Creating Controversy in Proxy Voting Advice*

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Abstract

We analyze the design of recommendations (available to all shareholders) and research reports (available only to subscribers) by a proxy advisor, who maximizes profits from selling information to shareholders. We show that the advisor benefits from biasing its recommendations against the a priori more likely alternative, thereby "creating controversy" for the vote. In contrast, it serves the advisor's interest to produce precise and unbiased research reports. Our results help reinterpret empirical patterns of shareholders' voting behavior, suggesting that proxy advisors' recommendations may not be a suitable benchmark for evaluating shareholders' votes. Our model also rationalizes the one-size-fits-all approach in recommendations.

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1 Introduction

Proxy advisory firms have emerged as major players in corporate governance. They make recommendations on how to cast votes and provide research reports for subscribing shareholders that contain the rationales for recommendations, including information on various aspects of firms' governance practices. The clients of the largest proxy advisor, Institutional Shareholder Services (ISS), include about 1,500 institutional investors, which cast more than 12 million ballots in 45,000 shareholder meetings around the world.[1] While proxy advisors' research reports are only available to their subscribers, their recommendations are frequently made public, either by the media or by the party supported by the proxy advisor – the company or the activist investor.[2] Through both these public recommendations and private research reports, proxy advisors have a substantial impact on voting outcomes (e.g., Alexander et al., 2010; Iliev and Lowry, 2015; Malenko and Shen, 2016).

Given the strong influence of proxy advisors, the quality and information content of their reports and recommendations have become an important topic of discussion among market participants and policymakers. For example, over 2018–2020, the SEC adopted a number of regulatory changes to ensure "that investors who use proxy voting advice receive more transparent, accurate, and complete information on which to make their voting decisions."[3] Do proxy advisors have incentives to produce unbiased and informative research and recommendations? In particular, as the SEC's concept release on the U.S. proxy system questioned, "Does the lack of a direct pecuniary interest in the effects of their recommendations on shareholder value affect how they formulate recommendations," "what criteria and processes do proxy advisory firms use," and are their "proxy research reports ... materially accurate and complete"?[4]

[1]Source: ISS website. The second largest proxy advisor, Glass Lewis, has more than 1,300 clients, who collectively manage more than $40 trillion in assets (source: Glass Lewis website).

[2]For example, it is common for an activist running a proxy fight to announce proxy advisors' support for its directors through the campaign website or press release distribution services, and for management to announce proxy advisors' support for its proposals. For examples of the media publicizing the recommendations, see "ISS recommends investors support Goldman Sachs on executive pay" (Reuters, April 14, 2022), "ISS says Wells Fargo pay reforms insufficient to justify support" (Reuters, April 13, 2022), and "Proxy adviser ISS backs all Exxon Mobil director candidates" (Reuters, May 10, 2022), which says "In a report sent by a representative late Monday ISS also recommended votes "for" the company's executive pay, but backed several shareholder resolutions focused on climate concerns." For an example of announcements by management, see the press release of Osisko "Leading Independent Proxy Advisory Firms Recommend Osisko Shareholders Vote for all Proposed Items at the Upcoming Annual Meeting" (April 28, 2022).

[3]See, e.g., https://www.sec.gov/rules/interp/2019/34-86721.pdf and https://www.sec.gov/rules/final/2020/34-89372.pdf. In June 2021, the SEC suspended the enforcement of these new regulations, given concern that they might potentially impair the independence and quality of the proxy voting advice.

[4]See p. 125 in https://www.sec.gov/rules/concept/2010/34-62495.pdf.

Motivated by these questions, this paper solves the information design problem of a proxy advisor who aims to maximize its profits from information sale to voters. Our main result is that even if all shareholders are unbiased and aligned at maximizing the value of their shares, the profit-maximizing proxy advisor often has incentives to produce *public recommendations that are biased* against the a priori more likely alternative. At the same time, the advisor has incentives to produce informative and *unbiased research reports* for its subscribers.

In our model, shareholders vote on a proposal whose value depends on the unknown state. The proxy advisor, who observes the state, faces an information design problem, modeled as Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). Specifically, it designs two signals about the state: one, which we refer to as the "research report," is available only to its subscribers, and the other, which we refer to as the "voting recommendation," is publicly observed by all shareholders. One potential way to interpret the design of recommendations are the proxy voting guidelines, which proxy advisors revise each year and announce publicly. Importantly, as we discuss in the paper, the chosen information design policies are time-consistent: because the advisor maximizes its ex-ante profit from information sales, it has no ex-post incentive to deviate from the design of either the report or recommendations. In addition to designing the two signals, the advisor sets a fee for subscribing to its research report. Each shareholder then decides whether to pay the fee and get the report or whether to only observe the public recommendation. The state is then realized, the proxy advisor's report and recommendation are produced, and shareholders cast their votes based on the information they receive. The proposal is approved if it receives a majority of the votes.

The fact that the proxy advisor maximizes its profit from information sales highlights a fundamental conflict of interest. If, instead, the advisor were maximizing the value for the downstream entities (asset managers and operating companies), then its public recommendations would perfectly reveal all the advisor's information—but then no investors would need to purchase the reports and the advisor would earn zero revenue. This raises the question of which design of recommendations and reports maximizes the fees that can be obtained from the subscribers. There is no obvious answer to this problem. For example, it may be natural to expect that the advisor will produce totally uninformative public recommendations, so as not to dilute the value of the private reports, and only give informative signals for a fee. Or, to the extent that public recommendations reveal some information, it may be natural to expect that they will be unbiased since all shareholders are aligned at maximizing firm value.

Nevertheless, we show that the proxy advisor's profit from information sale is maximized if (1) it designs a fully informative and unbiased research report, and (2) provides a public recommendation that is partially informative but biased against the alternative that is a priori more likely to be value-increasing (as long as the a priori likelihood of it being value-increasing is high enough). We refer to this bias against the more likely alternative as "creating controversy." By manipulating the public signal and recommending for the unexpected alternative too frequently, the advisor increases the probability that the vote will be close and thereby raises each shareholder's willingness to pay for the research report.

To see the intuition, consider one of the most frequent issues on which shareholders vote: the approval of directors proposed by the board's nominating committee. Suppose that the prior probability that a director nominee is good for the firm is sufficiently high. If the proxy advisor only provides information about the director in the research report but issues uninformative public recommendations, then shareholders who do not subscribe to the report base the decision on their positive priors and predominantly vote in favor of the director. This, however, implies that all non-subscribers tend to vote in the same way, so the aggregate vote outcome is unlikely to be close. Hence, a shareholder who is deciding whether to subscribe to the research report has little incentive to do so, because the probability that his informed vote will matter and sway the outcome towards the value-increasing decision is small.[5]

Suppose, instead, that the advisor issues partially informative recommendations to "create controversy:" it always recommends voting against directors who are value-decreasing, but sometimes recommends even against directors who are value-increasing. In this case, non-subscribing shareholders who see a negative recommendation infer that the director could be either good or bad and are unsure how to vote. This leads to a high chance of a close vote, which, in turn, increases the incentives of other shareholders to pay for information. Thus, by recommending for the unexpected alternative too often, the advisor increases shareholders' willingness to pay for its reports. Of course, the fact that negative recommendations are frequent implies that a positive recommendation is very informative about the director being value-increasing, leading non-subscribing shareholders to vote in favor. Hence, a higher probability of a close vote after a negative recommendation comes with a trade-off of a lower probability of a close vote after a positive recommendation. Nevertheless, we show that a recommendation appropriately biased this way is often optimal for the advisor and dominates any other information design, such as issuing completely uninformative recommendations, partially

[5] Maug and Rydqvist (2009), Filali Adib (2020), and Michaely, Ordonez-Calafi, and Rubio (2022) provide evidence that shareholders are strategic in that they focus on the events where their votes can sway the outcome.

informative but unbiased recommendations, or biasing the recommendations toward the more likely alternative.

At the same time, we show that the advisor will produce fully informative and unbiased research reports because it helps maximize the revenue from the fees charged to the subscribers. In this sense, the interest of the advisor is aligned with those of the shareholders to whom it sells subscriptions. The combination of public recommendations and private reports is thus central to the mechanism in our paper: the advisor serves the needs of its clients (subscribers), while limiting and biasing the information released through recommendations to obtain maximum revenue from selling the subscriptions. These results are consistent with the evidence on how institutional investors view proxy advisory services. Based on their survey of large asset managers, Bew and Fields (2012) write: "virtually unanimously, research participants highlighted the value they derive from ... [reports] ... digest[ing] and normaliz[ing] the vast quantities of data present in proxy statements in a short period of time," but conclude that the "value of ... voting recommendations is distinctly secondary" (see Section 5.4 for details).

Proxy advisors are often criticized for following a one-size-fits-all approach, i.e., giving recommendations that do not take into account firm-specific factors (e.g., Iliev and Lowry, 2015; Hayne and Vance, 2019). The one-size-fits-all approach can be rationalized by our model because it can help the advisor implement recommendations that create controversy. For example, one of ISS guidelines concerns busy directors: "Generally vote against or withhold from individual directors who sit on more than five public company boards." While a director's busyness, other things equal, is likely to be negatively correlated with his contribution to firm value, leading to partially informative recommendations (e.g., Fich and Shivdasani, 2006), this guideline does not take into account other director characteristics. Consider a nominee who sits on six boards but is an industry expert, has many years of experience, and whose other board seats are not too demanding. Whereas the research report will describe all the qualifications of the director allowing the subscribers to infer that the director is likely good for the firm, the recommendation will be negative, consistent with the "one-size-fits-all" criticism. Without reading the report, a non-subscribing shareholder will not know whether the negative recommendation reflects the true quality of the director or is purely given because of the director's six board seats.

We also show that the incentives to create controversy do not arise if the prior probability that the proposal is beneficial is close to 50%. In this case, the advisor designs an informative and unbiased report, but makes its recommendations completely uninformative. Intuitively, with priors close to 50%, an uninformative recommendation will naturally lead to a close vote, giving shareholders sufficient incentives to subscribe to the report. One way to implement such an uninformative recommendation policy is to always recommend the same action (always vote against or always in favor) on a given type of proposal. For example, both ISS and Glass Lewis 2022 guidelines specify a general recommendation against proposals to classify the board.

Our model has several implications for empirical research. First, it predicts that deviations from the proxy advisor's recommendations take a specific form: compared to the advisor, shareholders are more predisposed towards the a priori more likely alternative, essentially counteracting the bias in recommendations. Consider the above example of director elections, and suppose the advisor biases its recommendations against directors. Then, a positive recommendation implies that the director is good, and hence both non-subscribing and subscribing shareholders will vote in favor, leading to "rubberstamping" of positive recommendations. In contrast, a negative recommendation is sometimes given to good directors as well, so the informed subscribers will deviate from the recommendation and vote in their favor. Moreover, a negative recommendation generates a lot of uncertainty for the uninformed non-subscribers, leading some of them to vote in favor and others to vote against. As a result, the votes following negative recommendations will be much more dispersed. This prediction is consistent with the empirical evidence if we assume that management proposals are a priori sufficiently likely to be value-increasing. Management proposals that receive a positive recommendation typically pass with very high voting support, i.e., are "rubberstamped." In contrast, proposals that receive a negative recommendation often generate a lot of disagreement among shareholders. We discuss this evidence and other empirical implications in Section 5.

