyze the aggregate level of information in markets, as determined by both the amount of information produced by analysts and the manner in which their private information is communicated to markets.

2 The model

A firm is covered by two competing analysts, A and B, $i \in \{A, B\}$. The firm's earnings e are stochastic and can be either high or low, $e \in \{1, 0\}$. The earnings depend on the firm's fundamental as captured by the random variable ϕ, which can be high or low, $\phi \in \{H, L\}$, with equal probabilities,

$$\Pr(\phi = H) = \Pr(\phi = L) = \frac{1}{2}. \tag{1}$$

The fundamental ϕ depends on a whole gamut of information including macro economic, geopolitical, and firm-specific information. Firm's earnings represent the firm's fundamental with precision θ,

$$\Pr(e = 1|H) = \Pr(e = 0|L) = \theta \in \left(\frac{1}{2}, 1\right). \tag{2}$$

Thus, for higher values of θ, firm's earnings are more indicative of firm's fundamental. Importantly, both analysts can exert effort to learn the firm's fundamental. Formally, an analyst can obtain a private signal $s_i \in S = \{h, l\}$ with precision γ_i at $t = 1$,

$$\Pr(s_i = h|H) = \Pr(s_i = l|L) = \gamma_i \ \in \left(\frac{1}{2}, 1\right) \tag{3}$$

by paying a private cost $C_i(\gamma_i)$ at $t = 0$,

$$C_i(\gamma_i) = \frac{\beta c_i}{2}\left(\gamma_i - \frac{1}{2}\right)^2 \quad \text{where } \beta, c_i > 0 \text{ for } i \in \{A, B\}. \tag{4}$$

Conditional on ϕ, $\langle e, s_A, s_B \rangle$ are independently distributed. Note, however, that the correlation between analysts' signals, conditional on earnings, is positive for any $\theta \in \left(\frac{1}{2}, 1\right)$.[2]

At time $t = 1$, after obtaining their private signals $\langle s_A, s_B \rangle$, analysts simultaneously issue forecasts $m_i \in M = \{1, 0\}$ about the firm's earnings e. An analyst's forecasting strategy σ_i is a probability distribution over all pure strategies mapping from S to M and is defined as $\sigma_i^h = \Pr(m_i = 1|s_i = h)$ and $\sigma_i^l = \Pr(m_i = 1|s_i = l)$.

Analysts are risk neutral. An analyst's forecasting payoff is $u_i(m_i, m_j, e)$, which depends on his own forecast, his competitor's forecast and the publicly disclosed earnings at $t = 2$. We consider a *winner-takes-all* forecasting contest in which an analyst wins a reward W when his

[2] Even though analysts' signals are independent conditional on ϕ, they are positively correlated conditional on e. When the earnings perfectly represent the fundamental, i.e., $\theta = 1$, analysts' signals are also independent conditional on earnings, and thus, the conditional correlation is zero. We will show later that the correlation is increasing in analysts' precisions $\langle \gamma_A, \gamma_B \rangle$. When both analysts perfectly learn the fundamental ϕ, i.e., when $\gamma_A = \gamma_B = 1$, the conditional correlation becomes one.

forecast matches the earnings but his competitor's forecast does not, that is, $m_i = e \neq m_j$. There are no prizes for any other realizations of forecasts or earnings. Formally, an analyst's forecast payoff is

$$u_i(m_i, m_j, e) = \begin{cases} W & \text{if } m_i = e \neq m_j \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Winner-takes-all forecasting contests represent a widely used forecasting tournament on Wall Street and have been used in prior research (Banerjee, 2021; Lichtendahl Jr et al., 2013; Ottaviani and Sørensen, 2006). Popular press and academic research suggests that one of the top two components of (sell-side) analysts' compensation is their annual "All-Star" rankings by *Institutional Investor* (Brown et al., 2015). Top ranked "All-Star" analysts receive substantially larger bonuses (Groysberg et al., 2011) and have superior career choices in industry (Leone and Wu, 2007).[3] Figure 1 shows the sequence of events of the game.

— **Figure 1 here** —

The solution concept we employ is Perfect Bayesian Nash Equilibrium, which is referred to as equilibrium in this paper. For each analyst $i \in \{A, B\}$, we use the notation σ_i for an analyst's strategy profile $\left(\sigma_i^h, \sigma_i^l\right)$ in the forecasting stage. An equilibrium is a set of precision choices $\left\{\gamma_i^*, \gamma_j^*\right\}$ at $t = 0$ and a set of forecasting strategies $\left\{\sigma_i^*, \sigma_j^*\right\}$ at $t = 1$ such that (i) analysts choose their signal precisions optimally, correctly anticipating their equilibrium strategies in the forecasting stage, (ii) analysts choose their forecasting strategies optimally given their signal precisions, and (iii) given the information acquisition and fore-

[3]Our results naturally extend to a more general payoff structure in which other contestants can also receive some rewards, albeit less than the winner. In such a payoff structure, each analyst receives a base reward if both analysts' forecasts match the earnings, but receives no reward when none of their forecasts matches the earnings. An analyst receives the highest reward when his forecast matches the earnings but his opponent's does not. The analyst whose forecast does not match the earnings but his opponent's does, receives the lowest reward, often a penalty.

casting strategies, all beliefs are consistent with the Bayes' rule. Next, we define informative and uninformative equilibria in the forecasting subgame.

Definition 1 *An uninformative equilibrium is defined as an equilibrium in which equilibrium strategies satisfy* $\sigma_i^h = \sigma_i^l$ *for each* $i \in \{A, B\}$*. An informative equilibrium is an equilibrium that is not uninformative.*

For all informative equilibria in the forecasting subgame, we focus on symmetric strategies by assuming

$$\sigma_i^h = 1 - \sigma_i^l \equiv \sigma_i, \tag{6}$$

that is, $\Pr(m_i = 1 | s_i = h) = \Pr(m_i = 0 | s_i = l)$, for each $i \in \{A, B\}$. This assumption is without loss of generality, because, as we show in Lemma C.1 (see appendix), all informative forecasting equilibria have strategies $\sigma_i^l = 1 - \sigma_i^h$ and $\sigma_j^l = 1 - \sigma_j^h$.

To derive the equilibrium outcome, we start by characterizing analysts' forecasting equilibria $\langle \sigma_A^*, \sigma_B^* \rangle$ at $t = 1$, taking their signal precision choice $\langle \gamma_A^*, \gamma_B^* \rangle$ at $t = 0$ as given. Having derived the forecasting equilibria at $t = 1$, we go back to $t = 0$ to obtain analysts' optimal precision choices.

3 Benchmark

To isolate the impact of forecasting competition, we first consider a payoff structure *without competition*. Each analyst's payoff is determined solely by his own forecast accuracy without any concern for relative performance with respect to his opponent. Specifically, each analyst receives a payoff $w_0 > 0$ if his forecast matches the earnings and a zero payoff if it does not match the earnings, regardless of whether his opponent's forecast matches the earnings or not.

Given signal precision γ_i, the unique equilibrium in the forecasting stage is a pure strategy truth telling equilibrium, in which each analyst's forecast perfectly reveals his private signal (i.e., $\sigma_i^{h*} = 1$). Working backward, for a truth telling forecasting equilibrium, an analyst's ex-ante expected benefit of acquiring a signal with precision γ_i is given by

$$B_i(\gamma_i) = \mathbb{E}_{s_i}\left[\mathbb{E}_e\left[u_i(m_i, e)\,|s_i\right]\right] = \frac{w_0}{2}\left[1 + (2\theta - 1)(2\gamma_i - 1)\right]. \tag{7}$$

This leads to the net ex-ante expected payoff, taking into account the cost of information production,

$$V_i(\gamma_i) = \frac{w_0}{2}\left[1 + (2\theta - 1)(2\gamma_i - 1)\right] - \frac{\beta c_i}{2}\left(\gamma_i - \frac{1}{2}\right)^2. \tag{8}$$

As can be seen from the above expression of $V_i(\gamma_i)$, an analyst's incentive to produce more information by improving his signal precision γ_i increases in his payoff from forecast accuracy w_0 and earnings quality θ (i.e., $\Pr(e = 1|\phi = H) = \Pr(e = 0|\phi = L)$), and decreases in the cost of information production, given by the product βc_i. Proposition 1 summarizes the unique equilibrium. All proofs are in Appendix C.

Proposition 1 *Suppose each analyst receives a payoff of $w_0 > 0$ if $m_i = e$ and a zero payoff if $m_i \neq e$, without any concern for relative ranking. Then, for each analyst $i \in \{A, B\}$, there exists a unique information acquisition equilibrium with optimal precision γ_i^* at $t = 0$ and a unique truth telling forecasting equilibrium with strategy $\sigma_i^* = 1$ at $t = 1$ such that,*

$$\gamma_i^* = \begin{cases} \frac{1}{2}\left[1 + \frac{2w_0(2\theta-1)}{\beta c_i}\right] \in \left(\frac{1}{2}, 1\right) & \textit{for } \beta c_i > 2w_0(2\theta - 1) \\ 1 & \textit{otherwise} \end{cases} \tag{9}$$

In this benchmark case, an analyst's optimal information production is increasing in his payoff from forecast accuracy w_0 and earnings quality θ w_0, and is decreasing in the cost of information production βc_i.

4 Winner-takes-all contest

In this section, we consider our main model in which analysts compete in a winner-takes-all forecasting contest. We solve the model using backward induction. We start at t=1, taking as given the signal precision choices made by the analysts at t=0. After characterizing the equilibrium outcome at the forecasting subgame, we work backwards to t=0 to determine the analysts' optimal information choices.

4.1 Forecasting subgame

For any given set of signal precisions $\langle \gamma_A, \gamma_B \rangle$ and his private signal $s_i \in \{h, l\}$, each analyst decides his optimal forecast. An analyst can either issue a forecast that is consistent with his private signal (i.e., $m_i = 1$ when $s_i = h$ or $m_i = 0$ when $s_i = l$) or counter to his signal (i.e., $m_i = 0$ when $s_i = h$ or $m_i = 1$ when $s_i = l$) or randomize between these forecasts (i.e., $\sigma_i \in (0, 1)$). Suppose an analyst's receives a high signal, $s_i = h$. His likelihood of winning the contest with a consistent forecast, $m_i = 1$, is,

$$\Pr(e = 1, m_j = 0 | s_i = h) = \frac{\Pr(e = 1, m_j = 0, s_i = h)}{\Pr(s_i = h)}.$$

Analyst i's winning likelihood depends not only on the likelihood that his forecast matches the earnings but also on the likelihood that the competing analyst j's forecast does not match the earnings, which is $(1 - \gamma_j)\sigma_j$ if analyst j obtains the wrong signal and issues a consistent forecast or $\gamma_j(1-\sigma_j)$ if analyst j obtains a correct signal and issues an inconsistent forecast. We thus denote the latter likelihood as,

$$z_j \equiv \gamma_j + \sigma_j - 2\sigma_j\gamma_j, \text{ for } j = A, B.$$

It follows then that an analyst's likelihood of winning the contest by forecasting truthfully is

$$\Pr\left(e=1, m_j=0 | s_i=h\right)=\theta\gamma_i+\left(1-z_j\right)\left(1-\gamma_i-\theta\right).$$

Similarly, the likelihood of winning when issuing an inconsistent forecast is

$$\Pr\left(e=0, m_j=1 | s_i=h\right)=(1-\theta)\gamma_i-z_j(\gamma_i-\theta)$$

Comparing the two likelihoods implies that issuing a truthful forecast dominates (i.e., $\sigma_i=1$) when

$$\theta\gamma_i+\left(1-z_j\right)\left(1-\theta-\gamma_i\right)>(1-\theta)\gamma_i-z_j(\gamma_i-\theta),$$

or $z_j>1-\theta$, which further implies that

$$\sigma_j<\frac{\theta+\gamma_j-1}{2\gamma_j-1}.$$

Lemma 1 summarizes analyst i's best response strategy σ_i as a function of analyst j's strategy σ_j. Figure 2 shows plots of analysts' best response functions for different intervals of θ.