Moreover, these results suggest a reinterpretation of the empirical evidence on funds' voting behavior. Voting in favor of management when proxy advisors recommend "against" is often interpreted as lack of monitoring and pro-management bias. In contrast, our model emphasizes that proxy advisors' recommendations may not be the right benchmark since they can be biased against management to create controversy (see also related discussion in Spatt, 2021). Shareholders who deviate from negative recommendations and support management could be simply correcting the bias in recommendations, rather than voting in a biased way themselves. Instead, our model predicts that the votes of institutional investors managing large portfolios will be both informed and unbiased in equilibrium, and in this sense, could be considered a more suitable benchmark than proxy advisory recommendations (see Section 5.1 for details).

Our results are important for policy discussions of proxy advisors' biases. The focus of policy proposals has been on biases that arise if the proxy advisor also provides consulting to corporations (like ISS; see Section 4.1). In contrast, our paper emphasizes a different type of bias, which is inherent in selling advice to shareholders and emerges even if this is the only business of the advisor. One way to remove the controversy bias in the information provided to shareholders is to ban the advisor from issuing public recommendations. As we discuss in Section 4.1, this can be implemented by allowing the research report to contain relevant facts and analysis but not the vote recommendation per se. We show that even though the research report would still be fully informed and unbiased, the value implications of such a ban are not clear cut. The downside is that the ban on recommendations reduces the information available to non-subscribers, which makes their voting less informed and reduces firm value, whereas the upside is that the ban increases the fraction of subscribing, i.e., fully informed, shareholders.

We consider several extensions of the basic model. First, we assume that some shareholders obtain information about the proposal from other sources (e.g., via independent research). Second, we allow shareholders to have other, non-informational motives to become proxy advisors' clients (e.g., to reduce their litigation risk or to obtain vote execution services). We show that both the presence of other information sources and the additional motives to subscribe to proxy advisory services not only affect the size of the proxy advisor's client base, but also have interesting effects on the frequency of recommendations against the prior and the information content of the recommendations.

Related literature

Our paper contributes to the literature on shareholder voting,[6] including the growing literature on proxy advisors. Malenko and Malenko (2019) and Buechel, Mechtenberg, and Wagner (2021) analyze how the presence of proxy advisors affects shareholders' independent research; both papers take the quality of recommendations as given and assume they are unbiased. Levit and Tsoy (2020) show how one-size-fits-all recommendations arise in a cheap talk setting where a biased expert (e.g., proxy advisor) wants to convince other agents (e.g., shareholders) to accept a proposal. Unlike these papers, we focus on the information design problem of an advisor who maximizes profits from selling information. Ma and Xiong (2021) also study information design and show that biased recommendations can arise and even be associated with higher firm value. Unlike our paper, they do not distinguish between a public (recommendation) and

[6] E.g., Maug (1999), Brav and Mathews (2011), Levit and Malenko (2011), Esö, Hansen, and White (2014), Bar-Isaac and Shapiro (2020), Cvijanovic, Groen-Xu, and Zachariadis (2020), Bouton et al. (2021), and others.

private (research report) signal. As a consequence, in their model, the advisor's recommendations are biased only if shareholders are biased; if shareholders maximize firm value, then recommendations are unbiased. In contrast, in our setting, biased recommendations arise even though all shareholders maximize firm value, as a way to increase the probability of a close vote through manipulation of public information. This also distinguishes our paper from Matsusaka and Shu (2021), who study how proxy advisors cater their recommendations to biased shareholders such as socially responsible investing funds, and analyze the industry structure that emerges in equilibrium.

In the literature on Bayesian Persuasion, the closest papers to ours examine information design by a biased expert who wants to manipulate the elections to achieve his preferred outcome (Alonso and Camara, 2016; Bardhi and Guo, 2018; Chan et al., 2019; Kerman et al., 2020). In contrast, in our paper, the designer is unbiased in that it does not get any benefit from the vote outcome being in a particular direction; instead, it maximizes the ex-ante profits from information sale. This implies, in particular, that its information design policies are time-consistent, which is different from most other papers on Bayesian Persuasion. Another feature that distinguishes our paper is that the designer designs two signals for two different audiences – one public (for all shareholders) and one private (only for subscribers); furthermore, the composition of the latter audience is endogenously chosen by the designer. Inostroza (2021) also considers a designer (regulator) designing two signals for multiple audiences, but unlike our paper, these are both public signals on two different dimensions of the bank's fundamentals. In Leitner and Yilmaz (2019), the designer (bank) designs two signals, one is observed by the receiver (regulator), and the other is possibly only observed by the designer himself. Michaeli (2017) studies a manager who discloses the same signal to multiple users but, as in our paper, restricts access to information by optimally choosing the fraction of users who observe the signal. Goldstein and Huang (2016), Inostroza and Pavan (2020), and Alonso and Zachariadis (2021) analyze a regulator designing information (stress test) for multiple receivers who have private information, but unlike our paper, assume that the designer sends one signal to all receivers.[7] Chang and Szydlowski (2020) study persuasion in a matching market with multiple heterogeneous senders (investment advisors) and multiple heterogeneous receivers (their customers).

Our paper is also related to studies of the sale of information to traders in financial markets

[7]Other papers studying information design in the context of stress tests include Goldstein and Leitner (2018), Leitner and Williams (2022), and Orlov, Zryumov, and Skrzypacz (2020).

(e.g., Admati and Pfleiderer, 1986, 1990; Fishman and Hagerty, 1995; Cespa, 2008; Garcia and Sangiorgi, 2011).[8] One important conclusion in this literature is that the seller may benefit from adding noise to the information it sells, as a way to decrease the leakage of information through prices. In contrast, we show that the proxy advisor benefits from selling the most precise information to those subscribing to its report, but to increase the value of this information, it strategically biases the public information it reveals. These different results come from different interactions between the users of information: while traders compete with each other, shareholders in our model have interests on value maximization but want to free-ride on information acquisition of other shareholders. This feature also distinguishes proxy advisors from credit rating agencies, another type of information provider to investors (see Sangiorgi and Spatt (2017) for an overview of the literature and the comparison to proxy advisors). Another relevant difference between the two types of information providers is the pricing model – whereas credit rating agencies are paid by the issuers, proxy advisors are paid by investors. There are also certain similarities, such as the issue of multiple (albeit different from the proxy advisory setting) audiences explored in Frenkel (2015) and Bouvard and Levy (2018), and the provision of both paid and unpaid signals explored in Fulghieri, Strobl, and Xia (2014).

2 Model setup

Proposal to be voted on. The firm is owned by $N \geq 3$ shareholders, where N is odd. Each shareholder owns one share in the firm and has one vote. There is a proposal to be voted on, which is approved if at least $\frac{N+1}{2}$ shareholders vote in favor. Let $d \in \{0, 1\}$ denote the decision on the proposal, where $d = 1$ ($d = 0$) corresponds to proposal approval (rejection).

The value of the proposal to shareholder i, $u_i(d, \theta)$, depends on the unknown state $\theta \in \{0, 1\}$ and on the importance of the proposal to the shareholder, v_i, as follows:

$$u_i(d, \theta) = v_i \cdot u(d, \theta),$$

where

[8] Another related paper is Lizzeri (1999) who studies an intermediary choosing a policy of information disclosure to potential buyers when the informed seller pays for the services of the intermediary.

$$
\begin{aligned}
u(1,\theta) &= \begin{cases} 1, & \text{if } \theta = 1, \\ -1, & \text{if } \theta = 0, \end{cases} \\
u(0,\theta) &= 0.
\end{aligned}
$$

In other words, approving the proposal increases (decreases) shareholder value if $\theta = 1$ ($\theta = 0$), while rejecting the proposal and maintaining the status quo leaves firm value unchanged. The ex-ante probability that the proposal is value-increasing is $\Pr(\theta = 1) = \mu \in (0,1)$.

Thus, all shareholders' interests are perfectly aligned, but the extent to which they care about the proposal, v_i, may differ across them. The role of heterogeneous v_i is to produce variation across shareholders in their incentives to pay for advice so as to make more informed voting decisions. There are multiple reasons for this heterogeneity in practice. First, v_i can depend on the sensitivity of the fund manager's compensation to the value of its portfolio firm, which differs significantly across funds: it is the highest for hedge funds, the lowest for index funds, and intermediate for actively-managed mutual funds. Second, heterogeneity in v_i can be due to the fact that shareholders' voting practices are scrutinized by regulators and market participants to a different extent – for example, mutual funds, which are required to disclose their votes, may have higher v_i than other institutional investors. Differences in v_i can also reflect differences in the size of asset managers. For example, v_i is likely to be higher for shareholders with a larger number of firms in their portfolio.[9] In addition, v_i can indirectly capture the shareholder's position in the firm, although this interpretation needs to be used cautiously given our assumption of equal stakes across shareholders.

We assume that v_i is an independent (across shareholders) draw from a distribution with a continuous and differentiable c.d.f. $H(\cdot)$ over $[\underline{v}, \overline{v}]$ with $0 \leq \underline{v} < \overline{v} \leq \infty$. When making his information acquisition decision, each shareholder knows his own v_i, but not v_j of other shareholders (in practice, shareholders indeed do not perfectly know the ownership structure of their portfolio firms). This assumption is made for simplicity, as it allows us to focus on symmetric equilibria at the information acquisition stage.

Shareholders maximize $u_i(d,\theta)$ minus any costs of information acquisition. While our baseline model features one proposal in one firm, Section 5.5 discusses how our analysis can be extended to a setting where the subscription covers multiple proposals and firms.

[9] More precisely, assuming that all firms have the same prior probability that a certain type of proposal is value-increasing, v_i would be proportional to the number of firms in the shareholder's portfolio that have this proposal on the agenda.

Information structure. Each shareholder is initially uninformed and has prior μ that the proposal is value-increasing. There is a seller of information, the proxy advisor, who maximizes its profits from information sale to the shareholders. The proxy advisor has an informative signal about the state; for simplicity, we assume that it knows the state with certainty. The advisor prepares two signals, a private signal available only to the subscribers and a public signal available to everyone, and sets the fee it charges for the private signal. The private signal, denoted $\mathcal{R} = (R, \{\phi(\cdot|\theta)\}_{\theta\in\{0,1\}})$, consists of a finite signal space R and two distributions $\{\phi(\cdot|\theta)\}_{\theta\in\{0,1\}}$ of signal realizations $r \in R$, which describe the probability of signal realization r in each state. The public signal, denoted $\mathcal{S} = \left(S, \{\gamma(\cdot|r)\}_{r\in R}\right)$, consists of a finite signal space S and a family of distributions $\{\gamma(\cdot|r)\}_{r\in R}$ of signal realizations $s \in S$, which describe the probability of signal realization s for each realization $r \in R$. We will refer to the private signal policy $\mathcal{R}$ as the *research report* of the proxy advisor and to the public signal policy $\mathcal{S}$ as the *voting recommendation* of the proxy advisor. This formulation means that the research report is informative about the state θ, while policy $\mathcal{S}$ determines how the content of the research report is mapped into the voting recommendation.

This setup corresponds to the observed voting practices. Prior to the shareholder meeting, proxy advisors deliver to their subscribers a research report that presents a detailed analysis of the proposals on the agenda. In addition, for each proposal, the report contains a recommendation on whether to vote in favor or against. As discussed in the introduction, these recommendations are often made public by the media (especially for contentious meetings in which proxy advisors recommend against management), as well as by the party supported by the advisor. One way to think of policy $\mathcal{S}$ is that it represents the voting guidelines that proxy advisors publicly disclose and update every year. These guidelines describe, for various types of proposals, detailed rules and criteria that the advisor plans to use when making its recommendations for each individual firm. However, the information design problem of the advisor should be understood more generally than just the design of the guidelines.

We first conjecture and later verify (see Section 3.3.3) that it is optimal for the proxy advisor to design a fully informative research report, i.e., $R = \{0, 1\}$ and $r = \theta$. Thus, by subscribing to the advisor's services, a shareholder learns the state with certainty. Given this, the proxy advisor's problem is how to design the public recommendation for each possible realization of state θ: $\mathcal{S} = (S, \{\gamma(\cdot|\theta)\}_{\theta\in\{0,1\}})$. For example, in the case of a binary recommendation space $S = \{0, 1\}$, which we will show to be optimal, the recommendation policy is characterized by two probabilities, $\Pr(s = 1|\theta = 1) = \gamma(1|1)$ and $\Pr(s = 1|\theta = 0) = \gamma(1|0)$.

The advisor chooses policies $\mathcal{R}$ and $\mathcal{S}$ to maximize its expected payoff. Note that the advisor's ex-ante optimal information policies will be dynamically consistent for the advisor, in contrast to Kamenica and Gentzkow (2011) and most other models of Bayesian persuasion. This is because the advisor is only interested in maximizing ex-ante profits from information sale and does not obtain any ex-post benefit from a vote outcome in either direction. Thus, once the state is realized, the advisor does not gain from deviating and reporting a different signal for either the research report or the recommendation.

Timeline. The timeline is shown in Figure 1. At stage 1, the advisor chooses the information policy $(\mathcal{R}, \mathcal{S})$ and fee f that it charges to shareholders for subscribing to its research report. At stage 2, having observed the information policy $(\mathcal{R}, \mathcal{S})$, fee f, and their realizations of proposal importance v_i, shareholders simultaneously and non-cooperatively decide on whether to pay fee f to subscribe to the report. At stage 3, the advisor observes θ and issues report $r \in R$ and recommendation $s \in S$ based on the information policy $(\mathcal{R}, \mathcal{S})$. At stage 4, all shareholders observe the realization of s, and shareholders that became the subscribers also observe r. Shareholders then simultaneously decide whether to vote "for" $(a_i = 1)$ or "against" $(a_i = 0)$ the proposal, and the proposal gets implemented if it is approved by the majority.

We assume that shareholders cannot abstain from voting. This assumption is consistent the idea that many institutional investors, such as mutual funds, do not abstain for reputational and regulatory reasons. However, as we explain in footnote 15 below, the equilibrium of our model would also be an equilibrium in a model in which abstention is allowed.

The equilibrium concept is a symmetric Bayes-Nash equilibrium: at the information acquisition stage, shareholders with the same v_i follow the same subscription strategy, and at the voting stage, shareholders with the same information follow the same voting strategy. We focus on equilibria in weakly undominated strategies at the voting stage. In particular, when we write that there is a unique equilibrium, we mean a unique equilibrium in this class. We also assume that if, for a given fee f, there exist multiple equilibria at the information acquisition and voting stages, the advisor can induce his preferred equilibrium.[10]

[10] This equilibrium selection allows us to abstract from equilibria of the following form: the advisor charges any positive fee, no shareholder buys the report, and at the voting stage all shareholders vote the same way (for example, all vote in favor or all vote against). This is an equilibrium because a shareholder is never pivotal, so there is no profitable deviation at the voting stage and, since the value of information is zero, no shareholder wants to deviate and buy the report even if the fee is arbitrarily small.

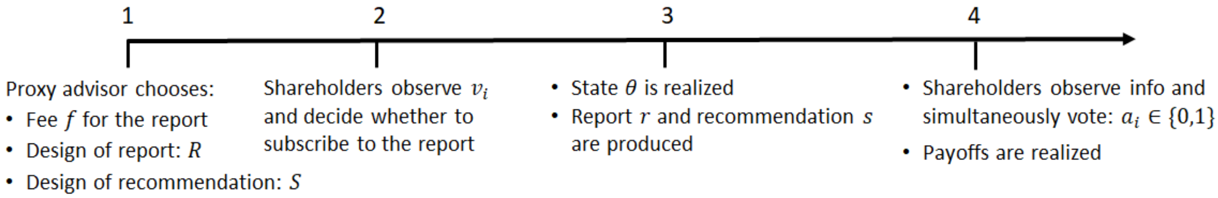

Figure 1. Timeline of the model.

2.1 Benchmark case: One shareholder

We start by considering the benchmark case of a single shareholder. This case helps capture the setting in which shareholders can perfectly coordinate with each other.[11]

The shareholder estimates the probability of making the correct decision given the information in the research report, $\Pr(d = \theta|\mathcal{R})$, and given the information in the public recommendation, $\Pr(d = \theta|\mathcal{S})$, and subscribes to the report if and only if

$$v_i \left[\Pr(d = \theta|\mathcal{R}) - \Pr(d = \theta|\mathcal{S})\right] \geq f.$$

It follows that the advisor wants to maximize $\Pr(d = \theta|\mathcal{R})$ and to minimize $\Pr(d = \theta|\mathcal{S})$. The former is achieved by designing a fully informative research report, and the latter by designing an uninformative public recommendation. The advisor's optimal fee solves the standard problem of a monopolist, who faces a trade-off between quantity sold (i.e., probability of the shareholder subscribing to the report) and the price paid by the customers (i.e., the fee charged for the report). We conclude:

Proposition 1 (Benchmark case). *If $N = 1$, the proxy advisor always designs an uninformative recommendation and a fully informative research report.*

Thus, when there are no coordination frictions among shareholders, the proxy advisor does not share any information for free and shares all its information through the report that is available for a fee. In practice, however, coordination among shareholders is not perfect (see, e.g., the discussion about frictions to communication in Section 5.5). Then, as our subsequent

[11] If shareholders perfectly coordinate and act as one, their willingness to pay for information is determined by the sum of their individual concerns about the voting outcome, $\sum_{i=1}^{N} v_i$.

analysis demonstrates, the advisor's information design problem is no longer trivial, and in particular, it may now be optimal to share some information for free.

3 Analysis

We solve the model by backward induction. We focus on the case in which the research report $\mathcal{R}$ is fully informative and solve for the equilibrium in the voting game, the equilibrium subscription decisions, pricing of information, and the public recommendation design. In Section 3.3.3, we complete the solution by proving that making the research report fully informative is optimal for the advisor.

3.1 Voting stage

Consider the voting stage following any given realization of the state θ and public recommendation s. Since the payoff of a shareholder is proportional to the importance v_i of the proposal to him, his vote does not depend on v_i, and depends only on his information set.

First, consider a shareholder who subscribed to the report. As we conjecture above, the report conveys the state perfectly, so the shareholder knows the state with certainty. Hence, it is a weakly dominant strategy for him to vote based on the state: $a_i = \theta$.

Next, consider shareholder i who did not subscribe to the report. This shareholder observes the public recommendation s and forms his posterior belief $\mu_s = \Pr(\theta = 1|s)$ about the state knowing the information policy $\mathcal{S}$. In addition, as in the literature on strategic voting (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998), the shareholder takes into account that his vote is only pivotal for the outcome if the number of "for" votes among other shareholders is exactly $\frac{N-1}{2}$, i.e., if the votes of others are split (we denote this set of events by Piv). In particular, the shareholder rationally conditions his decision not only on the recommendation s but also on the information that must be true when he is pivotal.[12] The shareholder finds it optimal to vote "for" ("against") if his posterior belief $\Pr(\theta = 1|s, Piv)$ is

[12]Such strategic voting is related to the idea of the "winner's curse" in auctions: both in auctions and in voting, an agent's action only matters in a particular situation – when his bid is the highest and when his vote is pivotal, respectively. Since other agents (other bidders and subscribing shareholders, respectively) have valuable information that the agent does not know, the agent rationally conditions his decision on the information that would be true when his decision matters.

above (below) $\frac{1}{2}$, and is indifferent if it is exactly equal to $\frac{1}{2}$, where by Bayes' rule,

$$\Pr(\theta = 1|s, Piv) = \frac{\Pr(Piv|\theta = 1, s)\,\mu_s}{\Pr(Piv|\theta = 1, s)\,\mu_s + \Pr(Piv|\theta = 0, s)\,(1-\mu_s)}. \tag{1}$$

Intuitively, the shareholder updates his beliefs based on two pieces of information: the recommendation s and the information he learns from the fact that he is pivotal. The former corresponds to terms μ_s and $1-\mu_s$ in (1), and the latter corresponds to terms $\Pr(Piv|\theta = 1, s)$ and $\Pr(Piv|\theta = 0, s)$. To find these two latter terms, let $\tilde{q}(v)$ denote the probability, as perceived by the shareholder, with which another shareholder subscribes to the report if his type is v, and let $q \equiv \int_{\underline{v}}^{\bar{v}} \tilde{q}(v)\,dH(v)$ denote the shareholder's unconditional perceived probability that each other shareholder subscribes to the report. In addition, let π denote the probability, as perceived by the shareholder, with which each non-subscribing shareholder votes for the proposal given recommendation s. In equilibrium, these perceived probabilities coincide with the actual probabilities of subscribing to the report and voting in favor given s.

Note that if $q = 0$ or $q = 1$, the advisor's profit is zero: if $q = 0$, no shareholder buys the report, and $q = 1$ is only possible if the fee is zero.[13] It follows that $q = 0$ and $q = 1$ never arise on equilibrium path, and we can focus on $q \in (0, 1)$.[14]

Given q and π, the shareholder's perceived probability that each other shareholder votes "for" conditional on state θ and signal s is $q + (1-q)\pi$ if $\theta = 1$ and $(1-q)\pi$ if $\theta = 0$, so the probability that there are exactly $\frac{N-1}{2}$ "for" votes among other shareholders is

$$\Pr(Piv|\theta = 1, s) = C_{N-1}^{\frac{N-1}{2}} \left[q + (1-q)\pi\right]^{\frac{N-1}{2}} \left[(1-q)(1-\pi)\right]^{\frac{N-1}{2}}, \tag{2}$$

$$\Pr(Piv|\theta = 0, s) = C_{N-1}^{\frac{N-1}{2}} \left[(1-q)\pi\right]^{\frac{N-1}{2}} \left[1 - \pi(1-q)\right]^{\frac{N-1}{2}}, \tag{3}$$

where $C_n^k = \frac{n!}{k!(n-k)!}$ is the binomial coefficient.

The next result characterizes the equilibrium in the voting game for any realization s:

Proposition 2 (Shareholders' voting strategies). *Consider the voting game that follows*

[13] Indeed, suppose $q = 1$, so that all shareholders know the state with certainty. In any equilibrium in which a shareholder is pivotal with a strictly positive probability, he must vote according to the state, but then all shareholders always vote the same way, so such an equilibrium does not exist. Hence, the only possible equilibria are those where no shareholder is ever pivotal. In these equilibria, the value of information is zero, so shareholders have no incentive to pay for the report if $f > 0$.