— **Figure 2 here** —

Lemma 1 ***[Forecasting response functions]*** *For each $i, j \in \{A, B\}$, $i \neq j$, analyst i's best response strategy σ_i to analyst j's strategy σ_j is given by*

$$\sigma_i=\begin{cases}1 & \text{for all } \sigma_j<\frac{\theta+\gamma_j-1}{2\gamma_j-1}\\ (0,1) & \text{for all } \sigma_j=\frac{\theta+\gamma_j-1}{2\gamma_j-1}\\ 0 & \text{for all } \sigma_j>\frac{\theta+\gamma_j-1}{2\gamma_j-1}\end{cases}. \qquad (10)$$

Now, with the above response functions, we explore conditions for the existence of different forecasting equilibria. But before we proceed, we discuss analysts' signal correlation conditional on earnings, because correlation will play an important role in our understanding of analysts' strategic behavior in forecasting as well as information acquisition. For any $i, j \in \{A, B\}, i \neq j$, let ρ be analysts' signal correlation conditional on earnings:

$$\rho \equiv Corr(s_i, s_j|e) = \frac{Cov(s_i, s_j|e)}{\sqrt{Var(s_i|e)Var(s_j|e)}}. \tag{11}$$

Replacing the values of conditional covariance and conditional variances in the definition (see derivation in appendix B.1),

$$\rho = \frac{\theta\,(1-\theta)\,(2\gamma_i - 1)\,(2\gamma_j - 1)}{\sqrt{[1-\theta+\gamma_i\,(2\theta-1)]\,[\theta-\gamma_i\,(2\theta-1)]\,[1-\theta+\gamma_j\,(2\theta-1)]\,[\theta-\gamma_j\,(2\theta-1)]}}. \tag{12}$$

Conditional signal correlation ρ in signal precisions γ_i, γ_j. With endogenous precisions, the conditional correlation is determined by analysts' equilibrium behavior in the information production stage and in the forecasting subgame. At one extreme, when one or more analysts have no information about the firm's fundamental ϕ, their signals are not correlated at all, i.e., if either $\gamma_i \to \frac{1}{2}$ or $\gamma_j \to \frac{1}{2}$, $\rho \to 0$. At another extreme, when analysts have perfect information about the firm's fundamental ϕ, signals are perfectly correlated, i.e., $\gamma_i \to 1$, $\gamma_j \to 1$, $\rho \to 1$. Further, signal correlation ρ decreases in θ. As θ increases, the variance of the common component of the signals, i.e., $Var\,(e|\phi) = \theta\,(1-\theta)$ decreases and so does ρ. When earnings perfectly represent the firm's fundamental, signals are then independent conditional on earnings (because they are conditional on fundamental), i.e., at $\theta \to 1$, $\rho \to 0$. On the other hand, when earnings are completely uninformative about the firm's fundamental, correlation naturally does not depend on θ, i.e., at $\theta \to \frac{1}{2}$, $\rho \to (2\gamma_i - 1)\,(2\gamma_j - 1)$.

Next we characterize conditions for the existence of forecasting equilibria, starting with a truth telling equilibrium. Specifically, for both analysts to issue truthful forecasts, it is required that their precisions are sufficiently small,

$$\sigma_A^* = \sigma_B^* = 1 \Leftrightarrow \max(\gamma_A, \gamma_B) < \theta. \tag{13}$$

Once the precision level of one of the analysts exceeds this threshold, e.g., $\gamma_A > \theta$, where $\gamma_A \geqslant \gamma_B$ (w.l.o.g), his opponent's best response to his truthful forecast is to issue a contrarian forecast and thus, the truth telling forecasting equilibrium no longer exists. Instead, there exists a second pure strategy equilibrium, in which the analyst with the higher signal precision issues a truthful forecast while his opponent with the lower precision issues a contrarian forecast. We refer to this equilibrium outcome as a *flipping* equilibrium,

$$\sigma_A^* = 1,\, \sigma_B^* = 0 \;\Leftrightarrow \max(\gamma_A, \gamma_B) > \theta. \tag{14}$$

Besides the pure strategy equilibria there is also a mixed strategy equilibrium when analysts are indifferent between reporting a truthful forecast and flipping. Specifically, it follows from Lemma 1, for a given set of precisions $\langle\gamma_A, \gamma_B\rangle$, analyst A's best response to σ_B is $\sigma_A \in (0,1)$ when $\sigma_B = \frac{\theta+\gamma_B-1}{2\gamma_B-1}$, which precisely defines the equilibrium mixed strategy σ_B. Similarly, analyst B's best response to σ_A is $\sigma_B \in (0,1)$ when $\sigma_A = \frac{\theta+\gamma_A-1}{2\gamma_A-1}$, which defines the equilibrium mixed strategy σ_A. Both conditions taken together leads to a unique mixed strategy equilibrium. Proposition 2 characterizes the equilibria of the forecasting subgame for a given set of signal precisions $\langle\gamma_A, \gamma_B\rangle$.[4]

Proposition 2 ***[Forecasting equilibria]*** *For any set of signal precisions* $\langle\gamma_A, \gamma_B\rangle$,[5]

[4] The forecasting subgame analyzed here is reminiscent of Banerjee (2021), but extends the analysis to the case of heterogenous signal precisions.

[5] In addition, there are two trivial uninformative pure strategy equilibria with strategies $\left(\sigma_A^{h*} = 1 = \sigma_A^{l*}; \sigma_B^{h*} = 0 = \sigma_A^{l*}\right)$ and $\left(\sigma_A^{h*} = 0 = \sigma_A^{l*}; \sigma_B^{h*} = 1 = \sigma_A^{l*}\right)$ for any $\theta \in \left(\frac{1}{2}, 1\right)$. In these equilibria, one analyst always forecasts high and his opponent always forecasts low, regardless of their private signals. Furthermore, there also exists a flipping equilibrium in which it is the analyst with lower precision that issues a truthful forecast whereas the analyst with a higher precision flips. For now, we disregard these inefficient subgame outcomes.

i) If $\max(\gamma_A, \gamma_B) \leqslant \theta$, *the unique equilibrium is truth telling with strategies* $(\sigma_A^* = 1 = \sigma_B^*)$;

ii) If $\min(\gamma_A, \gamma_B) < \theta < \max(\gamma_A, \gamma_B)$, *there exists a flipping equilibrium with strategies* $(\sigma_A^* = 1, \sigma_B^* = 0)$;

iii) If $\min(\gamma_A, \gamma_B) \geqslant \theta$, *there are multiple equilibria—a flipping equilibrium with strategies* $(\sigma_A^* = 1, \sigma_B^* = 0)$ *and a mixed-strategy equilibrium with strategies,*

$$\sigma_A^* = \frac{\theta + \gamma_A - 1}{2\gamma_A - 1},\ \sigma_B^* = \frac{\theta + \gamma_B - 1}{2\gamma_B - 1}. \tag{15}$$

The intuition is that when analysts' signal precisions are low, given a fixed θ, their signal correlation, conditional on earnings, is also low. For a small signal correlation, analysts are aware that their signals are not very likely to be same and thus they have enough room to differentiate by truthfully reporting their signals, leading to the existence of a truth telling equilibrium. However, when at least one of the signal precisions is high, correlation becomes larger, leaving little room for the analysts to differentiate by truthful reporting. While analysts cannot report their signals truthfully, they are able to differentiate in a way such that the strong analyst (the one with a high signal precision) reports truthfully and the weak analyst (the one with a low signal precision) 'flips' that is, issues a forecast counter to his signal, resulting in a flipping equilibrium. The strong analyst has a higher cost of deviating from his own signal and thus reports truthfully. The weak analyst has a lower cost of deviating and thus he is the one that flips.

The intuition of the mixed strategy equilibrium is that when analysts' signal precisions are sufficiently high, given a fixed θ, their signal correlation is also very high, analysts' best bet to differentiate is to mix their pure strategy reports, resulting in a mixed strategy equilibrium. Naturally, analysts tend to bias their forecasts more as their signal precisions increase. That is, for every $i \in \{A, B\}$, σ_i^* decreases in γ_i, or the *bias* away from one's own signal, $1-\sigma_i^* = \Pr(m_i = 0|s_i = h) = \Pr(m_i = 1|s_i = l)$, increases in γ_i. At very high precisions, there are multiple equilibria: the flipping equilibrium as well as the mixed strategy equilibrium.

4.2 Information production

Building on Proposition 2, here we analyze analysts' optimal information production or the choice of precisions $\langle \gamma_A, \gamma_B \rangle$ to yield the equilibrium level of information production. Specifically, at $t = 0$, analysts simultaneously choose their signal precisions, each based on their conjectured behavior of their opponent and assuming that each analyst acts optimally in the forecasting subgame at $t = 1$.[6] The best response of analyst i to the precision level γ_j of analyst j, for each $i, j \in \{A, B\}$, $i \neq j$, is given by,

$$\gamma_i(\gamma_j) \in \arg\max B_i(\gamma_i, \gamma_j) - C_i(\gamma_i), \tag{16}$$

where analyst i's ex-ante expected payoff (benefit) is given by

$$B_i(\gamma_i, \gamma_j) = \mathbb{E}_{s_i}\left[\mathbb{E}_{e,m_j}[u_i(m_i(\sigma_i^*(\gamma_i, \gamma_j)), m_j\left(\sigma_j^*(\gamma_i, \gamma_j)\right), e)|s_i]\right], \tag{17}$$

taking equilibrium forecasting strategies $\left\langle \sigma_i^*(.), \sigma_j^*(.) \right\rangle$ given.

Without loss of generality we express our next set of results with the convention that analyst A is the more efficient in information production than analyst B such that $c_A \leqslant c_B$. We call analyst A interchangeably the efficient or the strong analyst, and analyst B the inefficient or the weak analyst.

[6]Results do not change whether we assume that analysts choose their precisions privately or we allow analysts' signal precisions to be common knowledge among the analysts. This is because analysts' equilibrium forecasting strategies do not depend on their opponents' signal precisions. As such, to save clutter, we use γ_j (without a 'hat') and not $\hat{\gamma}_j$ which is analyst i's conjectured precision of analyst j's chosen precision.

4.2.1 Truth telling forecasting equilibrium

We begin by deriving the conditions for an equilibrium with truthful forecasts. From Proposition 2 part (i), this requires that both analysts optimally choose information production levels such that $\gamma_A \leq \theta$, $\gamma_B \leq \theta$. Specifically, in a truth telling equilibrium, the optimal level of information production for analyst A is given by (see derivation in (B.15)),

$$\gamma_A(\gamma_B) = \arg\max W\left[\theta\gamma_A + \gamma_B(1-\theta-\gamma_A)\right] - \frac{\beta c_A}{2}\left(\gamma_A - \frac{1}{2}\right)^2. \tag{18}$$

Thus, analyst A's best response precision level to analyst B's precision level γ_B is

$$\gamma_A(\gamma_B) = \frac{1}{2} + \frac{W}{\beta c_A}\left[\theta - \gamma_B\right] \text{ for } \gamma_B \leq \theta. \tag{19}$$

Note that analysts respond to their opponents' higher information production by lower information production. In other words, the incentive to produce information is decreasing in the level of information production chosen by the opponent analyst—there is *substitutability* in information production. This is intuitive, because in a winner-takes-all contest, analysts' incentives to differentiate lead to substitutability of their forecasting behaviors (Colombo et al., 2014; Hellwig and Veldkamp, 2009). Taking together the best response precision level of analyst B, given analyst A's precision, we obtain the unique equilibrium precisions for a truth telling forecasting equilibrium summarized in Proposition 3. Furthermore, the feasibility of the solution, i.e., precisions satisfy $\max(\gamma_A^*, \gamma_B^*) \leq \theta$, implies that the incentive to produce information is not too high, which requires that either the cost of information production is sufficiently high or the contest reward is sufficiently low. Formally, the feasibility condition is

$$W \leqslant \beta \min(c_A, c_B). \tag{20}$$

Proposition 3 characterizes optimal signal precisions for a truth telling forecasting equilibrium. Figure 3 shows different forecasting equilibria with endogenous information in the W-θ space.

— **Figure 3 here** —

Proposition 3 ***[Optimal precisions in a truth telling forecasting equilibrium]*** *For costs $c_A \leqslant c_B$ (w.l.o.g) and the reward level $W \leqslant \beta c_A$, there exists a truth telling forecasting equilibrium $\langle \sigma_A^*, \sigma_B^* \rangle = \langle 1, 1 \rangle$ with the following optimal precision levels $\left\langle \gamma_{A,TT}^*, \gamma_{B,TT}^* \right\rangle$:*

$$\gamma_{A,TT}^* = \frac{1}{2}\left[1 + \frac{W(\beta c_B - W)}{\beta^2 c_A c_B - W^2}(2\theta - 1)\right];\ \gamma_{B,TT}^* = \frac{1}{2}\left[1 + \frac{W(\beta c_A - W)}{\beta^2 c_A c_B - W^2}(2\theta - 1)\right]. \quad (21)$$

Unlike in the benchmark case without competition, a truth telling forecasting equilibrium exists only for relatively small reward levels, and optimal precisions need not be monotonically increasing in the reward. In equilibrium, at first both analysts produce information as the reward level W increases, but after a certain threshold, the efficient (inefficient) analyst responds by producing more (less) information. Eventually, when the reward level reaches $W = \beta c_A$, the inefficient analyst produces no information, i.e., $\gamma_B = \frac{1}{2}$, and the efficient analyst produces $\gamma_A = \theta$. We summarize these properties of optimal precisions in Lemma 2. Figure 4(a) shows plots of optimal precisions as functions of reward levels for truth telling forecasting equilibrium.