[14] Formally, this is because the advisor can set fee f for which there is an equilibrium with $q \in (0, 1)$ and the advisor's profit is positive (we show this statement below). Thus, given our assumption that the advisor can induce his preferred equilibrium for a given fee, the advisor will not find it optimal to induce $q = 0$ or $q = 1$.

a recommendation realization s, and suppose that each shareholder believes that other shareholders subscribe to the report with probability $q \in (0,1)$. This game has a unique equilibrium in undominated strategies, which takes the following form. If a shareholder is a subscriber, he votes according to the state, $a_i = \theta$. If a shareholder is not a subscriber and $\mu_s \in (0,1)$, he votes in favor with probability $\frac{1}{2}$ if $\mu_s = \frac{1}{2}$ and with probability

$$\pi(q, \mu_s) = \frac{z_s(1-2q) - 1 + \sqrt{(z_s - 1)^2 + 4q^2 z_s}}{2(z_s - 1)(1-q)} \tag{4}$$

if $\mu_s \neq \frac{1}{2}$, where $z_s \equiv \left(\frac{\mu_s}{1-\mu_s}\right)^{\frac{2}{N-1}}$, $\pi(q, \mu_s) \in (0,1)$ and is increasing in μ_s. If a shareholder is not a subscriber and $\mu_s = 1$ ($\mu_s = 0$), he votes in favor with probability one (zero).

Thus, all shareholders with precise information (subscribers) vote according to their information, whereas shareholders with imprecise information (non-subscribers) randomize between voting for and against, and the expected voting support by non-subscribers is increasing in the belief μ_s that the proposal is value-increasing.[15]

Figure 2 illustrates Proposition 2 and highlights that the sensitivity of non-subscribing shareholders' votes to their posterior μ_s is affected by the expected fraction of subscribers, q. Intuitively, a shareholder who observes s expects other non-subscribers to likely vote along the posterior belief μ_s. But then, the fact that the vote is split implies that the subscribing shareholders (who know the state) are relatively more likely to have voted against belief μ_s. Thus, by conditioning on being pivotal, the shareholder updates his beliefs in the direction opposite of μ_s, so his vote becomes less reliant on μ_s. The extent of this learning from being pivotal depends on q. If q is low, the shareholder learns little from the fact that the vote is split, since almost all other shareholders are non-subscribers and have the same information as him. As a result, his voting strategy relies heavily on whether posterior μ_s is above or below $\frac{1}{2}$, as illustrated by the solid (red and green) lines in Figure 2 for $q = 0.0001$ and 0.01, respectively. In particular, the bold red line shows that in the limit when $q \to 0$, $\pi(q, \mu_s)$ converges to a step integer function $1\left\{\mu_s > \frac{1}{2}\right\}$. In contrast, if q is relatively high, a non-subscriber learns

[15] This equilibrium would also be an equilibrium of the model in which shareholders can abstain, and in the event of a tie, the proposal is implemented randomly. Indeed, consider a non-subscribing shareholder. Conditional on being pivotal and the public recommendation, his posterior belief about the state is $\frac{1}{2}$. If the shareholder does not abstain, he randomizes between voting in favor and against. If he deviates and abstains, there is a tie, and the proposal is implemented randomly. Since the shareholder's posterior belief is $\frac{1}{2}$, his assessed probability of the correct decision being made is $\frac{1}{2}$ regardless of whether he abstains or not. Thus, he is indifferent between abstaining and not abstaining, so the equilibrium continues to exist. The same argument applies if, in the event of a tie, the proposal is always accepted or is always rejected.

quite a bit from the fact that the votes of others are split, since the probability that each voter is perfectly informed is now higher. As a consequence, the probability that he votes "for" becomes less sensitive to posterior μ_s around $\mu_s = \frac{1}{2}$. This is illustrated by the dashed and dash-dotted lines in Figure 2 for $q = 0.1$ and $q = 0.6$, respectively.

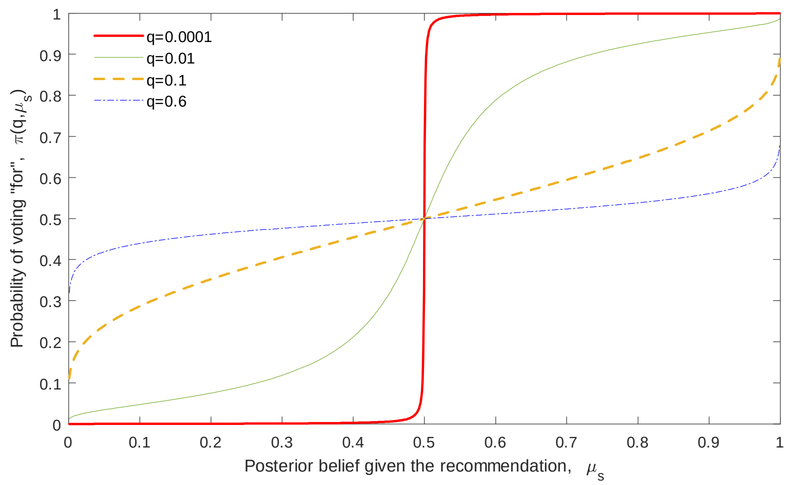

Figure 2. Shareholders' voting strategies. The figure illustrates Proposition 2 by plotting the probability $\pi(q, \mu_s)$ that a non-subscribing shareholder votes for the proposal as a function of his posterior belief μ_s for $N = 25$ and four different values of q: 0.0001, 0.01, 0.1, and 0.6.

3.2 Information acquisition stage

Since the recommendation of the proxy advisor is publicly observable, the shareholder's incentive to purchase the research report reflects his incremental value from the additional information contained in the report, i.e., from learning the state.[16] In particular, it captures the incremental value to the shareholder from a more efficient voting outcome due to his own voting being more informed. This value is a function of probability q, with which he expects other shareholders to subscribe to the report, the recommendation policy $\mathcal{S}$, and the shareholder's concern about the voting outcome v_i. Let $V_i(q, \mathcal{S})$ denote this value.

To derive $V_i(q, \mathcal{S})$, consider shareholder i's value from buying the report, and thus learning the state with certainty, for a particular realization of the public recommendation $s \in S$. If the posterior belief μ_s induced by this recommendation is one or zero, the shareholder knows

[16] A traditional challenge confronting information producers, such as proxy advisory firms, is the free-rider problem, which leads to difficulty in being paid for the information that they generate (e.g., Arrow (1962)).

the state with certainty from observing the recommendation, so his incremental value from the report is zero. If $\mu_s \in (0,1)$, the shareholder's value from the report is positive. His vote changes the decision of the firm only if the votes of other $N-1$ shareholders are split. We denote the probability of this event by $\Pr(Piv|q,\mu_s)$ and note that

$$\Pr(Piv|q,\mu_s) = \Pr(Piv|\theta=1,s)\,\mu_s + \Pr(Piv|\theta=0,s)\,(1-\mu_s)\,, \tag{5}$$

where $\Pr(Piv|\theta,s)$ is given by (2)–(3) with $\pi = \pi(q,\mu_s)$ given by (4). Conditional on the shareholder being pivotal, learning the state with certainty changes his probability of voting correctly from $\frac{1}{2}$ to 1, and his expected value from doing so is $\frac{v_i}{2}$.[17] Thus, for any realization s, the shareholder's value from the report is $\frac{v_i}{2}\Pr(Piv|q,\mu_s)$. Aggregating over all possible realizations of $s \in S$, the value of the report to shareholder i is

$$V_i(q,\mathcal{S}) = v_i \cdot V(q,\mathcal{S})\,, \tag{6}$$

where

$$V(q,\mathcal{S}) = \frac{1}{2}\sum_{s\in S}\Pr(Piv|q,\mu_s)\,\tau_s \tag{7}$$

and $\tau_s \equiv \mu\gamma(s|1) + (1-\mu)\gamma(s|0)$ is the probability of recommendation s. Intuitively, $V(q,\mathcal{S})$ is the average (before the recommendation is realized) probability that a shareholder is pivotal multiplied by the benefit $\frac{1}{2}$ of learning the state conditional on being pivotal.

Hence, shareholder i buys the report if and only if $v_i \cdot V(q,\mathcal{S}) \geq f$, or equivalently,

$$v_i \geq \frac{f}{V(q,\mathcal{S})}. \tag{8}$$

It follows that, given fee f, the probability that each shareholder subscribes to the advisor's report is $1 - H\left(\frac{f}{V(q,\mathcal{S})}\right)$. We can equivalently rewrite this expression as an inverse demand function. Specifically, to ensure that, on average, fraction q of shareholders subscribe to the

[17] This is because, as explained above, the shareholder rationally conditions his decisions on the information that is true when he is pivotal. As Proposition 2 and its proof show, if $\mu_s \in (0,1)$, then the shareholder randomizes between voting for and against, and his posterior belief $\Pr(\theta=1|s,Piv)$ conditional on s and being pivotal is $\frac{1}{2}$. Thus, the shareholder believes that without the research report, he votes correctly ($a_i = \theta$) with probability $\frac{1}{2}$. With the research report, the shareholder votes correctly with probability 1. Since $u(\theta,\theta) - u(1-\theta,\theta) = 1$, the shareholder's expected value from learning the state (i.e., his willingness to pay for the report) conditional on s and being pivotal is $\frac{v_i}{2}$. In particular, this conditional willingness to pay does not depend on the prior μ. See Section A.8 of the Online Appendix for a detailed derivation.

report, the fee must be:

$$f = V(q, \mathcal{S}) H^{-1}(1-q), \tag{9}$$

where $H^{-1}(\cdot)$ is an increasing function. We summarize these arguments as follows:

Proposition 3 (Shareholders' information acquisition). *For a given fee f and public recommendation policy $\mathcal{S}$, shareholder i subscribes to the proxy advisor's report if and only if the proposal is sufficiently important to him, $v_i \geq f/V(q, \mathcal{S})$, where $V(q, \mathcal{S})$ is given by (7). If the equilibrium fraction of subscribers q is in $(0,1)$, it satisfies (9).*

3.3 Proxy advisor's problem

The proxy advisor chooses fee f and information policy $\mathcal{S}$ to maximize its expected profit. Since q is the expected fraction of subscribers and each subscriber pays fee f, the advisor's expected profit is Nqf. Using (9), the advisor's problem is to choose q and $\mathcal{S}$ to solve

$$\max_{q,S} qH^{-1}(1-q) \left(\sum_{s \in S} \Pr(Piv|q, \mu_s) \tau_s \right). \tag{10}$$

We solve this problem by decomposing it into two steps. First, in Section 3.3.1, we take the fraction of subscribers q as given and find the advisor's optimal public recommendation policy $\mathcal{S}^*(q)$ for any q. This optimal policy maximizes the average probability of a split vote, $\sum_{s \in S} \Pr(Piv|q, \mu_s) \tau_s$, which, according to (6), is exactly what determines shareholders' willingness to pay for the report. Second, in Section 3.3.2, we find the advisor's optimal fraction of subscribers and the fee that induces it, taking into account $\mathcal{S}^*(q)$.

3.3.1 Public recommendation design

Note that any information policy $\mathcal{S}$ can be represented as a combination of the set of possible recommendations S, posterior beliefs $\{\mu_s, s \in S\}$ that these recommendations induce, and the frequencies with which each of these recommendations is produced $\{\tau_s, s \in S\}$. Since shareholders use Bayesian updating, the average posterior belief must be equal to the prior:

$$\sum_{s \in S} \mu_s \tau_s = \mu. \tag{11}$$

Condition (11) is referred to in the information design literature as the Bayes plausibility constraint (e.g., Kamenica and Gentzkow, 2011). Kamenica and Gentzkow (2011) show that the reverse is also true: any combination $\{S,\ \{\mu_s, \tau_s\}_{s\in S}\}$ that satisfies (11) can be induced by some information policy $\mathcal{S} = (S, \gamma(\cdot|\theta=0), \gamma(\cdot|\theta=1))$. It follows that for a given q, we can rewrite the advisor's problem as

$$\begin{array}{c}\max_{S,\{\mu_s,\tau_s\}} \sum_{s\in S} \Pr(Piv|q,\mu_s)\,\tau_s \\ s.t.\ \sum_{s\in S} \mu_s \tau_s = \mu.\end{array} \tag{12}$$

In other words, taking q as given, how should the advisor design its recommendations to maximize the average probability that the vote is split? The intuition behind the Bayes plausibility constraint is that the advisor cannot systematically deceive the shareholders, which, in turn, limits its ability to inflate the probability of a split vote. Suppose, for example, that one of the advisor's recommendations induces $\mu_s \approx 0.5$, i.e., a lot of uncertainty, and is thereby likely to result in a split vote. If the prior belief μ is sufficiently high (e.g., $\mu = 0.8$), then the Bayes plausibility constraint implies that this "controversial" recommendation cannot be given too frequently: other recommendations, which induce a more positive posterior belief (above 0.8) and are less likely to result in a split vote, must be sufficiently frequent as well.

We solve (12) using the standard approach in the information design literature, referred to as "concavification" (e.g., Kamenica and Gentzkow, 2011). This approach considers the objective of the designer as a function of any given posterior belief (in our setting the objective function is $\Pr(Piv|q,\cdot)$, the probability of a shareholder being pivotal), and then takes the concave closure of this function, which we denote $P(q,\cdot)$. Then, the largest expected probability of a split vote that the advisor can achieve given prior belief μ is $P(q,\mu)$, and the information policy that achieves this maximum can be found graphically. To apply this approach to our setting, we start by deriving the convexity/concavity properties of $\Pr(Piv|q,\cdot)$.

Lemma 1. *There exist $\underline{\mu} \in \left(0, \frac{1}{2}\right)$, $\bar{\mu} \in \left(\frac{1}{2}, 1\right)$, and $\varepsilon > 0$, such that: (i) if $0 < q < \frac{1}{2}$, $\Pr(Piv|q,\mu_s)$ is strictly convex in μ_s for $\mu_s \in \left(0, \underline{\mu}\right)$ and $\mu_s \in (\bar{\mu}, 1)$, and strictly concave in μ_s for $\mu_s \in \left(\frac{1}{2}-\varepsilon, \frac{1}{2}+\varepsilon\right)$; (ii) if $\frac{1}{2} < q < 1$, $\Pr(Piv|q,\mu_s)$ is strictly concave in μ_s for $\mu_s \in \left(0, \underline{\mu}\right)$, $\mu_s \in (\bar{\mu}, 1)$, and $\mu_s \in \left(\frac{1}{2}-\varepsilon, \frac{1}{2}+\varepsilon\right)$.*

These properties are illustrated in Figure 3: the solid (blue) line in each panel plots $\Pr(Piv|q,\mu_s)$ as a function of μ_s for $N = 25$ and different values of the fraction of subscribers q. To see the intuition, recall from Proposition 1 and Figure 2 that non-subscribers' voting strategy $\pi(q, \mu_s)$ is less sensitive to μ_s around $\mu_s = \frac{1}{2}$ when q is higher. If q is very small, most shareholders do not subscribe to the report, and each non-subscribing shareholder bases his vote mostly on μ_s: if $\mu_s > 0.5$ ($\mu_s < 0.5$), he is very likely to vote in favor (against). Hence, the probability of a split vote is small (zero in the limit of $q \to 0$) except in a narrow interval around $\mu_s = 0.5$. This case is illustrated in panels A and B for $q = 0.01$. As q increases, non-subscribing shareholders' votes become less sensitive to μ_s around $\frac{1}{2}$ and, in addition, there is a higher fraction of subscribers, who vote according to the true state. As a result, the probability of a split vote becomes less sensitive to μ_s around $\frac{1}{2}$ (see panels C and D for $q = 0.1$ and $q = 0.6$).

To see how we can use Lemma 1 to derive the optimal information policy using the concavification approach, consider several information policies, illustrated in panels A and B of Figure 3 for $q = 0.01$. First, consider a *fully informative* policy, which can be implemented by giving two recommendations: one negative (given if $\theta = 0$) and one positive (given if $\theta = 1$). This policy corresponds to two posterior beliefs, $\mu_0 = 0$ and $\mu_1 = 1$, and is represented by two orange circles in panel A. Since the probability of a split vote is zero for both posterior beliefs, the average probability of a split vote is zero as well, so such a policy is never optimal.

An alternative information policy is an *uninformative* recommendation. For example, it can be implemented by always recommending against, always recommending in favor, or randomizing between recommending in favor and against in a way uncorrelated with the state. This policy corresponds to a single posterior belief equal to the prior μ, and is illustrated by the blue circle in panel A. If μ is close to 0.5 (as in panel A), there is a lot of a priori uncertainty, so the probability of a split vote given an uninformative recommendation is relatively high, and moreover (as we show in Proposition 5), is higher than for any other information policy. However, if μ is sufficiently far from 0.5, for example, $\mu = 0.8$ (as in panel B), the proposal is a priori likely to be value-increasing, so the probability of a split vote upon an uninformative recommendation is small. In this case, the advisor can increase the average probability of a split vote by making its recommendations *partially informative*.

To see this, suppose the advisor gives two recommendations: positive (denoted $s = 1$) and negative (denoted $s = 0$), which induce posteriors $\mu_1 = 1$ and μ_0 close to 0.5, illustrated in panel B. The advisor can implement this policy by always giving a negative recommendation if the proposal is value-decreasing, but also sometimes giving a negative recommendation if it is value-increasing.[18] Bayes plausibility implies that the frequencies of the two recommendations,

[18] More specifically, this information policy has $\gamma(0|0) = 1$ and $\gamma(0|1)$ such that $\mu_0 = \Pr(\theta = 1|s = 0) =$

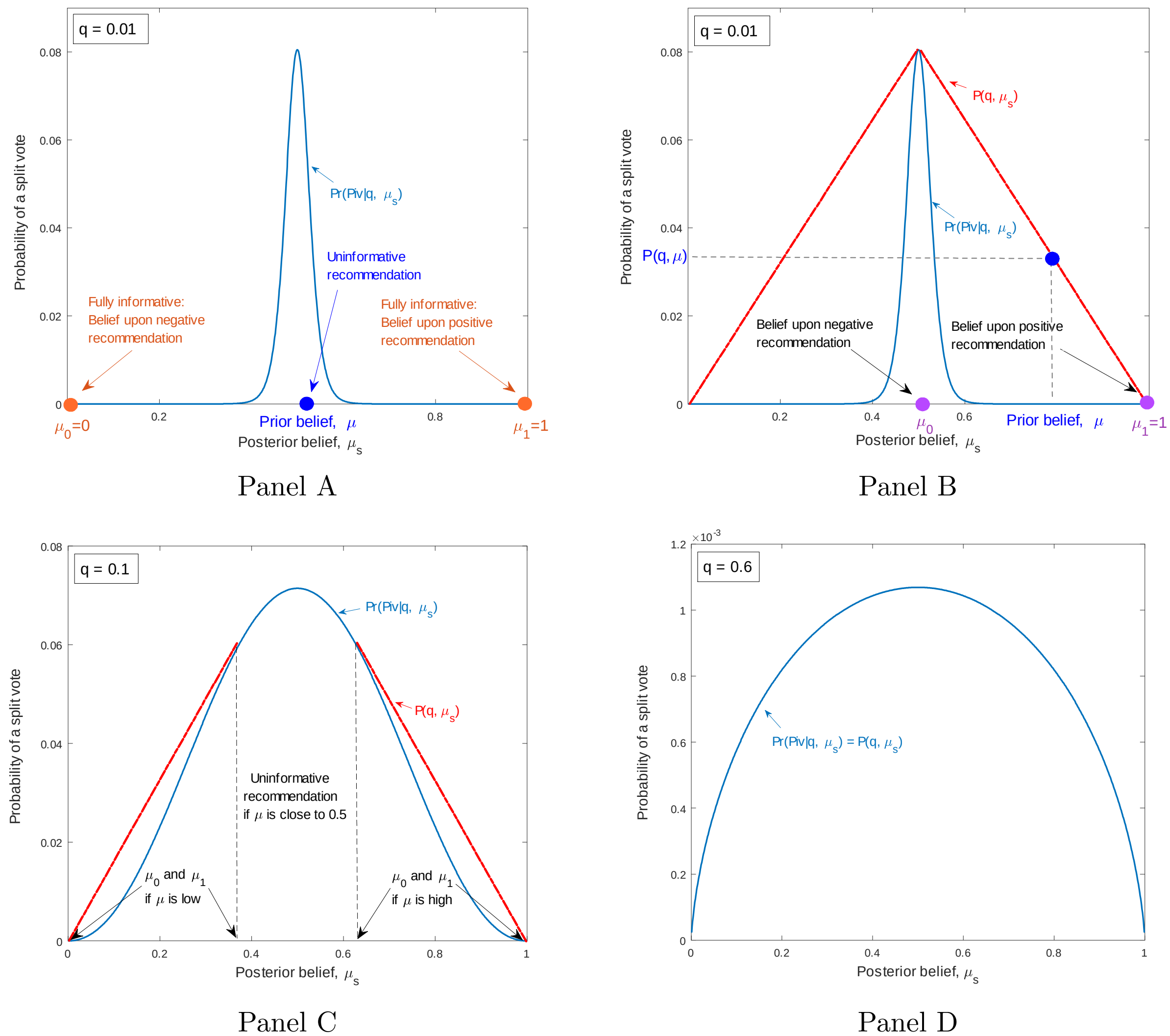

Figure 3. Design of public recommendations for different values of q. The solid (blue) lines in all panels plot the probability of a shareholder being pivotal, $\Pr(Piv|q, \mu_s)$, as a function of the posterior belief μ_s for N=25. The dashed (red) lines in panels B and C present the concave closure of this function, $P(q, \mu_s)$. Panels A and B illustrate the case of q=0.01. Panel A shows a fully informative recommendation and an uninformative recommendation for μ close to 0.5. Panel B shows the optimal (partially informative) recommendation for μ=0.8. Panel C illustrates the optimal recommendation for q=0.1 and different μ: if μ is low (below 0.38), it induces posteriors 0 and 0.38; if μ is high (above 0.62), it induces posteriors 0.62 and 1; if μ is between the two cut-offs, it is uninformative. Panel D illustrates the case of q=0.6, in which the optimal recommendation is uninformative for all μ.