Lemma 2 *For costs $c_A \leqslant c_B$ (w.l.o.g.), and optimal precisions defined in* (21),

i) The efficient analyst's optimal precision $\gamma_{A,TT}^*$ increases in the reward level W for all $W \in (0, \beta c_A)$;

ii) The inefficient analyst's optimal precision $\gamma_{B,TT}^*$ has an inverted U-shaped relationship with the reward level W. Formally, there exists a reward threshold $W_{TT} \in (0, \beta c_A)$ such *that* $\frac{d\gamma^*_{B,TT}}{dW} > 0$ *for* $W \in (0, W_{TT})$ *and* $\frac{d\gamma^*_{B,TT}}{dW} < 0$ *for* $W \in [W_{TT}, \beta c_A)$, *where*

$$W_{TT} = \beta \left[c_B - \sqrt{c_B \left(c_B - c_A \right)} \right] \tag{22}$$

4.2.2 Flipping forecasting equilibrium

We continue by deriving equilibrium information production at reward levels for which a truth telling forecasting equilibrium no longer exists. This requires that at least one of the analysts optimally chooses a sufficiently high signal precision, i.e., $\max(\gamma_A, \gamma_B) > \theta$. We begin by deriving the conditions for analysts' best response precision levels for the flipping equilibrium $(\sigma^*_A = 1, \sigma^*_B = 0)$ consistent with the above convention that $c_A \leq c_B$. This implies that $z_A = 1 - \gamma_A$ and $z_B = \gamma_B$ and that the optimal levels of information production are given by (see derivation in (B.16))

$$\gamma_A(\gamma_B) = \arg\max W \left[\gamma_A \gamma_B \theta + \left(1 - \gamma_A \right) \left(1 - \gamma_B \right) \left(1 - \theta \right) \right] - \frac{\beta c_A}{2} \left(\gamma_A - \frac{1}{2} \right)^2 .$$
$$\gamma_B(\gamma_A) = \arg\max W \left[\left(1 - \gamma_A \right) \left(1 - \gamma_B \right) \theta + \gamma_A \gamma_B \left(1 - \theta \right)) \right] - \frac{\beta c_B}{2} \left(\gamma_B - \frac{1}{2} \right)^2 .$$

Thus, the best response precision levels of the analysts as a function of their competitors' precision levels are:

$$\gamma_A(\gamma_B) = \min \left(\frac{1}{2} + \frac{W}{\beta c_A} \left[\theta + \gamma_B - 1 \right], 1 \right) \text{ and } \gamma_B(\gamma_A) = \min \left(\frac{1}{2} + \frac{W}{\beta c_B} \left[\gamma_A - \theta \right], 1 \right). \tag{23}$$

Unlike in the truth telling forecasting equilibrium, analysts respond to competitors' high information production by producing more information. Analysts' incentive to produce information is increasing in their opponents' information production—there is *complementarity* in information production. The complementarity result is somewhat surprising given that in a winner-takes-all contest, analysts' objective is to differentiate themselves and substitutability in action generally leads to substitutability in information production (Colombo et al., 2014; Hellwig and Veldkamp, 2009, e.g.,). The intuition here is that a flipping equilibrium has an endogenous mechanism of differentiation—the stronger analyst with the higher signal precision truthfully reveals his private signal, whereas the weaker analyst with a lower signal precision flips. Paradoxically, what is required for this endogenous differentiation to succeed is the coordination among analysts' forecasts, for which the complementarity of information acquisition is optimal.

Whether the consequent equilibrium precision levels are interior depends on the incentive to produce information. Specifically, this is determined in part by the reward cutoff W_{FP} defined below (defined for $c_A \leqslant c_B$),

$$W_{FP} = \left[\frac{\beta c_B}{4(1-\theta)}\right]\left[\left((2\theta-1)^2 + 8(1-\theta)(c_A/c_B)\right)^{\frac{1}{2}} - (2\theta-1)\right] \in \left(\beta c_A, \beta\sqrt{c_A c_B}\right). \quad (24)$$

Proposition 4 ***[Optimal precisions in a flipping forecasting equilibrium]*** *For costs $c_A \leqslant c_B$ (w.l.o.g.) and W_{FP} as defined in* (24)*, there exists a flipping forecasting equilibrium $\langle \sigma_A^*, \sigma_B^* \rangle = \langle 1, 0 \rangle$ with the following optimal precision levels $\left\langle \gamma_{A,FP}^*, \gamma_{B,FP}^* \right\rangle$:*

i) For $W \in (\beta c_A, W_{FP})$, precision levels are given by (25a)*, which satisfy $\frac{1}{2} < \gamma_{B,FP}^* < \gamma_{A,FP}^* < 1$;*

$$\gamma_{A,FP}^* = \frac{1}{2}\left[1 + \frac{W(\beta c_B - W)}{\beta^2 c_A c_B - W^2}(2\theta-1)\right], \gamma_{B,FP}^* = \frac{1}{2}\left[1 + \frac{W(W - \beta c_A)}{\beta^2 c_A c_B - W^2}(2\theta-1)\right]; \quad (25a)$$

ii) For $W \in \left[W_{FP}, \frac{\beta c_B}{2(1-\theta)}\right)$, precision levels are given by (25b)*, which satisfy $\frac{1}{2} < \gamma_{B,FP}^* < \gamma_{A,FP}^* = 1$;*

$$\gamma_{A,FP}^* = 1, \gamma_{B,FP}^* = \frac{1}{2}\left[1 + \frac{2(1-\theta)W}{\beta c_B}\right]; \quad (25b)$$

iii) For $W \geqslant \frac{\beta c_B}{2(1-\theta)}$, precision levels are $\gamma_{B,FP}^* = \gamma_{A,FP}^* = 1$.

In the flipping forecasting equilibrium, both analysts produce more information as the reward level increases, although the marginal rate of increase is higher for the efficient analyst, that is, $\frac{\partial \gamma^*_{A,FP}}{\partial W} > \frac{\partial \gamma^*_{B,FP}}{\partial W}$ at $W \in (\beta c_A, W_{FP})$. Figure 4(b) shows plots of optimal precisions as functions of reward levels for truth telling as well as flipping forecasting equilibria. Plots at $W \in (0, \beta c_A)$ represent optimal precisions for the truth telling equilibrium and plots at $W \geqslant \beta c_A$ represent optimal precisions for the flipping equilibrium.

— Figure 4 here —

4.2.3 Symmetric cost

Here we discuss forecasting equilibria with endogenous information when analysts have the same information acquisition cost, i.e., $c_A = c_B = c$. Corollary 1 characterizes analysts' equilibrium forecasting behavior with endogenous information. While the equilibria types are similar to those with asymmetric costs discussed in earlier sections, truth telling forecasting equilibrium exists across all reward levels. Importantly, part (ii)(a) of the corollary shows that at the intermediate levels of reward, there exists an equilibrium in which analysts with the *symmetric* information acquisition cost acquire *asymmetric* levels of information, and the analyst with higher signal precision reports truthfully while the other analyst with lower precision flips. Figure 5 shows plots of analysts' best response precision levels to their opponents' precision level. There are multiple equilibria. While in the truth telling forecasting equilibrium, optimal precisions are the same, in the flipping forecasting equilibrium, optimal precisions are different.

— Figure 5 here —

Corollary 1 ***[Optimal precisions with symmetric cost]*** *When analysts have the same information acquisition cost c, then:*

i) For any $W > 0$, there exists a truth telling forecasting equilibrium $\langle \sigma_A^*, \sigma_B^* \rangle = \langle 1, 1 \rangle$ with the following optimal precision levels $\left\langle \gamma_{A,TT}^*, \gamma_{B,TT}^* \right\rangle$:

$$\gamma_{A,TT}^* = \gamma_{B,TT}^* = \frac{1}{2}\left[1 + \frac{W\,(2\theta - 1)}{W + \beta c}\right] \in \left(\frac{1}{2}, \theta\right); \tag{26}$$

ii) For any $W > \beta c$, there exists a flipping forecasting equilibrium $\langle \sigma_A^*, \sigma_B^* \rangle = \langle 1, 0 \rangle$ with the following optimal precision levels $\left\langle \gamma_{A,TT}^*, \gamma_{B,TT}^* \right\rangle$:

a) if $W \in \left(\beta c, \frac{\beta c}{2(1-\theta)}\right)$, precision levels are given by

$$\gamma_{A,FP}^* = 1; \quad \gamma_{B,FP}^* = \frac{1}{2}\left[1 + \frac{2W\,(1 - \theta)}{W + \beta c}\right] \in \left(\frac{1}{2}, 1\right); \tag{27}$$

b) if $W \geqslant \frac{\beta c}{2(1-\theta)}$, precision levels are: $\gamma_{A,FP}^* = 1 = \gamma_{B,FP}^*$.

4.2.4 Noisy forecasting

Here we show that an information production equilibrium does not exist with mixed forecasting strategies. The mixed strategy equilibrium $\langle \sigma_A^*, \sigma_B^* \rangle = \left\langle \frac{\theta+\gamma_A-1}{2\gamma_A-1}, \frac{\theta+\gamma_B-1}{2\gamma_B-1} \right\rangle$ is derived such that the expected payoffs from issuing a truthful forecast equals the expected payoffs from issuing a false forecast, or $z_A = z_B = 1 - \theta$. It follows from the above that the ex-ante expected payoff (benefit) from this strategy for analyst $i \in \{A, B\}$ is,

$$B_i = W\left[\theta\gamma_i + \theta(1 - \theta - \gamma_i)\right] = \theta\,(1 - \theta)\,W,$$

which does not depend on analyst i's signal precision γ_i. Thus, analyst i has no incentive to produce information at level $\gamma_i > \frac{1}{2}$ and the feasibility condition for the mixed strategy equilibrium to exist, i.e., $\gamma_i > \theta$, is not satisfied.

Proposition 5 ***[Non-existence of noisy forecasting equilibrium]*** *There exists no mixed strategy forecasting equilibrium with endogenous information.*

5 Price efficiency

In this section we study the price efficiency of analyst forecasts. To measure price efficiency, we adopt the commonly used expected squared deviation between the market price and the firm's fundamental. We define price efficiency of analyst forecasts as the inverse of the expected squared deviation between the market price and the fundamental, i.e., $\left[\mathbb{E}_{m_i,m_j,e}\left[(\phi - P(m_i,m_j,e))^2\right]\right]^{-1}$, normalized by the inverse of the expected squared deviation between the market price without analyst forecasts and the fundamental, i.e., $\left[\mathbb{E}_{e}\left[(\phi - P(e))^2\right]\right]^{-1}$, that is,

$$\Pi \equiv \frac{\mathbb{E}_{e}\left[(\phi - P(e))^2\right]}{\mathbb{E}_{m_i,m_j,e}\left[(\phi - P(m_i,m_j,e))^2\right]}.$$

Assuming a risk neutral market, the price with and without analyst forecasts are $P(m_i,m_j,e) = \mathbb{E}_{m_i,m_j,e}[\phi|m_i,m_j,e]$ and $P(e) = \mathbb{E}_e[\phi|e]$, the price efficiency of analyst forecasts can be expressed as[7]

$$\Pi = \frac{\mathbb{E}_e\left[Var(\phi|e)\right]}{\mathbb{E}_{m_i,m_j,e}\left[Var(\phi|m_i,m_j,e)\right]}. \tag{28}$$

Higher values of Π correspond to lower variance ratios which mean that analyst forecasts are more informative about the firm's fundamental. At an extreme, when analyst forecasts are completely uninformative of the fundamental, the price efficiency equals one.

Expanding (28), for any $\alpha,\beta,\delta \in \{1,0\}$, the price efficiency can be expressed as (see

[7] Our results remain the same if we measure price (information) efficiency with respect to earnings e, i.e.,

$$\Pi = \frac{Var(e)}{\mathbb{E}_{m_i,m_j}\left[Var(e|m_i,m_j)\right]}.$$

derivation is appendix B.4)

$$\Pi = \left[\sum_{\alpha,\beta,\delta \in \{1,0\}} \left[\frac{2\theta\left(1-\theta\right)}{\frac{1}{\Pr(m_i=\alpha,m_j=\beta,e=\delta|\phi=H)} + \frac{1}{\Pr(m_i=\alpha,m_j=\beta,e=\delta|\phi=L)}} \right] \right]^{-1}, \tag{29}$$

where, $\Pr\left(m_i = \alpha, m_j = \beta, e = \delta | \phi = \zeta\right) = \Pr\left(m_i = \alpha, m_j = \beta | \phi = \zeta\right) \Pr\left(e = \delta | \phi = \zeta\right)$ for any $\zeta \in \{H, L\}$.