τ_1 and τ_0, are such that $\tau_1 + \tau_0\mu_0 = \mu$, that is, $\tau_0 = \frac{1-\mu}{1-\mu_0}$. The positive recommendation reveals the state with certainty and thus never results in a split vote, whereas the negative recommendation generates a lot of uncertainty and results in a split vote with a high probability. The average probability of a split vote is the weighted average between 0 and $\Pr(Piv|q,\mu_0)$ with weights τ_1 and τ_0, and the Bayes plausibility constraint pinpoints this average probability to the one illustrated in panel B: it is the value of the linear function (depicted by the red dashed line connecting the two points) at prior belief μ. As is clear from panel B, this partially informative policy leads to a higher average probability of a split vote than an uninformative policy. Finally, note that for the average probability of a split vote to be maximized, the dashed red line should be tangent to the solid blue curve at point μ_0, that is, it should concavify function $\Pr(Piv|q,\cdot)$. Hence, the maximum average probability of a split vote is $P(q,\mu)$.

The following result uses Lemma 1 and formalizes this intuition.

Proposition 4 (Optimality of creating controversy for a fixed q). *The optimal public recommendation is binary, $S = \{0,1\}$. If $q \in \left(0, \frac{1}{2}\right)$ and priors are sufficiently asymmetric, the optimal recommendation creates controversy in the following sense:*

1. *If $\mu \leq \mu_1(q) \equiv \frac{(1-4q^2)^{\frac{N-1}{2}}}{1+(1-4q^2)^{\frac{N-1}{2}}}$, recommendation $s=0$ induces belief $\mu_0 = 0$, and recommendation $s=1$ induces belief $\mu_1(q)$ and is given with probability $\frac{\mu}{\mu_1(q)}$.*

2. *If $\mu \geq \mu_0(q) \equiv \frac{1}{1+(1-4q^2)^{\frac{N-1}{2}}}$, recommendation $s=0$ induces belief $\mu_0(q)$ and is given with probability $\frac{1-\mu}{1-\mu_0(q)}$, and recommendation $s=1$ induces belief $\mu_1 = 1$.*

Proposition 4 implies that if prior beliefs are sufficiently asymmetric and the proxy advisor does not have too many subscribers, the optimal recommendation is partially informative. For example, consider a large enough μ (the case of a small μ is analogous). Then a positive recommendation reveals that the state is 1 and leads shareholders to *rubberstamp* the proposal: all shareholders vote in favor, regardless of whether they are subscribers or non-subscribers, so the research report is irrelevant. In contrast, a negative recommendation is always given if $\theta = 0$ but is also often given if $\theta = 1$, and hence is "*controversial*:" it only reveals that the probability of $\theta = 1$ is close to 50%. The frequency of this "controversial" recommendation is $\Pr(s=0) = \frac{1-\mu}{1-\mu_0(q)}$, which exceeds the probability that the proposal is value-decreasing

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$\frac{\gamma(0|1)\mu}{\gamma(0|1)\mu+1-\mu}$, which implies $\gamma(0|1) = \frac{1-\mu}{\mu}\frac{\mu_0}{1-\mu_0}$. As discussed in the introduction, it can be implemented by proxy voting guidelines that specify giving a negative recommendation if a certain condition is satisfied (e.g., a director has too many board seats).

($\Pr(\theta = 0) = 1 - \mu$) by a factor of $\frac{1}{1-\mu_0(q)}$. In this sense, the advisor's optimal recommendation policy is biased against the a priori more likely alternative. The extent to which the optimal recommendation is biased depends on the expected fraction of subscribers q as follows:

Corollary 1. *For any μ, there exists $\bar{q}(\mu)$ such that a partially informative recommendation policy of the form in Proposition 4 is optimal if and only if $q \leq \bar{q}(\mu)$. For $q \leq \bar{q}(\mu)$, if q increases:*

(i) there is a higher frequency of recommendations against the prior (measured by $\frac{\Pr(s=1)}{\Pr(\theta=1)}$ or $\Pr(s = 1) - \Pr(\theta = 1)$ for small μ, and by $\frac{\Pr(s=0)}{\Pr(\theta=0)}$ or $\Pr(s = 0) - \Pr(\theta = 0)$ for large μ);

(ii) recommendations against the prior become "less convincing," in the sense that the posterior belief upon them is closer to the prior.

These comparative statics results can be seen in Figure 3; we explain them for the case of large μ for simplicity. The comparison between panels B and C reveals that as q increases, the posterior belief upon the "controversial" recommendation (i.e., belief μ_0 upon $s = 0$) moves farther from 0.5 and closer to μ. Intuitively, the advisor faces the following trade-off when picking μ_0. One option is to induce a very uncertain posterior (i.e., μ_0 close to 0.5), so that the probability of a split vote upon the controversial recommendation is very high. However, Bayes plausibility implies that such a recommendation cannot be given too frequently, which is costly for the advisor because the "rubberstamped" recommendation $s = 1$, which never results in a split vote, must be given more frequently. An alternative is to induce a less uncertain, i.e., closer to the prior, posterior μ_0, which leads to a lower probability of a split vote but is given more frequently. Hence, the advisor's trade-off is between a higher probability of a split vote conditional on the controversial recommendation and a higher frequency of the split vote taking place. When q increases, shareholders' votes become less sensitive to their posterior beliefs around $\frac{1}{2}$ (see Figure 2), so the probability of a split vote is relatively high even if the posterior is not too close to $\frac{1}{2}$. As a result, the advisor finds it optimal to pick a less uncertain posterior belief but with a higher frequency. Overall, as q increases, controversial recommendations become more frequent, but "less convincing" (in the sense of inducing a posterior closer to the prior) and result in a lower likelihood of a split vote. Once q becomes high enough (such that $\mu_0(q) = \mu$), this partially informative recommendation policy becomes completely uninformative: $s = 0$ is given with probability one and induces belief μ.

While partially informative recommendations are optimal when priors are sufficiently asymmetric, they are not always optimal. If the prior belief is close to $\frac{1}{2}$, there is already a lot of uncertainty about the correct decision, so the probability of a split vote is high and shareholders have incentives to become informed. In this case, there is no benefit to manipulating public information, as formalized in the following proposition:

Proposition 5 (Optimality of uninformative recommendations). *There exists $\varepsilon > 0$ such that if $\mu \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$, the optimal public recommendation is uninformative.*

This result is illustrated for $q = 0.1$ and $N = 25$ in panel C of Figure 3: it shows that when μ is between the two cutoffs, $\mu_1(q)$ and $\mu_0(q)$, the optimal recommendation is uninformative. The comparison of panels B–D of Figure 3 also shows that as the expected fraction of subscribers increases, the range of priors for which the optimal recommendation is uninformative widens (the function $\Pr(Piv|q, \mu_s)$ is concave over a wider interval around $\mu_s = \frac{1}{2}$). Moreover, if q increases to over $\frac{1}{2}$ (see Lemma 1), it is suboptimal to give information for free regardless of the priors, as can be seen for $q = 0.6$ in panel D. Intuitively, as q increases, shareholders' votes become less sensitive to the posterior belief μ_s around $\frac{1}{2}$, and hence the marginal benefit from inducing beliefs closer to 50% upon a negative recommendation decreases. Since there is also a cost of doing so — the split vote never occurs upon a positive recommendation, the range of priors for which the optimal recommendation is informative shrinks.

3.3.2 Pricing of information

We now analyze the fee charged by the proxy advisor for the research report. The advisor faces the standard trade-off between price and quantity: a lower fee attracts shareholders with lower valuations (i.e., lower v_i) and increases quantity sold, but leaves more rents to shareholders with high v_i. In addition, the unique feature of the proxy advisory setting is that a shareholder's valuation of the seller's product depends on the number of other shareholders purchasing the product (i.e., on q) and on the seller's design of public recommendations (see Eq. (6)–(7)). These are the two margins the advisor uses to further increase its revenues.

Suppose the advisor chooses a fee such that it sells, in expectation, to a fraction q of shareholders. As shown above, the highest probability of a split vote that the advisor can achieve (when the prior is μ and the expected fraction of subscribers is q) is $P(q, \mu)$, where $P(q, \cdot)$ is the concave closure of $\Pr(Piv|q, \cdot)$. Thus, using (10), the advisor's expected profit is

$\frac{N}{2}qH^{-1}(1-q)P(q,\mu)$, so the optimal fee induces q that solves[19]

$$\max_{q} qH^{-1}(1-q)P(q,\mu). \tag{13}$$

The next result provides sufficient conditions under which the advisor's optimal fee and information design create controversy.

Proposition 6 (Optimality of creating controversy under endogenous pricing). *Let q^* be the maximum of $qH^{-1}(1-q)$. If $q^* < \frac{1}{2}$ and priors are sufficiently asymmetric, the advisor sets the fee and designs public recommendations to create controversy:*

1. *If $\mu \leq \mu_1(q^*)$, recommendation $s = 0$ induces belief $\mu_0 = 0$, and $s = 1$ induces belief $\mu_1(q^*)$ and is given with probability $\frac{\mu}{\mu_1(q^*)}$. The price of the research report is $f = 2^{1-N}\mu C_{N-1}^{\frac{N-1}{2}}H^{-1}(1-q^*)$, and each shareholder subscribes to it with probability q^*.*

2. *If $\mu \geq \mu_0(q^*)$, recommendation $s = 0$ induces belief $\mu_0(q^*)$ and is given with probability $\frac{1-\mu}{1-\mu_0(q^*)}$, and $s = 1$ induces belief $\mu_1 = 1$. The price of the research report is $f = 2^{1-N}(1-\mu)C_{N-1}^{\frac{N-1}{2}}H^{-1}(1-q^*)$, and each shareholder subscribes to it with probability q^*.*

The condition $q^* < \frac{1}{2}$ in Proposition 6 implies that the distribution $H(\cdot)$ is such that the advisor finds it optimal to sell the report to a relatively small fraction of the shareholders and demand a high price from them. Moreover, to increase these shareholders' willingness to pay for the report, the advisor designs public recommendations to create controversy. The condition $q^* < \frac{1}{2}$ is material: as shown in Lemma 1, the function $\Pr(Piv|q,\mu_s)$ is strictly concave around $\mu_s = 0$ and $\mu_s = 1$ when $q > \frac{1}{2}$, and hence a controversial recommendation of the form described by Proposition 4 may no longer be optimal. The next result shows this formally.

Proposition 7. *Let q^* be the maximum of $qH^{-1}(1-q)$, and suppose that $q^* \geq \frac{1}{2}$ and the distribution $H(\cdot)$ has an increasing hazard rate. Then, for any prior belief μ, any pair $(q, \mathcal{S}^*(q))$, where $\mathcal{S}^*(q)$ is a partially informative recommendation policy of the form in Proposition 4, is dominated by some pair $(\hat{q}, \mathcal{S}_{uninf})$, where $\mathcal{S}_{uninf}$ is an uninformative recommendation.*

[19] In general, a given fee f can induce multiple equilibria at the information acquisition and voting stages, corresponding to different values of q. However, given our earlier assumption that the advisor can induce his preferred equilibrium, we simply optimize over q. The fee that induces this optimal q is then given by (9).

To illustrate the intuition for Propositions 6 and 7, consider an example in which the intensity of shareholders' concerns about the proposal v_i follows the power distribution.

Example 1: $H(x) = x^{\alpha}$. Note that $qH^{-1}(1-q) = q(1-q)^{\frac{1}{\alpha}}$ is increasing in q if and only if $q < \frac{\alpha}{\alpha+1}$, so $q^* = \frac{\alpha}{\alpha+1}$. Thus, if $\alpha < 1$ and the prior belief is sufficiently asymmetric, the advisor finds it optimal to design a partially informative and biased recommendation and sell the research report to an expected fraction $\frac{\alpha}{\alpha+1} < \frac{1}{2}$ of shareholders. Intuitively, $\alpha < 1$ means that the distribution of v_i has a positive skew: most shareholders care little about the proposal, but some care quite a lot. In this case, it is optimal to sell the report to a relatively small fraction of shareholders who care significantly about the proposal, and to increase these shareholders' private value of the report by inducing controversy via a biased public recommendation. In contrast, if $\alpha > 1$, the distribution of v_i has a negative skew: there are many shareholders who care a lot about the proposal and some who care very little. In this case, it is optimal to sell the report to a large fraction of shareholders, and any recommendation that induces controversy (of the form in Proposition 4) is dominated by an uninformative recommendation.

Properties of strategic recommendation design. We conclude this section by exploring the comparative statics of the public recommendation policy designed by the advisor.

First, consider the effect of the distribution of shareholders' concerns about the proposal, $H(\cdot)$. By changing the trade-off between price and quantity sold, a change in $H(\cdot)$ leads to a change in q^*, the optimal expected fraction of shareholders who subscribe to the research report. Corollary 1 and Propositions 6–7 imply that as q^* increases, recommendations against the prior become more frequent but less convincing. Moreover, once q^* increases sufficiently, such controversial recommendation design is no longer optimal. The reason, as explained earlier, is that a higher q^* leads to stronger learning from being pivotal, which in turn leads to a lower sensitivity of shareholders' votes to posterior μ_s around $\frac{1}{2}$.

A related intuition leads to the following comparative statics in the number of shareholders:

Corollary 2. *Suppose $q^* < \frac{1}{2}$. Then for any μ, there exists $\bar{N}(\mu)$ such that a partially informative recommendation policy of the form in Proposition 4 is optimal if and only if $N \leq \bar{N}(\mu)$. For $N \leq \bar{N}(\mu)$, if N increases, recommendations against the prior become more frequent but less convincing (in the sense of inducing posterior beliefs closer to the prior).*

The reason for this result is that as N increases, shareholders learn more from being pivotal, so their voting strategies become less sensitive to their posterior belief μ_s around $\frac{1}{2}$. To see the intuition, consider a non-subscribing shareholder observing recommendation s. If $\mu_s > \frac{1}{2}$, the shareholder expects other non-subscribing shareholders to be relatively more likely to support the proposal. By conditioning on the event that the vote is split, the shareholder infers that sufficiently many shareholders voted against the proposal, i.e., against μ_s. If the overall number of shareholders is small, these opposing shareholders could be other non-subscribing shareholders (since they randomize between voting for and against). However, if N is large, there is a much higher chance that at least some of these opposing shareholders are the subscribers, who observe the state with certainty and vote against only if the proposal is value-decreasing. Hence, a larger N leads to a stronger inference from being pivotal, and thus a lower sensitivity to μ_s. As a result, the probability of a split vote is less sensitive to μ_s around $\frac{1}{2}$, so the vote is likely to be split for a larger set of posterior beliefs. This reduces the benefits of creating controversy for the same reasons as before.

Together, Proposition 1 and Corollary 2 imply that controversial recommendations arise only with multiple shareholders, but that the number of shareholders cannot be too large.

3.3.3 Private research report design

So far, we have solved for the optimal public recommendation design, the fee charged for the research report, and the equilibrium at the information acquisition and voting stages conjecturing that the advisor finds it optimal to design the private research report that perfectly reveals the state. The next proposition verifies that this conjecture is indeed correct:

Proposition 8 (Optimality of a fully informative research report). *A fully informative private research report, i.e., one with $R = \{0, 1\}$ and $r = \theta$, is optimal for the advisor.*

Loosely speaking, a fully informative research report is optimal because adding noise to the report dilutes its value and lowers the willingness to pay of shareholders who subscribe to it. The more precise intuition, which is formalized in the proof, is a combination of two steps. The first step points out that for any report with an arbitrary number of signals, there is a report with two signals for which each shareholder's willingness to pay is not lower. Intuitively, any signal induces one of the three best responses from the subscribers: they either vote for the proposal, or against the proposal, or randomize between voting for and against. In the latter case, the subscriber's posterior belief must be $\frac{1}{2}$, so the value from buying the report is zero. Hence, the advisor is better off designing a report that avoids such signals. As for the former two types of signals, the advisor could equivalently combine all signals that induce subscribers to vote "for" into one, and all signals that induce subscribers to vote "against" into the other.

The second step points out that, since the advisor is the only source of information about the state, one can think of the game induced by the report as the basic game (with a fully informative report) in which the signals in the report are referred to as "states". In this redefined game, an imperfectly informative report means that the value upon taking the "correct" decision (voting according to the report) is lower than one. The arguments in Section 3.2 then imply that a shareholder's value from the report is lower than if it were fully informative.

4 Policy implications and extensions

This section discusses the policy implications of our results and presents several extensions.

4.1 Ban on recommendations

Policy discussions of proxy advisors' biases typically focus on the concern that some proxy advisors, like ISS, receive consulting fees from the companies that they evaluate, which may lead to a pro-management bias in their recommendations (Li, 2018). Our paper highlights a very different bias, which arises even if providing voting advice is the only business of the proxy advisor (e.g., as in the case of Glass Lewis). The controversy bias we identify cannot be alleviated by separating the two businesses or disclosing the advisor's consulting relationships with companies, which has been the focus of policy proposals.

One policy that could address the bias highlighted in our paper is to prohibit the advisor from issuing public recommendations. A potential way to implement this policy is to ban proxy advisors from including voting recommendations in their reports, and only allowing the reports to contain relevant analysis and facts. Since, in practice, such information is continuous and multidimensional, it would be less likely to become public.

Formally, suppose the advisor is not allowed to generate any public signal $\mathcal{S}$. Note that the absence of a public signal is equivalent to an uninformative recommendation policy $\mathcal{S}_{uninf}$, i.e., the one satisfying $\Pr(\mu_s = \mu) = 1$. Hence, this constrained model can be analyzed in the same way as the general model analyzed above. In particular, a fully informative private research report is still optimal if there is a ban on recommendations (the proof of Proposition 8 applies to any policy $\mathcal{S}$, including an uninformative one). In this sense, the ban on recommendations indeed removes the controversy bias in the information provided to the shareholders.

Let q_{uninf} denote the expected fraction of subscribers that the advisor targets if the recommendation is uninformative, and let $(q^*, \mathcal{S}^*)$ denote the optimal expected fraction of subscribers and recommendation policy in the basic model. We are interested in comparing $\Pr(d = \theta | q^*, \mathcal{S}^*)$ and $\Pr(d = \theta | q_{uninf}, \mathcal{S}_{uninf})$, where $\Pr(d = \theta | q, \mathcal{S})$ is the probability of a correct decision being made when the recommendation policy is $\mathcal{S}$ and the probability of becoming a subscriber is q. Since the ban on recommendations only has bite if $\mathcal{S}^*$ is partially informative, we assume that the conditions of Proposition 6 are satisfied, so that $\mathcal{S}^*$ generates controversy.

Our model suggests that a ban on recommendations has both an upside and a downside, and the effectiveness of the ban depends on the trade-off between the two. The downside is that the ban on recommendations reduces information of non-subscribers and leads to less informative voting by them (biased but partially informative recommendations are better than no information at all). If the fraction of subscribers stayed the same, the reduction of information of non-subscribers would lead to lower-quality voting outcomes. The upside of the ban is that it induces the proxy advisor to target a higher fraction of subscribers. Intuitively, when priors are asymmetric and there are no public recommendations, the probability of a split vote is relatively small because non-subscribing shareholders are all likely to vote according to the priors. By marginally increasing q, the advisor makes non-subscribing shareholders' votes less sensitive to the priors (see Figure 2) and, in addition, increases the probability that the votes of subscribers will counteract the votes of non-subscribers. Both effects increase the probability of a split vote and thereby shareholders' willingness to pay for the report. The next proposition formalizes this trade-off:

Proposition 9 (Ban on public recommendations). *Suppose that $\mathcal{S}^*$ is a partially informative recommendation from Proposition 6, and suppose distribution $H(\cdot)$ has an increasing hazard rate. Then a ban on public recommendations:*

(i) decreases the quality of decision-making if the fraction of subscribers is kept constant: $\Pr(d = \theta | q^*, \mathcal{S}^*) > \Pr(d = \theta | q^*, \mathcal{S}_{uninf})$*;*

(ii) increases the probability with which a shareholder becomes a subscriber, i.e., $q_{uninf} > q^*$*.*

To illustrate this trade-off most starkly and show how either of the two effects can dominate, we consider an example in which there are only two types.

Example 2: Suppose $N = 25$, $\mu = 0.9$, and $v_i \in \{v_L, v_H\}$, where $v_L = 1 < v_H$ and $\Pr(v_i = v_H) = 0.1$. We can think of this distribution as the limit of a continuous distribution with two modes, and all the above arguments apply (the derivations for this example are in Section A.5 of the Online Appendix). If v_H is sufficiently high (above 5), then without a ban on recommendations, the advisor finds it optimal to set a high fee, so that only shareholders of type v_H subscribe to the report (i.e., $q = 0.1$), and to produce partially informative recommendations. In particular, the positive recommendation induces a posterior of 1, while the negative recommendation induces a posterior of 0.62, as in Panel C of Figure 3. Then, voting results in the correct decision with probability 92.5%. If a ban on public recommendations is introduced, there are two cases. If v_H is not too high, for example, $v_H = 5$, the advisor finds it optimal to sell the report to some of the low types as well. Specifically, we show that the optimal fee induces $q = 0.257$ (0.1 of type v_H and 0.157 of type v_L), in which case voting results in the correct decision with probability $95.65\% > 92.5\%$. Intuitively, since all low types have the same valuation, it is easy for the advisor to attract many additional subscribers by decreasing the fee by a small amount. As a result, the positive effect from additional subscribers, corresponding to part (ii) of Proposition 9, is very high and dominates the negative effect from the loss of a partially informative public signal. In contrast, if v_H is high, for example, $v_H = 7$, the advisor finds it optimal to sell the report to high types only, as without a ban. As a result, the positive effect from additional subscribers is non-existent, and the probability of the correct decision falls to $90.3\% < 92.5\%$.

4.2 Exogenously informed shareholders

The basic model assumes that the proxy advisor is the only source of information for shareholders. In reality, large institutional investors perform their own research and thus are likely to vote informatively irrespectively of research reports and recommendations of the advisor. Does the presence of such "exogenously" informed shareholders lead a profit-maximizing advisor to design more or less biased recommendations?

To study this question, consider the following simple extension of the basic model. Suppose that if a shareholder's draw of v_i is above a certain cutoff $\hat{v}$, then the shareholder is informed about the state without buying the research report. Intuitively, we interpret such shareholders as large institutional investors, which are informed on their own by virtue of having strong incentives to invest in information. Denote $\chi \equiv 1 - H(\hat{v})$ the expected fraction of such exogenously informed shareholders. The case of $\hat{v} = \bar{v}$ (i.e., $\chi = 0$) captures the basic model.