As we can see, to derive the value of price efficiency, we need the values of $\Pr\left(m_i = \alpha, m_j = \beta | \phi = \zeta\right)$, which requires the knowledge of how the market interprets analyst forecasts, i.e., how the market maps forecasts to analysts' signals: $(m_i, m_j) \longmapsto (s_i, s_j)$. The market's interpretation depends on its belief about whether an analyst is efficient or inefficient in acquiring information. As discussed earlier, analysts can be of two types: "efficient" (E) with a low cost of information acquisition c_E or "inefficient" (I) with a high cost of information acquisition c_I such that $c_E < c_I$. The market does not know analysts' information cost for certain—it knows only the probability distribution of information acquisition costs. We assume that while the market may not know perfectly identify an efficient (inefficient) analyst, analysts themselves know each others' information cost. Analysts work in the same industry and have a better understanding of each other's information cost than the market.[8] We state this assumption formally below.

Assumption 1 ***[Market's interpretation of analyst forecasts]*** *Analysts know each other's information acquisition cost, but the market does not know these costs. The market believes that analyst A is an efficient analyst and analyst B is an inefficient analyst with a probability p, that is,*

$$\Pr\left(c_A = c_E, c_B = c_I\right) = 1 - \Pr\left(c_A = c_I, c_B = c_E\right) = p \in [\frac{1}{2}, 1),$$

[8] Alternatively, we could have assumed that analysts know their opponents' information cost only partially or not at all. While that assumption is more general, it would complicate our analysis without contributing additional insights.

and $\Pr(c_A = c_E = c_B) = \Pr(c_A = c_I = c_B) = 0$.

At one extreme, when $p = 1$, the market perfectly knows analyst types (analyst A is efficient and analyst B is inefficient, which we have assumed so far in our discussion until this section. At the other extreme, when $p = \frac{1}{2}$, the market does not have any information about analyst types, i.e., both analysts have equal likelihood of being an efficient analyst. We denote the precision of an efficient analyst as γ_E and the precision of an inefficient analyst as γ_I.

To see how the market's interpretation of analyst forecasts affect price efficiency, consider forecasts: $(m_A = 1, m_B = 0)$. In a flipping forecasting equilibrium, the market maps forecasts to analyst signals such that, for any $\phi \in \{H, L\}$,

$$\Pr(m_A = 1, m_B = 0|\phi) = p \Pr(s_{A=E} = h, s_{B=I} = h|\phi) + (1-p)\Pr(s_{A=I} = l, s_{B=E} = l|\phi).$$

The forecasts are interpreted as $(s_A = h, s_B = h)$ with probability p (when analyst A is an efficient type and analyst B is an inefficient type) or as $(s_A = l, s_B = l)$ with probability $1 - p$ (when analyst A is an inefficient type and analyst B is an efficient type). This is because in a flipping forecasting equilibrium, an efficient type makes a forecast that truthfully reveals his private signal, whereas the inefficient type makes a forecast counter to his private signal. If, for example, $\phi = H$, $\Pr(s_{A=E} = h, s_{B=I} = h|\phi = H) = \gamma_E\gamma_I$ and $\Pr(s_{A=I} = l, s_{B=E} = l|\phi = H) = (1-\gamma_I)(1-\gamma_E)$, then

$$\Pr(m_A = 1, m_B = 0|\phi = H) = p\gamma_E\gamma_I + (1-p)(1-\gamma_E)(1-\gamma_I). \tag{30}$$

Similarly, for forecasts: $(m_A = 1, m_B = 1)$, given $\phi = H$,

$$\Pr(m_A = 1, m_B = 1|\phi = H) = \gamma_E\gamma_I,$$

which does not depend on p, because, in this case, the market is certain that the efficient analyst received a high signal and the inefficient one a low signal regardless of whether analyst A or B is the efficient one. For forecasts: $(m_A = 1, m_B = 0)$, however, the market is not certain whether the efficient or the inefficient analyst received a high signal, and hence the expression (30) depends on p. In general, for both flipping and truth telling equilibria, the market *can* perfectly infer the private signal of the efficient and the inefficient analysts when their forecasts *match*, but *cannot* perfectly infer when their forecasts *do not match* (see derivations in appendix B.5). This is important because the market's learning from analyst forecasts, and hence the price efficiency, depends on knowing the private signals of the efficient and the ineffıcent analysts, regardless of whether analyst A or B is the efficient analyst.

Proposition 6 summarizes the properties of price efficiency and the comparative statics of the threshold W_{FP}. Figure 6 shows Π as a function of the reward level W. Panel (a) shows a plot for $p = 0.5$ and panel(b)shows plots for $p = 0.5$, $p = 0.7$ and $p = 0.9$, highlighting that Π increases in p.

— Figure 6 here —

Proposition 6 ***[Price efficiency of analyst forecasts: Properties]*** *Suppose analysts' forecasts are interpreted as in Assumption 1. Then:*

i) Price efficiency is nonmonotone in reward level—it first increases in W at $W \in (0, W_{FP})$, then it decreases in W at $W \in \left(W_{FP}, \frac{\beta c_I}{2(1-\theta)}\right)$, and finally it remains constant in W at $W \geqslant \frac{\beta c_I}{2(1-\theta)}$, where W_{FP} is defined in (24);

ii) W_{FP} increases in β, c_E and c_I but decreases in θ.

At $W \in (0, W_{FP})$, as the reward level increases, analysts' signal precisions increase and more information is impounded in the market leading to an increase in price efficiency.[9] At $W \in \left(W_{FP}, \frac{\beta c_I}{2(1-\theta)}\right)$, the efficient analyst's optimal precision is constant in W, and the inefficient analyst's precision increases in W. At $\gamma^*_{E,FP} = 1$ and $\gamma^*_{I,FP} \in \left(\frac{1}{2}, 1\right)$,

$$\Pi = \frac{1}{\gamma^*_{I,FP}(2\theta - 1)^2 + 4\theta(1-\theta)},$$

which decreases in $\gamma^*_{I,FP}$, and since $\gamma^*_{I,FP}$ increases in W, Π decreases in W. To understand the intuition, consider an extreme case, $p = \frac{1}{2}$. The market learns whether the efficient (inefficient) analyst received a high (low) signal only when analysts' forecasts match; it does not learn anything when forecasts do not match. Even when analysts' forecasts match, an increase in $\gamma^*_{I,FP}$ decreases the market's learning, because given the fixed $\gamma^*_{E,FP} = 1$, the higher the value of $\gamma^*_{I,FP}$, the lower the difference in weights the market places on the private signals of the two analysts, leading to a lower level of learning by the market. At the other extreme case, $p \to 1$, the market can perfectly identify an efficient analyst in matching as well as non-matching forecasts. When $\gamma^*_{E,FP} = 1$, the efficient analyst perfectly learns the fundamental, which, in turn, is learnt by the market. Thus, price efficiency reaches its maximum value at $W = W_{FP}$ and remains constant at $W > W_{FP}$. This result shows that as long as there is an uncertainty in the market, whatever small it may be, about the analysts' information cost, i.e., $p \in [\frac{1}{2}, 1)$, the price efficiency decreases at $W \in \left(W_{FP}, \frac{\beta c_I}{2(1-\theta)}\right)$.

6 Conclusions

Security analysts play an important information intermediary role in financial markets. One of the key objectives of (sell-side) analysts is to attain the much coveted " All-Star" status in the annual ranking based on their relative value of research assessed by the portfolio

[9] The effect of the decreasing optimal precision of the inefficient analyst at $W \in [W_{TT}, \beta c_E)$ is dominated by the increasing optimal precision of the efficient analyst, leading to the overall effect of an increasing price efficiency in W.

managers. In this paper, we analyze the price efficiency of analyst forecasts when competing analysts strategically acquire and communicate information to win a forecasting contest. One key distinguishing feature of our paper is to allow analysts to gather information by paying an information cost and become differently informed.

We have shown that, analysts cannot truthfully reveal their private opinions about the firm they cover if contest rewards are sufficiently high. While greater rewards can induce more information production, more precise information increase the conditional correlation among the signals of the competing analysts making differentiation to win the contest harder. If rewards are above a certain threshold, there is no equilibrium in which analysts acquire information and truthfully report their private information. At higher reward levels, there is a flipping forecasting equilibrium in which analysts with lower information cost acquire more precise information and truthfully reveal their private information, whereas analysts with higher information cost acquire less precise information and forecast counter to their private information (flip). Increasing contest competitiveness by boosting the rewards generally encourages information production but can discourage information acquisition by weaker analysts who have higher information cost. Increasing contest rewards can hurt the price efficiency of analyst forecasts if reward levels are sufficiently high. We also find flipping equilibria in which two analysts with exactly the same information cost become differently informed and use different forecasting strategies.

While our paper is not about designing optimal forecasting contests, results from our analysis caution against using very high rewards in a contest or tournament setting. High rewards can deter truthful reporting, discourage information production by weaker analysts, and reduce price efficiency of analyst forecasts.

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A Figures

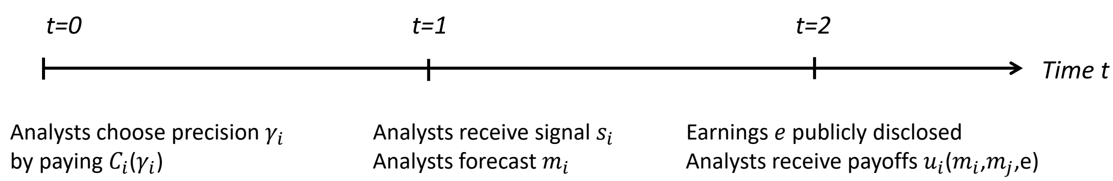
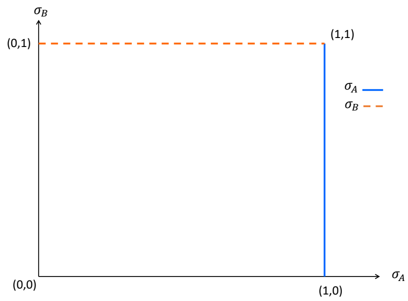
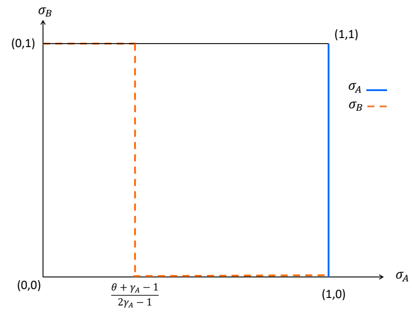


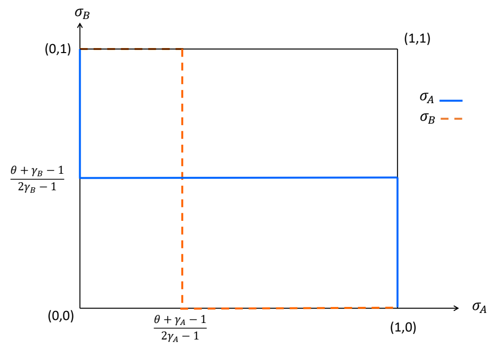
Figure 1: The sequence of events in the model.



(a) $\theta > \gamma_A$



(b) $\gamma_B < \theta < \gamma_A$



(c) $0.5 < \theta < \gamma_B$

Figure 2: Plots of best response functions of analyst A and analyst B in the forecasting subgame. Blue (solid) curve represents analyst A's response function, σ_A, and orange (dashed) curve represents analyst B's response function, σ_B. Parameter values are: $\gamma_A = 0.8$, $\gamma_B = 0.7$.

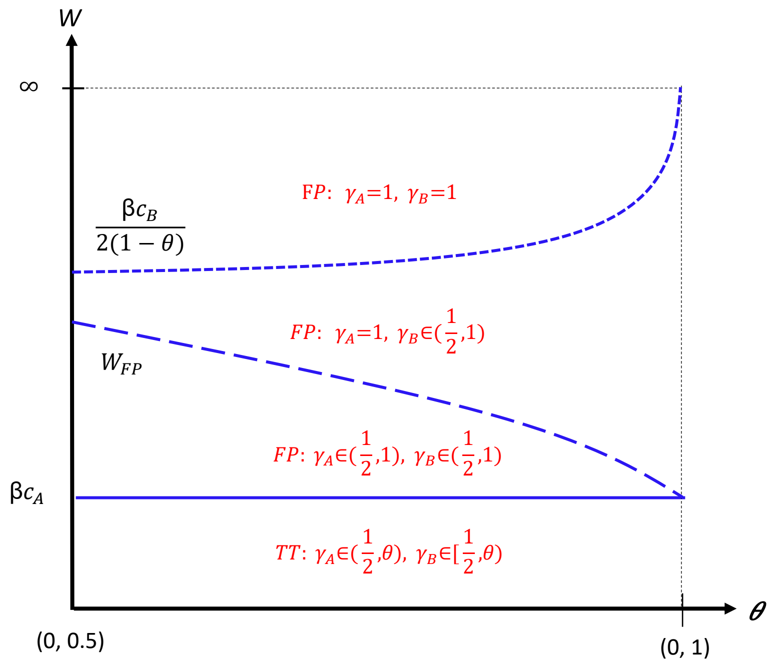
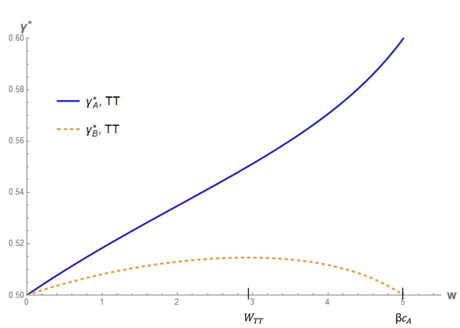
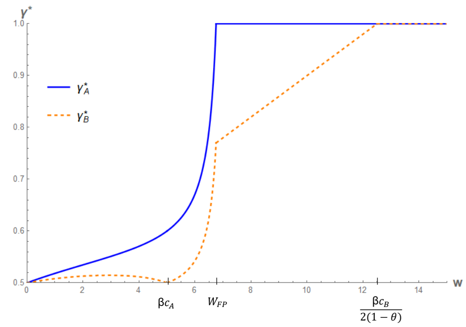


Figure 3: Forecasting equilibria with endogenous information as functions of θ and W. The notation “TT” implies truth telling forecasting equilibrium and “FP” implies flipping forecasting equilibrium.



(a) Equilibrium TT



(b) Equilibria TT and FP

Figure 4: Equilibrium γ's as a function of W. Panel (a) shows γ's for the truth telling forecasting equilibrium. Panel (b) shows γs for both truth telling (TT) and flipping (FP) forecasting equilibria. In panel (b), plots at $W \in (0, \beta c_A)$ represent precisions for truth telling equilibrium and plots at $W \geqslant \beta c_A$ represent precisions for flipping equilibrium. In both panels, blue (solid) curve represents analyst i's signal precision γ_i and orange (dashed) curve represents analyst j's signal precision γ_j. Parameters are set to $\theta = 0.6$, $\beta = 1$, $c_A = 5$, and $c_B = 10$.

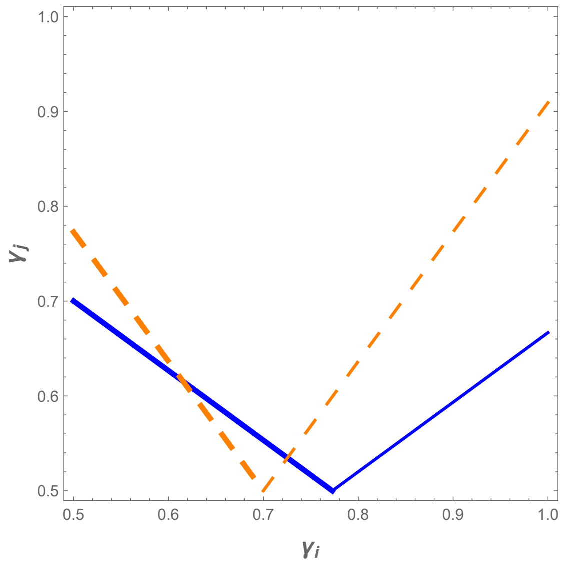
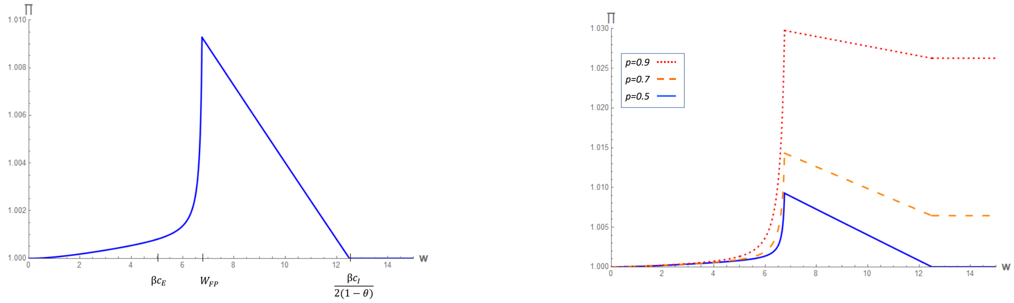


Figure 5: Plots of analysts' best response precision levels at the information acquisition stage with symmetric cost. Blue (solid) curves represent analyst i's response function γ_i to analyst j's choice of signal precision γ_j, and orange (dashed) curves represent analyst j's response function γ_j to analyst i's choice of signal precision γ_i. The thicker curves (solid and dashed) represent response functions in the truth telling forecasting equilibrium; the finer curves (solid and dashed) represent response functions in the flipping equilibrium. For the truth telling equilibrium, $\gamma_i^* = 0.62 = \gamma_j^*$; for the flipping equilibrium, $\gamma_i^* = 1$, $\gamma_j^* = 0.91$. Other parameters are set at $\theta = 0.7$, $\beta = 1$, $W = 1.5$, and $c = 1.1$.



(a) $p = 0.5$

(b) $p = 0.5, 0.7, 0.9$

Figure 6: Equilibrium Π as a function of W. Panel (a) shows the plot of Π as a function of W for $p = 0.5$. Panel (b) shows plots of Π vs. W for $p = 0.5$ (blue, solid), $p = 0.7$ (orange, large-dashed) and $p = 0.9$ (red, dotted). Other parameters are set to $\theta = 0.6$, $\beta = 1$, $c_E = 5$, and $c_I = 10$.

B Derivations

B.1 Conditional correlation

We first derive the expression of ρ and then discuss the properties of ρ.

Fix $e = 1$. The expression for analysts' signal correlation conditional on $e = 1$ is

$$\rho = \frac{Cov(s_i, s_j|e = 1)}{\sqrt{Var(s_i|e = 1)Var(s_j|e = 1)}}, \tag{B.1}$$

where

$$\begin{aligned} Var(s_i|e = 1) =& E\left[s_i|e\right] - \left(E\left[s_i|e\right]\right)^2 \\ &= \Pr\left(s_i = 1|e = 1\right) - \left[\Pr\left(s_i = 1|e = 1\right)\right]^2 \\ &= \Pr\left(s_i = 1|e = 1\right)\left[1 - \Pr\left(s_i = 1|e = 1\right)\right] \\ &= \left[1 - \theta + \gamma_i\left(2\theta - 1\right)\right]\left[\theta - \gamma_i\left(2\theta - 1\right)\right], \end{aligned} \tag{B.2}$$

because

$$\begin{aligned} \Pr(s_i &= 1|e = 1) = \frac{\Pr(s_i = 1, e = 1)}{\Pr(e = 1)} \\ &= \frac{\Pr(\phi = H)\Pr(s_i = 1, e = 1|\phi = H) + \Pr(\phi = L)\Pr(s_i = 1, e = 1|\phi = L)}{\Pr(\phi = H)\Pr(e = 1|\phi = H) + \Pr(\phi = L)\Pr(e = 1|\phi = L)} \\ &= \theta\gamma_i + \left(1 - \theta\right)\left(1 - \gamma_i\right), \end{aligned}$$

and

$$\begin{aligned} Cov(s_i, s_j|e = 1) &= E\left[s_i, s_j|e = 1\right] - E\left[s_i|e = 1\right]E\left[s_j|e = 1\right] \\ &= \Pr\left(s_i = 1, s_j = 1|e = 1\right) - \Pr(s_i = 1|e = 1)\Pr(s_j = 1|e = 1) \\ &= \theta\left(1 - \theta\right)\left(2\gamma_i - 1\right)\left(2\gamma_j - 1\right), \end{aligned}$$

Replacing the values of conditional covariance and conditional variances in (11), we have, for $e = 1$,

$$\rho = \frac{\theta\left(1-\theta\right)\left(2\gamma_i - 1\right)\left(2\gamma_j - 1\right)}{\sqrt{\left[1-\theta+\gamma_i\left(2\theta-1\right)\right]\left[\theta-\gamma_i\left(2\theta-1\right)\right]\left[1-\theta+\gamma_j\left(2\theta-1\right)\right]\left[\theta-\gamma_j\left(2\theta-1\right)\right]}},$$

which is (12). Using an analogous argument, we can see that the value of ρ at $e = 0$ is the same as that at $e = 1$.

Properties of ρ.

(i) ρ increases in γ_i, because the numerator $Cov(s_i, s_j|e)$ increases in γ_i and the denominator $Var(s_i|e)$ decreases in γ_i. The same argument holds for γ_j.

(ii) Differentiating ρ with respect to θ,

$$\frac{\partial\rho}{\partial\theta} = \frac{N1 \times N2}{D},$$

where

$$N1 = -\left[\begin{array}{c} \gamma_i\left(2\left(\gamma_j\right)^2(1-2\theta)^2 - 2\gamma_j(1-2\theta)^2 + (\theta-1)\theta\right) \\ +\gamma_i\left(-2\left(\gamma_j\right)^2(1-2\theta)^2 + 2\gamma_j(1-2\theta)^2 - \theta^2 + \theta\right) + (\gamma_j - 1)\gamma_j(\theta-1)\theta \end{array}\right], \quad \text{(B.3)}$$

and

$$N2 = (2\gamma_i - 1)(2\gamma_j - 1)(2\theta - 1) > 0,$$

and

$$D = 2\left[\begin{array}{c} (\gamma_i(2\theta-1)-\theta)(\gamma_i(2\theta-1)-\theta+1) \\ \times(\gamma_j(2\theta-1)-\theta)(\gamma_j(2\theta-1)-\theta+1) \end{array}\right]^{3/2} > 0,$$

such that

$$sgn\left(\frac{\partial\rho}{\partial\theta}\right) = sgn\left(N1\right). \tag{B.4}$$

Let

$$N1 = f\left(\theta\right) = a_0 + a_1\theta + a_2\theta^2, \tag{B.5}$$

such that by (B.3),

$$a_2 = \gamma_i\left(1-\gamma_i\right)\left[1 - 8\left(\gamma_j\right)^2\left(1-\gamma_j\right)\right] + \gamma_j\left(1-\gamma_j\right) = -a_1 \tag{B.6a}$$

$$a_0 = -2\gamma_i\gamma_j\left(1-\gamma_i\right)\left(1-\gamma_j\right) < 0. \tag{B.6b}$$

By tedious algebra, we can show that $a_2 > 0$ for any $\gamma_i \in \left(\frac{1}{2}, 1\right)$ and $\gamma_j \in \left(\frac{1}{2}, 1\right)$.

By (B.6a), $a_1 = -a_2 < 0$, and thus, from (B.3),

$$N1 = a_0 + a_1\theta\left(1-\theta\right) < 0, \tag{B.7}$$

and, hence by (B.4),

$$\frac{\partial\rho}{\partial\theta} < 0. \tag{B.8}$$

B.2 Analysts' expected payoffs

Let the expected payoff of analyst $i \in \{A, B\}$ with a signal $s_i = \kappa \in \{h, l\}$ when he forecasts $m_i = \eta \in \{1, 0\}$ and his opponent $j \in \{A, B\}, j \neq i$, forecasts m_j, be

$$U_i\left(m_i = \eta | s_i = \kappa\right) \equiv \mathbb{E}_{e,m_j}[u_i(m_i = \eta, m_j, e) | s_\tau = \kappa]. \tag{B.9}$$

The net expected of analyst i with a signal $s_i = \kappa$ from issuing a forecast $m_i = 1$ relative to the forecast $m_i = 0$ is

$$\Delta U_{i,\kappa} \equiv U_i \left(m_i = 1 | s_i = \kappa\right) - U_i \left(m_i = 0 | s_\tau = \kappa\right). \tag{B.10}$$

Substituting analysts' payoffs for different values of signals, forecasts and earnings, we have, for analyst i,

$$\Delta U_{i,h} = W \left[\gamma_i \theta + (1-\gamma_i)(1-\theta) - \sigma_j^h \left[\gamma_i \gamma_j + (1-\gamma_i)(1-\gamma_j)\right] - \sigma_j^l \left[\gamma_i (1-\gamma_j) + (1-\gamma_i)\gamma_j\right]\right] \tag{B.11a}$$

$$\Delta U_{i,l} = W \left[(1-\gamma_i)\theta + \gamma_i (1-\theta) - \sigma_j^h \left[\gamma_i (1-\gamma_j) + (1-\gamma_i)\gamma_j\right] - \sigma_j^l \left[\gamma_i \gamma_j + (1-\gamma_i)(1-\gamma_j)\right]\right], \tag{B.11b}$$

and, for analyst j,

$$\Delta U_{j,h} = W \left[\gamma_j \theta + (1-\gamma_j)(1-\theta) - \sigma_i^h \left[\gamma_i \gamma_j + (1-\gamma_i)(1-\gamma_j)\right] - \sigma_i^l \left[\gamma_i (1-\gamma_j) + (1-\gamma_i)\gamma_j\right]\right] \tag{B.11c}$$

$$\Delta U_{j,l} = W \left[(1-\gamma_j)\theta + \gamma_j (1-\theta) - \sigma_i^h \left[\gamma_i (1-\gamma_j) + (1-\gamma_i)\gamma_j\right] - \sigma_i^l \left[\gamma_i \gamma_j + (1-\gamma_i)(1-\gamma_j)\right]\right], \tag{B.11d}$$

B.3 Analysts' ex-ante benefits

We show here that the ex-ante expected payoffs (benefit) of analyst i of acquiring a signal with precision γ_i when analyst j acquires a signal with precision γ_j is

$$\begin{aligned} B_i(\gamma_i, \gamma_j; \sigma_i, \sigma_j) =& W(\gamma_i\gamma_j\theta + (1-\gamma_i)(1-\gamma_j)(1-\theta))\sigma_i(1-\sigma_j) \\ &+ W(\gamma_i(1-\gamma_j)\theta + (1-\gamma_i)\gamma_j(1-\theta))\sigma_i\sigma_j \\ &+ W((1-\gamma_i)\gamma_j\theta + \gamma_i(1-\gamma_j)(1-\theta))(1-\sigma_i)(1-\sigma_j) \\ &+ W((1-\gamma_i)(1-\gamma_j)\theta + \gamma_i\gamma_j(1-\theta))(1-\sigma_i)\sigma_j, \end{aligned} \tag{B.12}$$

where σ_i and σ_j are equilibrium forecasting strategies, which are functions of (γ_i, γ_j).

For any (σ_i, σ_j) in the forecasting subgame, an analyst's ex-ante expected payoffs at $t = 0$ is

$$B_i(\gamma_i, \gamma_j) = \sum_{s_i, s_j, e} \Pr(s_i, s_j, e) \sum_{m_i, m_j} \Pr(m_i|s_i)\Pr(m_j|s_j) u_i(m_i, m_j, e). \tag{B.13}$$

Expanding (B.13) yields

$$\begin{aligned} &B_i(\gamma_i, \gamma_j; \sigma_i, \sigma_j) \\ &= \Pr(s_i = 1, s_j = 1, e = 1) \begin{bmatrix} \sigma_i\sigma_j u_i(m_i = 1, m_j = 1, e = 1) \\ +\sigma_i(1-\sigma_j)u_i(m_i = 1, m_j = 0, e = 1) \\ +(1-\sigma_i)\sigma_j u_i(m_i = 0, m_j = 1, e = 1) \\ +(1-\sigma_i)(1-\sigma_j)u_i(m_i = 0, m_j = 0, e = 1) \end{bmatrix} \\ &+ ... + \Pr(s_i = 0, s_j = 0, e = 0) \begin{bmatrix} (1-\sigma_i)(1-\sigma_j)u_i(m_i = 1, m_j = 1, e = 0) \\ +(1-\sigma_i)\sigma_j)u_i(m_i = 1, m_j = 0, e = 0) \\ +\sigma_i(1-\sigma_j)u_i(m_i = 1, m_j = 0, e = 0) \\ +\sigma_i\sigma_j u_i(m_i = 1, m_j = 0, e = 0) \end{bmatrix}. \end{aligned} \tag{B.14}$$

Notice that (B.14) has a total of $2^3 = 8$ terms of $\Pr(s_i, s_j, e)$, and for each of these terms, there are $2^2 = 4$ terms of $u_i(m_i, m_j, e)$ in the square brackets with a total of $2^5 = 32$ terms.

Substituting the payoff values $u_i(m_i, m_j, e)$ from (5) in (B.14),

$$
\begin{aligned}
&B_i(\gamma_i, \gamma_j; \sigma_i, \sigma_j) \\
&= \frac{1}{2}(\gamma_i\gamma_j\theta + (1-\gamma_i)(1-\gamma_j)(1-\theta))\,\sigma_i(1-\sigma_j)W + \ldots + \\
&\frac{1}{2}(\gamma_i\gamma_j\theta + (1-\gamma_i)(1-\gamma_j)(1-\theta))\,\sigma_i(1-\sigma_j)W,
\end{aligned}
$$

which, after collecting similar terms, yields (B.12).

For the truth telling forecasting equilibrium, for each analyst $i \in \{A, B\}$,

$$B_i(\gamma_i, \gamma_j; \sigma_i = 1, \sigma_j = 1) = W(\gamma_i(1-\gamma_j)\theta + (1-\gamma_i)\gamma_j(1-\theta)). \tag{B.15}$$

For the flipping forecasting equilibrium, for analysts i and j,

$$B_i(\gamma_i, \gamma_j; \sigma_i = 1, \sigma_j = 0) = W(\gamma_i\gamma_j\theta + (1-\gamma_i)(1-\gamma_j)(1-\theta)), \tag{B.16a}$$

$$B_j(\gamma_i, \gamma_j; \sigma_i = 1, \sigma_j = 0) = W((1-\gamma_i)(1-\gamma_j)\theta + \gamma_i\gamma_j(1-\theta)). \tag{B.16b}$$

For the reverse flipping forecasting equilibrium, for analysts i and j,

$$B_i(\gamma_i, \gamma_j; \sigma_i = 0, \sigma_j = 1) = W((1-\gamma_i)(1-\gamma_j)\theta + \gamma_i\gamma_j(1-\theta)), \tag{B.17a}$$

$$B_j(\gamma_i, \gamma_j; \sigma_i = 0, \sigma_j = 1) = W(\gamma_i\gamma_j\theta + (1-\gamma_i)(1-\gamma_j)(1-\theta)). \tag{B.17b}$$

B.4 Expected conditional variance

For any $\alpha, \beta, \delta \in \{1, 0\}$,

$$\begin{aligned}
&\mathbb{E}_{m_i,m_j,e}\left[Var\left(\phi|m_i,m_j,e\right)\right] \\
&= \sum_{\alpha,\beta,\delta\in\{1,0\}} \Pr\left(m_i=\alpha,m_j=\beta,e=\delta\right) Var\left(\phi|m_i=\alpha,m_j=\beta,e=\delta\right),
\end{aligned} \tag{B.18a}$$

where

$$Var\left(\phi|m_i=\alpha,m_j=\beta,e=\delta\right) = \Pr\left(\phi=H|m_i=\alpha,m_j=\beta,e=\delta\right)\Pr\left(\phi=L|m_i=\alpha,m_j=\beta,e=\delta\right), \tag{B.18b}$$

and

$$\Pr\left(\phi=H|m_i=\alpha,m_j=\beta,e=\delta\right) = \frac{\Pr\left(\phi=H\right)\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=H\right)}{\Pr\left(m_i=\alpha,m_j=\beta,e=\delta\right)}. \tag{B.18c}$$

Taken together (B.18a)-(B.18c) and , for any $\alpha, \beta, \delta \in \{1, 0\}$,

$$\begin{aligned}
& \Pr\left(m_i=\alpha,m_j=\beta,e=\delta\right) Var\left(\phi|m_i=\alpha,m_j=\beta,e=\delta\right) \\
=\ & \left[\Pr\left(\phi=H\right)\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=H\right)\right] \\
& \times\left[\frac{\Pr\left(\phi=L\right)\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=L\right)}{\Pr\left(m_i=\alpha,m_j=\beta,e=\delta\right)}\right] \\
=\ & \frac{\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=H\right)\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=L\right)}{2\left[\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=H\right)+\Pr\left(m_i=\alpha,m_j=\beta,e=\delta|\phi=L\right)\right]} \\
=\ & \frac{1}{2}\left[\frac{1}{\frac{1}{\Pr(m_i=\alpha,m_j=\beta,e=\delta|\phi=H)}+\frac{1}{\Pr(m_i=\alpha,m_j=\beta,e=\delta|\phi=L)}}\right],
\end{aligned}$$

because $\Pr(\phi = H) = \frac{1}{2}$, and

$$\begin{aligned}
& \Pr(m_i = \alpha, m_j = \beta, e = \delta) \\
= \; & \frac{1}{2}\left[\Pr(m_i = \alpha, m_j = \beta, e = \delta | \phi = H) + \Pr(m_i = \alpha, m_j = \beta, e = \delta | \phi = L)\right].
\end{aligned}$$

Thus, for any $\alpha, \beta, \delta \in \{1, 0\}$,

$$\begin{aligned}
& \mathbb{E}_{m_i, m_j, e}\left[Var(\phi | m_i, m_j, e)\right] \\
= \; & \frac{1}{2} \sum_{\alpha, \beta, \delta \in \{1,0\}} \left[\frac{1}{\frac{1}{\Pr(m_i=\alpha, m_j=\beta, e=\delta|\phi=H)} + \frac{1}{\Pr(m_i=\alpha, m_j=\beta, e=\delta|\phi=L)}}\right],
\end{aligned}$$

and after replacing $\mathbb{E}_e\left[Var(\phi|e)\right] = \theta(1-\theta)$, we have

$$\Pi = \left[\sum_{\alpha, \beta, \delta \in \{1,0\}} \left[\frac{2\theta(1-\theta)}{\frac{1}{\Pr(m_i=\alpha, m_j=\beta, e=\delta|\phi=H)} + \frac{1}{\Pr(m_i=\alpha, m_j=\beta, e=\delta|\phi=L)}}\right]\right]^{-1},$$

which is (29).

B.5 Interpretation of analyst forecasts

Truth telling equilibrium. Consider analyst forecasts: $(m_A = 1, m_B = 0)$. In a *truth telling forecasting equilibrium*, the market maps analyst forecasts to analyst signals as

$$(m_A = 1, m_B = 0) \longmapsto (s_{A=E} = h, s_{B=I} = l) \cup (s_{A=I} = h, s_{B=E} = l),$$

such that

$$\begin{aligned}
& \Pr(m_A = 1, m_B = 0 | \phi) \\
& = p \Pr(s_{A=E} = h, s_{B=I} = l | \phi) + (1-p) \Pr(s_{A=I} = h, s_{B=E} = l | \phi). \qquad \text{(B.19)}
\end{aligned}$$

This is because in a truth telling forecasting equilibrium, both analyst types make forecasts that truthfully reveal their private signals. For $\phi = H$,

$$\begin{aligned}
\Pr\left(s_{A=E} = h, s_{B=I} = l|\phi = H\right) &= \gamma_E\left(1-\gamma_I\right), \\
\Pr\left(s_{A=I} = h, s_{B=E} = l|\phi = H\right) &= \gamma_I\left(1-\gamma_E\right).
\end{aligned}$$

Therefore, from (B.19),

$$\Pr\left(m_A = 1, m_B = 0|\phi = H\right) = p\gamma_E\left(1-\gamma_I\right) + \left(1-p\right)\gamma_I\left(1-\gamma_E\right).$$

Using an analogous method, we derive the rest of the probabilities.

To summarize, in a *truth telling* forecasting equilibrium with endogenous information, for any $\alpha, \beta \in \{1, 0\}$ and $\zeta \in \{H, L\}$,

$$\begin{aligned}
&\Pr\left(m_i = \alpha, m_j = \beta|\phi = \zeta\right) \\
&= \begin{cases}
\gamma_E\gamma_I & \text{if } \alpha = \beta = 1\,(0) \text{ and } \zeta = H\,(L) \\
\left(1-\gamma_E\right)\left(1-\gamma_I\right) & \text{if } \alpha = \beta = 0\,(1) \text{ and } \zeta = H\,(L) \\
p\gamma_E\left(1-\gamma_I\right) + \left(1-p\right)\gamma_I\left(1-\gamma_E\right) & \text{if } \alpha \neq \beta = 0\,(1) \text{ and } \zeta = H\,(L) \\
p\left(1-\gamma_E\right)\gamma_I + \left(1-p\right)\left(1-\gamma_I\right)\gamma_E & \text{if } \alpha \neq \beta = 1\,(0) \text{ and } \zeta = H\,(L).
\end{cases}
\end{aligned} \tag{B.20}$$

Flipping equilibrium. Using an analogous method used in the text, we derive the rest of the probabilities. To summarize, in a *flipping* forecasting equilibrium with endogenous information, for any $\alpha, \beta \in \{1,0\}$ and $\zeta \in \{H, L\}$,

$$\begin{aligned}&\Pr\left(m_i = \alpha, m_j = \beta | \phi = \zeta\right) \\ &= \begin{cases} \gamma_E\left(1-\gamma_I\right) & \text{if } \alpha = \beta = 1\,(0) \text{ and } \zeta = H\,(L) \\ \left(1-\gamma_E\right)\gamma_I & \text{if } \alpha = \beta = 0\,(1) \text{ and } \zeta = H\,(L) \\ p\gamma_E\gamma_I + (1-p)\left(1-\gamma_E\right)\left(1-\gamma_I\right) & \text{if } \alpha \neq \beta = 0\,(1) \text{ and } \zeta = H\,(L) \\ p\left(1-\gamma_E\right)\left(1-\gamma_I\right) + (1-p)\gamma_E\gamma_I & \text{if } \alpha \neq \beta = 1\,(0) \text{ and } \zeta = H\,(L). \end{cases}\end{aligned} \tag{B.21}$$

C Proofs

Proof of Proposition 1. Suppose analyst i receives signal $s_i = h$. He will report $m_i = 1$ if and only if

$$E_e\left[u_i\left(m_i = 1, e\right) | s_i = h\right] \geqslant E_e\left[u_i\left(m_i = 0, e\right) | s_i = h\right],$$

that is,

$$\Pr\left(e = 1 | s_i = h\right) w_0 \geqslant \Pr\left(e = 0 | s_i = h\right) w_0,$$

which is *always* true, because

$$\Pr\left(e = 1 | s_i = h\right) = \theta\gamma_i + (1-\theta)\left(1-\gamma_i\right) > (1-\theta)\gamma_i + \theta\left(1-\gamma_i\right) = \Pr\left(e = 0 | s_i = h\right).$$

Similarly, an analyst with a signal $s_i = l$ will always reports $m_i = l$.

The first order condition for the optimality of an analyst's endogenous signal precision is $\frac{d}{d\gamma_i} V_i\left(\gamma_i\right) = 0$, which, by (8), is

$$w_0\left(2\theta - 1\right) - \beta c_i\left(\gamma_i - \frac{1}{2}\right) = 0,$$

and thus the optimal signal precision is

$$\gamma_i = \frac{1}{2}\left[1 + \frac{2w_0\left(2\theta - 1\right)}{\beta c_i}\right],$$

which is (9). The second order condition $\frac{d^2}{d\gamma_i^2}V_i\left(\gamma_i\right)|_{\gamma_i=\gamma_i} < 0$ is always satisfied because of the convexity of the cost function. ■

Lemma C.1 *Any informative equilibrium, if it exists, will have strategies:* $\sigma_i^l = 1 - \sigma_i^h$ *and* $\sigma_j^l = 1 - \sigma_j^h$.

Proof. An informative equilibrium can be of three types: (i) $\sigma_i^h = \sigma_i^l$ and $\sigma_j^h \neq \sigma_j^l$, (ii) $\sigma_i^h \neq \sigma_i^l$ and $\sigma_j^h = \sigma_j^l$, and (iii) $\sigma_i^h \neq \sigma_i^l$ and $\sigma_j^h \neq \sigma_j^l$. We start with showing that the first two types of equilibria cannot exist. Then we show that in equilibrium type (iii), the only equilibria that exist will have $\sigma_i^h = 1 - \sigma_i^l$ and $\sigma_j^h = 1 - \sigma_j^l$.

Consider equilibrium type (i). Let $\sigma_i^h = \sigma_i^l = \sigma_i$. If $\sigma_i = 1$, then using the expressions of $\Delta U_{j,h}$ and $\Delta U_{j,l}$ from (**??**), $\Delta U_{j,h} < 0$ and $\Delta U_{j,l} < 0$ implying $\sigma_j^h = 0 = \sigma_j^l$, a contradiction that $\sigma_j^h \neq \sigma_j^l$. Similarly, if $\sigma_i = 0$, then, B.11, $\Delta U_{j,h} > 0$ and $\Delta U_{j,l} > 0$ implying $\sigma_j^h = 1 = \sigma_j^l$, again a contradiction. If $\sigma_i \in (0,1)$, then given $\sigma_j^h \neq \sigma_j^l$, the two necessary conditions for equilibrium (i) to exist are: $\Delta U_{i,h}\left(\sigma_j^h \neq \sigma_j^l\right) = 0$, and $\Delta U_{i,l}\left(\sigma_j^h \neq \sigma_j^l\right) = 0$, for which the unique solution is:

$$\sigma_j^h = \frac{\theta + \gamma_j - 1}{2\gamma_j - 1} = 1 - \sigma_j^l \in (0,1)\,.$$

However, with the above solution, $\Delta U_{j,h}\left(\sigma_i\right) = 0 = \Delta U_{j,l}\left(\sigma_i\right)$, which cannot be true, because if $\Delta U_{j,h}\left(\sigma_i\right) = 0$, that is, $\Delta U_{j,h}\left(\sigma_i^h\right) = \gamma_j\theta + \left(1 - \gamma_j\right)\left(1 - \theta\right) - \sigma_i = 0$, then $\Delta U_{j,l}\left(\sigma_i\right) \neq 0$ or if $\Delta U_{j,l}\left(\sigma_i\right) = 0$ then $\Delta U_{j,h}\left(\sigma_i\right) \neq 0$. Thus, an equilibrium with $\sigma_i \in (0,1)$ and $\sigma_j^h \neq \sigma_j^l$ cannot exist. Taken all together, equilibrium type (i) cannot exist. By an analogous argument, equilibrium type (ii) cannot exist.

We are now left to consider only equilibria (iii) with strategies: $\sigma_i^h \neq \sigma_i^l$ and $\sigma_j^h \neq \sigma_j^l$. We prove in the following Claim C.2 that such equilibria will *always* have $\sigma_i^l = 1 - \sigma_i^h$ and $\sigma_j^l = 1 - \sigma_j^h$.

Claim C.2 *The only equilibrium in a generic informative equilibrium* $\left(\sigma_i^h \neq \sigma_i^l, \sigma_j^h \neq \sigma_j^l\right)$ *that exists will have strategies* $\sigma_i^l = 1 - \sigma_i^h$ *and* $\sigma_j^l = 1 - \sigma_j^h$.

Proof. Suppose there exists an equilibrium such that $\sigma_i^l > 1 - \sigma_i^h$ (this is w.l.o.g; we can assume $\sigma_i^l < 1 - \sigma_i^h$) and $\sigma_j^l = 1 - \sigma_j^h$. Then $\sigma_i^h + \sigma_i^l > 1$ and thus there exist no equilibria with $\left(\sigma_i^h = 1, \sigma_i^l = 0\right)$ or $\left(\sigma_i^h = 0, \sigma_i^l = 1\right)$. The only equilibria are of the form with strategies $\left(\sigma_j^h \in [0,1], \sigma_j^l \in [0,1]\right)$ and (a) $\left(\sigma_i^h \in (0,1), \sigma_i^l \in (0,1)\right)$, (b) $\left(\sigma_i^h = 1, \sigma_i^l \in (0,1)\right)$ and (c) $\left(\sigma_i^h \in (0,1), \sigma_i^l = 1\right)$ such that $\sigma_i^h + \sigma_i^l > 1$.

Equilibria (b) and (c) cannot exist. This is because, for equilibrium (b) to exist, the necessary conditions that must be satisfied are $\Delta U_{i,h}\left(\sigma_j^h, \sigma_j^l\right) > 0$ and $\Delta U_{i,l}\left(\sigma_j^h, \sigma_j^l\right) = 0$. However, both conditions cannot be simultaneously satisfied because with $\sigma_j^l = 1 - \sigma_j^h$, $\Delta U_{i,h}\left(\sigma_j^h, \sigma_j^l\right) = -\Delta U_{i,l}\left(\sigma_j^h, \sigma_j^l\right)$. Similarly, equilibrium (c) can be ruled out.

The only candidate equilibrium is equilibrium (a). There are three possible cases for equilibrium (a): $\left(\sigma_i^h \in (0,1), \sigma_i^l \in (0,1)\right)$ with (i) $\left(\sigma_j^h = 1, \sigma_j^l = 0\right)$, (ii) $\left(\sigma_j^h = 0, \sigma_j^l = 1\right)$, and (iii) $\left(\sigma_j^h \in (0,1), \sigma_j^l \in (0,1)\right)$. Cases (i) and (ii) are ruled out. For case (a), at $\sigma_j^h = 1, \sigma_j^l = 0$, $\Delta U_{i,h}\left(\sigma_j^h, \sigma_j^l\right) = -\Delta U_{i,l}\left(\sigma_j^h, \sigma_j^l\right)$, which implies that there exists no interval on θ such that the necessary conditions for the equilibrium to exist, that is, $\Delta U_{i,h}\left(\sigma_j^h, \sigma_j^l\right) = 0 = \Delta U_{i,l}\left(\sigma_j^h, \sigma_j^l\right)$, are simultaneously satisfied. The case (ii) is ruled out using a similar argument. The only candidate equilibrium is the third case with strategies $\left(\sigma_i^h \in (0,1), \sigma_i^l \in (0,1)\right)$ and $\left(\sigma_j^h \in (0,1), \sigma_j^l \in (0,1)\right)$. For this equilibrium to exist, four conditions are to be simultaneously satisfied: $\Delta U_{i,h}\left(\sigma_j^h, \sigma_j^l\right) = \Delta U_{i,l}\left(\sigma_j^h, \sigma_j^l\right) = \Delta U_{j,h}\left(\sigma_j^h, \sigma_j^l\right) = \Delta U_{j,l}\left(\sigma_i^h, \sigma_i^l\right) = 0$, of which the unique solution is

$$\sigma_i^h = \frac{\theta + \gamma_i - 1}{2\gamma_i - 1} = 1 - \sigma_i^l \text{ and } \sigma_j^h = \frac{\theta + \gamma_j - 1}{2\gamma_j - 1} = 1 - \sigma_j^l,$$

which contradicts the assumption that $\sigma_i^l > 1 - \sigma_i^h$. Taken together all cases, we conclude that there exists no informtive equilibrium with strategies $\sigma_i^l > 1 - \sigma_i^h$ and $\sigma_j^l = 1 - \sigma_j^h$. The symmetric case with strategies $\sigma_i^l = 1 - \sigma_i^h$ and $\sigma_j^l > 1 - \sigma_j^h$ can be similarly ruled out. ■

This completes the proof of Lemma C.1. ■

Proof of Lemma 1. Results follow directly from the discussion before the Lemma. ■

Proof of Proposition 2.

Part (i). Truth telling is an equilibrium if $\sigma_A^* = 1$ given $\sigma_B^* = 1$ and $\sigma_B^* = 1$ given $\sigma_A^* = 1$. By Lemma 1, $\sigma_A^* = 1$ if $\sigma_B^* < \frac{\theta+\gamma_B-1}{2\gamma_B-1}$, or equivalently, if $\sigma_B^* = 1 < \frac{\theta+\gamma_B-1}{2\gamma_B-1}$, or $\theta > \gamma_B$. Similarly, $\sigma_B^* = 1$ if $\sigma_A^* = 1 < \frac{\theta+\gamma_A-1}{2\gamma_A-1}$, or equivalently, if $\theta > \gamma_A$. Taken together, there exists a truth telling equilibrium if $\theta \geqslant \max(\gamma_A, \gamma_B)$.

The uniqueness of the truth telling equilibrium is guaranteed by showing that there exists no other informative equilibrium at $\theta \geqslant \max(\gamma_A, \gamma_B)$. We show later that the equilibrium $(\sigma_A^* = 1, \sigma_B^* = 0)$ exists only if $\theta < \max(\gamma_A, \gamma_B)$ in part (ii), and the equilibria $(\sigma_A^* \in (0,1), \sigma_B^* \in (0,1))$ exists only if $\theta < \min(\gamma_A, \gamma_B)$ in part (iii).

Parts (ii) and (iii). Without loss of generality, let $\gamma_A > \gamma_B$. For the flipping equilibrium to exist, two conditions must be satisfied simultaneously: $\sigma_A^* = 1$ given $\sigma_B^* = 0$, and $\sigma_B^* = 0$ given $\sigma_A^* = 1$. By Lemma 1, $\sigma_A^* = 1$ if $\sigma_B^* < \frac{\theta+\gamma_B-1}{2\gamma_B-1}$, or equivalently, if $\sigma_B^* = 0 < \frac{\theta+\gamma_B-1}{2\gamma_B-1}$, which is always true for any $\theta, \gamma_B > \frac{1}{2}$. Similarly, $\sigma_B^* = 0$ if $\sigma_A^* > \frac{\theta+\gamma_A-1}{2\gamma_A-1}$, or equivalently, if $\sigma_A^* = 1 > \frac{\theta+\gamma_A-1}{2\gamma_A-1}$, or if $\theta < \gamma_A$. Taken together, there exists a flipping equilibrium if $\theta < \max(\gamma_A, \gamma_B)$.

For the mixed strategy equilibrium to exist, two conditions must be satisfied simultaneously: $\sigma_A^* \in (0,1)$ given $\sigma_B^* \in (0,1)$, and $\sigma_A^* \in (0,1)$ given $\sigma_A^* \in (0,1)$. By Lemma 1,

$\sigma_A^* \in (0,1)$ if $\sigma_B^* = \frac{\theta+\gamma_B-1}{2\gamma_B-1} \in (0,1)$, because $\frac{\theta+\gamma_B-1}{2\gamma_B-1} < 1$ if $\theta < \gamma_B$. Similarly, $\sigma_B^* \in (0,1)$ if $\sigma_A^* = \frac{\theta+\gamma_A-1}{2\gamma_A-1} \in (0,1)$, because $\frac{\theta+\gamma_A-1}{2\gamma_A-1} < 1$ if $\theta < \gamma_A$. Taken together, there exists a mixed strategy equilibrium if $\theta < \min(\gamma_A, \gamma_B)$. ■

Proof of Proposition 3.

Endogenous precisions are derived by solving best response functions for the analysts as stated (19). The solution is given by (21). Next, we need to check whether these precisions satisfy the feasibility condition for the existence of the truth telling forecasting equilibrium, that is, $\theta \geqslant \max\left(\gamma_{A,TT}^*, \gamma_{B,TT}^*\right)$.

Note that $\gamma_{A,TT}^* \geqslant \gamma_{B,TT}^*$ because $c_A \leqslant c_B$. Thus, the feasibility condition is equivalent to

$$\theta \geqslant \gamma_{A,TT}^* = \frac{1}{2}\left[1 + \frac{W(\beta c_B - W)}{\beta^2 c_A c_B - W^2}(2\theta - 1)\right],$$

which, after some algebra, yields $(2\theta-1)\beta c_B(\beta c_A - W) \geqslant 0$, or $0 < W \leqslant \beta c_A$. It is easy to check that given these bounds of W, $\gamma_{A,TT}^* \in \left(\frac{1}{2}, \theta\right)$ for every $i \in \{A, B\}$. ■

Proof of Lemma 2.

Differentiating

$$\frac{\partial \gamma_{A,TT}^*}{\partial W} = \frac{\beta c_B (2\theta-1)\left(W^2 - 2\beta c_A W + \beta^2 c_A c_B\right)}{2\left(W^2 - \beta^2 c_A c_B\right)^2} > 0,$$

because, for $c_B > c_A$,

$$W^2 - 2\beta c_A W + \beta^2 c_A c_B = (W - \beta c_A)^2 + \beta^2 c_A (c_B - c_A) > 0.$$

Differentiating

$$\frac{\partial \gamma_{B,TT}^*}{\partial W} = \frac{\beta c_A (2\theta-1)\left(W^2 - 2\beta c_B W + \beta^2 c_A c_B\right)}{2\left(W^2 - \beta^2 c_A c_B\right)^2},$$

where $W^2 - 2\beta c_B W + \beta^2 c_A c_B \equiv f(W)$ is a quadratic polynomial of W. $f(W = 0) > 0$ and by Descartes' rule of signs, $f(W) = 0$ has two positive solutions: $\beta \left[c_B \pm \sqrt{c_B^2 - c_A c_B}\right]$. Since $\gamma^*_{B,TT}$ is defined only for $W < \beta c_A$, there exists a unique

$$W_{TT} = \beta \left[c_B - \sqrt{c_B^2 - c_A c_B}\right],$$

such that $f(W) > 0$ if $W < W_{TT}$ and $f(W) \leqslant 0$ if $W \geqslant W_{TT}$. We can show $W_{TT} < \beta c_A$, because $c_A > c_B - \sqrt{c_B^2 - c_A c_B}$, or $\sqrt{c_B^2 - c_A c_B} > c_B - c_A$, or $c_B > c_A$, which is true. ■

Proof of Proposition 4.

Solving the best response functions given in (23) yield the optimal precisions given in (25a). Both $\gamma^*_{A,FP} > \frac{1}{2}$ and $\gamma^*_{B,FP} > \frac{1}{2}$ only if $\beta^2 c_A c_B - W^2 > 0$ or $W < \beta\sqrt{c_A c_B}$. For the flipping forecasting equilibrium to exist, we must have $\theta < \max\left(\gamma^*_{A,FP}, \gamma^*_{B,FP}\right)$, that is, $W > \beta c_A$. Thus, we must have $\beta c_A < W < \beta\sqrt{c_A c_B}$ for the existence of a FP equilibrium such that $\gamma^*_{A,FP}, \gamma^*_{B,FP} > \frac{1}{2}$.

Next, we need to check whether, and at what vlaues of W, $\gamma^*_{A,FP} \leqslant 1$, that is,

$$\frac{1}{2}\left[1 + \frac{W(\beta c_B - W)}{\beta^2 c_A c_B - W^2}(2\theta - 1)\right] \leqslant 1,$$

which can be simplified to

$$2(1-\theta)W^2 + (2\theta - 1)\beta c_B - \beta^2 c_A c_B \leqslant 0. \tag{C.1}$$

Let

$$f(W) \equiv 2(1-\theta)W^2 + (2\theta - 1)\beta c_B - \beta^2 c_A c_B.$$

The polynomial $f(W)$ is a quadratic function of W; also, $f(W) = 0$ has one positive root, by Descartes' rule of signs; $f(W = 0) < 0$ and $f\left(W = \beta\sqrt{c_A c_B}\right) > 0$. Thus, by the Intermediate Value Theorem, there exists a unique $W_{RF} \in \left(\beta c_A, \beta\sqrt{c_A c_B}\right)$ such that $f\left(W = W_{RF}\right) = 0$. Thus, $\gamma^*_{A,FP} \in \left(\frac{1}{2}, 1\right)$ at $W \in \left(\beta c_A, W_{RF}\right)$ and $\gamma^*_{A,FP} \geqslant 1$ for any $W \in [W_{RF}, \beta\sqrt{c_A c_B})$. The value of W_{RF} is derived by solving $f\left(W_{RF}\right) = 0$ and choosing the root with the positive sign:

$$W_{RF} = \left[\frac{\beta c_B}{4\left(1-\theta\right)}\right]\left[\left(\left(2\theta-1\right)^2 + 8\left(1-\theta\right)\left(c_i/c_B\right)\right)^{\frac{1}{2}} - \left(2\theta-1\right)\right], \tag{C.2}$$

which is (24).

For part (ii), we take $\gamma^*_{A,FP} = 1$ and derive $\gamma^*_{B,FP}$ by replacing $\gamma^*_{A,FP} = 1$ in the response functions given in (23). The values of $\left\langle\gamma^*_{A,FP}, \gamma^*_{B,FP}\right\rangle$ must satisfy the feasibility condition for the existence of a flipping equilibrium, that is, $\theta < \max\left(\gamma^*_{A,FP}, \gamma^*_{B,FP}\right)$. The upper bound of this equilibrium is derived by setting $\gamma^*_{B,FP} = 1$, that is,

$$\frac{\beta c_B}{2\left(1-\theta\right)} = \left\{W : \gamma^*_{B,FP} = 1\right\}.$$

For part (iii), we fix $\gamma^*_{A,FP} = \gamma^*_{B,FP} = 1$, which satisfy the existence condition of the flipping equilibrium that $\theta < \max\left(\gamma^*_{A,FP}, \gamma^*_{B,FP}\right)$. ■

Proof of Corollary 1.

Part (i) follows from replacing $c_A = c_B = c$ in (21), and noting that $\max\left(\gamma^*_{A,TT}, \gamma^*_{B,TT}\right) \leqslant \theta$, the feasibility condition of the existence of the truth telling forecasting equilibrium, is satisfied for any $W > 0$.

Part (ii) follows directly from Proposition 4 after replacing $c_A = c_B = c$ and noting that $\beta c = W_{FP}$. ■

Proof of Proposition 5.

Shown in the main text. ■

Proof of Proposition 6.

Part (i). At $W \in \left(W_{FP}, \frac{\beta c_I}{2(1-\theta)}\right)$,

$$\Pi\left(\gamma_E = 1, \gamma_I < 1\right) = \frac{1/4}{\gamma_I\left(2\theta-1\right)^2 p\left(1-p\right) + \theta\left(1-\theta\right)},$$

which decreases in γ_I and since $\gamma_I = \gamma^*_{B,FP}$ increases in W (see Proposition 4), Π decreases in W. At $W \geqslant \frac{\beta c_I}{2(1-\theta)}$,

$$\Pi\left(\gamma_E = 1, \gamma_I = 1\right) = \frac{1/4}{\left(2\theta-1\right)^2 p\left(1-p\right) + \theta\left(1-\theta\right)},$$

which does not depend on W.

Part (ii). Substituting $c_A = c_E$ and $c_B = c_I$ in (24), we have

$$W_{FP} = \left[\frac{\beta c_I}{4\left(1-\theta\right)}\right]\left[\left(\left(2\theta-1\right)^2 + 8\left(1-\theta\right)\left(c_E/c_I\right)\right)^{\frac{1}{2}} - \left(2\theta-1\right)\right]. \tag{C.3}$$

By inspection, W_{FP}, increases in β and c_E. For $\frac{\partial W_{FP}}{\partial c_I} > 0$, implies

$$\frac{\beta\left(\sqrt{\frac{4(1-\theta)(2c_E-\theta c_I)}{c_I}+1} - (2\theta-1)\right)}{4(1-\theta)} - \frac{\beta c_E}{c_I\sqrt{\frac{4(1-\theta)(2c_E-\theta c_I)}{c_I}+1}} > 0.$$

Denoting $t \equiv \frac{4(1-\theta)(2c_E-\theta c_I)}{c_I} + 1$, the above inequality leads to $\frac{\left(\sqrt{t}-(2\theta-1)\right)}{4(1-\theta)} > \frac{c_E}{c_I\sqrt{t}}$ or $tc_I + 4(1-\theta)c_E > (2\theta-1)\,c_I\sqrt{t}$, which, after replacing the value of t, yields

$$4(1-\theta)(2c_E-\theta c_I) + c_I - 4(1-\theta)c_E > (2\theta-1)\,c_I\sqrt{\frac{4(1-\theta)(2c_E-\theta c_I)}{c_I}+1},$$

that is,

$$4(1-\theta)(c_E-\theta c_I) + c_I > (2\theta-1)\sqrt{c_I\left[4(1-\theta)(2c_E-\theta c_I]\right) + c_I^2},$$

which, after squaring both sides and some algebraic manipulation, is true.

Differentiating, $\frac{\partial W_{FP}}{\partial \theta} = -\beta \frac{N}{D}$, where

$$N = c_I \left(\sqrt{\frac{c_I(2\theta-1)^2 + 8c_E(1-\theta)}{c_I}} - 2\theta + 1 \right) - 4c_E(1-\theta),$$

$$D = 4(\theta-1)^2 \sqrt{\frac{c_I(2\theta-1)^2 - 8c_E(\theta-1)}{c_I}}.$$

It is enough to show that $N > 0$, which will be true if

$$c_I \sqrt{\frac{c_I(2\theta-1)^2 - 8c_E(1-\theta)}{c_I}} > c_I(2\theta-1) + 4c_E(1-\theta),$$

or equivalently, after squaring both sides and some algebraic manipulation, $16(1-\theta)^2 c_E\,(c_I - c_E) > 0$, which is true because $c_I > c_E$. ■