Let q denote the expected fraction of shareholders who are informed about the state, either because they become informed without the proxy advisor ($v_i > \hat{v}$) or because they subscribe to the advisor's research report. The advisor's design of recommendations is the same function of q as in the basic model (see the proof of Proposition 10), and thus induces controversy if priors are sufficiently asymmetric. Differently from the basic model, the optimal q in the range where controversial recommendations are optimal, $q_e^*(\chi)$, now maximizes $(q-\chi) H^{-1}(1-q)$. We show that $q_e^*(\chi)$ increases in χ and is greater than q^* in the basic model. In other words, a greater presence of exogenously informed shareholders leads to a higher overall fraction of informed shareholders even after accounting for strategic pricing of information by the advisor. Intuitively, an increase in χ means that the advisor can no longer target shareholders with the highest valuations v_i, and hence the trade-off between price and quantity leads the advisor to target shareholders with marginally lower valuations. Using our interpretation above, if more institutions start doing their own in-house research, the proxy advisor lowers its fees to sell to institutions with a lower willingness to pay.

Since the overall fraction of informed shareholders increases, the probability of a split vote becomes less sensitive to posterior belief μ_s around $\frac{1}{2}$. As a result, recommendations against the prior become more frequent but less convincing for the same reasons as in Corollary 1. The following proposition summarizes these results:

Proposition 10. *Suppose that a partially informative recommendation of the form in Proposition 4 is optimal. If the expected fraction χ of exogenously informed shareholders increases: (i) the overall expected fraction of informed shareholders $q_e^*(\chi)$ increases as well; and (ii) recommendations against the prior become more frequent but less convincing (in the sense of inducing posterior beliefs closer to the prior).*

4.3 Other motives to subscribe to proxy advisory services

In practice, shareholders have additional motives for subscribing to proxy advisory services in addition to their desire to make more informed voting decisions. First, both ISS and Glass Lewis provide vote execution services, which include submission of the votes and filing regulatory forms through special voting platforms. As Shu (2022) points out, the users of proxy advisors' platforms get access to proxy advisors' research reports. Second, becoming a subscriber can help asset managers reduce the litigation risk associated with their voting practices. According to former SEC commissioner Daniel M. Gallagher, "for the price of purchasing the proxy advisory firm's recommendations, an investment adviser could ward off potential litigation over its conflicts of interest" (Gallagher (2014, p. 5)). In this section, we study how the presence of such additional motives affects the design of recommendations.

Suppose that each shareholder gets an additional benefit ω from subscribing to the proxy advisor. Then (6) implies that the overall value of subscribing is $\omega + v_i V(q, \mathcal{S})$, where $V(q, \mathcal{S})$ is given by (7). An increase in ω allows the advisor to charge its clients higher fees. As a result, the price-quantity trade-off induces the advisor to target a larger client base, leading to an increase in the fraction $q^*(\omega)$ of shareholders who receive the report. A higher $q^*(\omega)$, for the same reasons as before, makes it optimal for the advisor to give controversial recommendations more frequently, but induce posteriors that are closer to the priors:

Proposition 11. *Suppose that a partially informative recommendation of the form in Proposition 4 is optimal. If shareholders' additional motives to subscribe become stronger (ω increases): (i) the expected fraction of informed shareholders increases; and (ii) recommendations against the prior become more frequent but less convincing (in the sense of inducing posterior beliefs closer to the prior).*

5 Empirical implications and discussion

In this section, we relate our results to the empirical evidence on shareholders' voting patterns, present new empirical predictions, and discuss several assumptions.

5.1 Bias in recommendations and the correct voting benchmark

In our model, the bias in the proxy advisor's recommendations arises because the advisor is maximizing its profits from information sales, rather than the value of the operating companies. One way to explore our conclusions is to compare the recommendations of the proxy advisors with the votes cast by large asset managers, whose interests are potentially more directly aligned with value maximization. Empirical evidence highlights that proxy advisors often make recommendations that are more anti-management than the votes of the major index funds. For example, Brav et al. (2020) show that large index funds, such as BlackRock and Vanguard, seem to be more supportive of management than ISS in proxy contests, and the estimates of investor ideology and corporate governance preferences in Bolton et al. (2020) and Bubb and Catan (2021) suggest a similar pattern for other types of proposals and both ISS and Glass Lewis recommendations.

Implication 1: **Reinterpreting the evidence**. Large index funds are frequently criticized because their greater support for management relative to proxy advisors' recommendations is viewed as a sign of passivity or pro-management bias. In contrast, our results suggest a different interpretation of such voting behavior and emphasize that proxy advisors' recommendations may not be the most suitable benchmark (see also discussion in Spatt, 2021). Empirically, management proposals are typically approved with a high support rate, suggesting that "for management" is often the a priori more likely alternative. For example, the average support rate is about 90% for say-on-pay proposals (Ertimur, Ferri, and Oesch, 2013; Malenko and Shen, 2016) and about 95% in director elections (Cai, Garner, and Walkling, 2009; Ertimur, Ferri, and Oesch, 2018).[20] Assuming that "for management" is indeed the a priori more likely alternative, shareholders who deviate from negative proxy advisors' recommendations and support the management, could instead be optimally correcting the "controversy" bias in these recommendations.

Implication 2: **Correct voting benchmark**. If proxy advisors' recommendations are not always the correct benchmark, what could be an alternative? Our results suggest that the votes of large asset managers could potentially reflect a more suitable benchmark. To see this, recall that one interpretation of v_i, the extent to which an asset manager cares about the proposal, is the asset manager's size. Under this interpretation, the model predicts that institutional investors that manage larger portfolios are more likely to to vote based on their analysis of proxy advisors' reports, rather than purely based on the proxy advisor's recommendations (see Eq. (8)). This prediction is consistent with the idea in Iliev and Lowry (2015), Iliev, Kalodimos, and Lowry (2021), and Gantchev and Giannetti (2021) that larger asset managers are more likely to be "active voters." Moreover, in our model, the votes of shareholders with large v_i are both informed and unbiased, and thus could be viewed as a more suitable benchmark than the biased recommendations of the proxy advisor.

5.2 Voting outcomes and proxy advisors' recommendations

Implication 3: Rubberstamping. Our model predicts a particular pattern of shareholders' deviations from proxy advisors' recommendations. In situations where the advisor's recommendation is in favor of the a priori expected alternative, shareholders should "rubberstamp" this recommendation (see the discussion following Proposition 4). In contrast, if the recommendation is against the a priori expected alternative, we expect a large disagreement in shareholders'

[20] This is expected, given that proposals are endogenously put forward by management and thus are more likely to be designed to appeal to shareholders. Additionally, many of these votes are about matters that are on average non-contentious (such as most director elections).

votes, with some shareholders voting in favor and some against.

Assuming, given the evidence discussed above, that "for management" is the a priori more likely alternative in say-on-pay votes and director elections, we expect shareholders to rubberstamp pro-management recommendations, but not to rubberstamp anti-management recommendations. The evidence is consistent with this prediction. Table 1 in Malenko and Shen (2016) shows that positive ISS recommendations on say-on-pay are accompanied by 93% average shareholder support and zero failed proposals (out of 1,764 proposals with a positive recommendation in their sample), consistent with "rubberstamping." In contrast, negative say-on-pay recommendations are accompanied by 69% average support and an 11% likelihood of the proposal being rejected. Table 1 in Ertimur, Ferri, and Oesch (2018) shows a similar pattern for director elections: all directors with a positive recommendation from either ISS or Glass Lewis received majority support (with the vast majority receiving more than 90% votes in favor), but there is much greater dispersion in votes upon a negative recommendation.

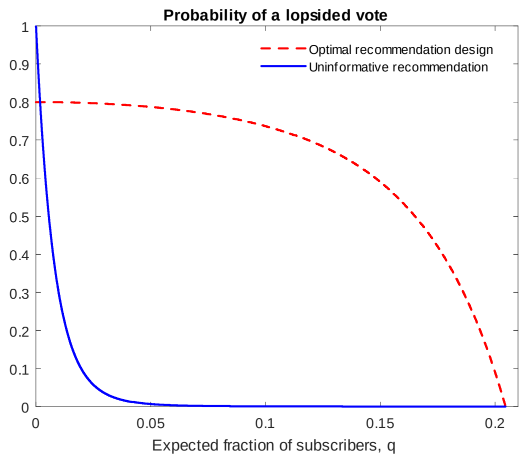
Implication 4: High probability of both close votes and lopsided votes. The rubberstamping of recommendations that are consistent with the priors leads to another implication: both close votes and lopsided votes (i.e., votes where almost all shareholders vote the same way) are relatively frequent. Lopsided votes are prevalent in practice (see the discussion above) but rarely arise in models of strategic voting, so their relatively high frequency is a distinguishing feature of our model. To show this formally, we compare the frequency of a vote tally of either 0 or N between two settings: (1) in equilibrium of our model when controversial recommendations are optimal, and (2) in a setting without public recommendations but with the same fraction of subscribers.[21] This comparison, as well the comparison of the frequency of a close vote (defined as a vote tally of $\frac{N-1}{2}$ or $\frac{N+1}{2}$) between the two settings, is shown in Figure 4.

First, the figure confirms that the probability of a close vote is higher under optimally designed recommendations. This implication directly follows from the preceding analysis because the advisor designs recommendations to maximize the probability that a shareholder is pivotal (see Eq. (12)).[22] Second, the figure shows that except when the fraction of subscribers is close to zero, optimally designed recommendations also generate a higher probability of a lopsided vote. The intuition is the following. In a setting without public recommendations,

[21] We deliberately set the fraction of subscribers to be the same in the two settings to simplify the comparison. The additional difference, which we do not account for in this comparison, is that when there are no public recommendations, the advisor targets a higher fraction of subscribers, $q_{uninf} > q^*$ (see Proposition 9).

[22] As we show in Section B of the Online Appendix, the probability of a close vote coincides, up to a constant that only depends on N, with the probability of a shareholder being pivotal.

the probability of a lopsided vote is close to zero except when q is very small. This is because a lopsided vote requires all N shareholders to vote the same way, and since each shareholder votes in favor with probability between zero and one, the chance of this happening goes down to zero very fast as N increases. In contrast, if public recommendations create controversy, then conditional on the recommendation that is consistent with the priors, both subscribing and non-subscribing shareholders vote in favor with probability one, and thus lopsided votes occur with a non-trivial probability.[23]



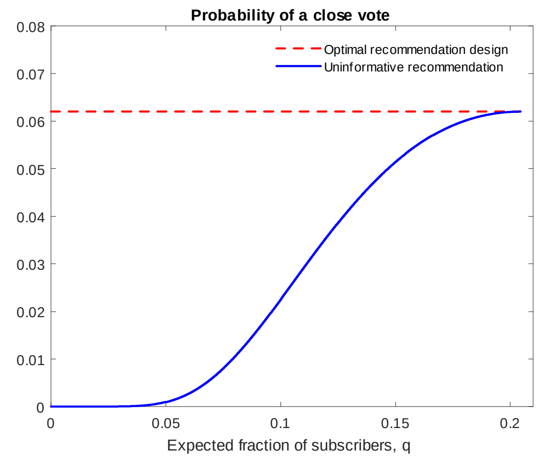


Figure 4. Probability of a close vote and of a lopsided vote. The figure plots the probability of a close vote (defined as a vote tally of either $\frac{N-1}{2}$ or $\frac{N+1}{2}$) and of a lopsided vote (defined as a vote tally of either 0 or N) as a function of the expected fraction of subscribers q in two settings: 1) the equilibrium in our model, and 2) the setting without public recommendations that features expected fraction of subscribers q. The figure focuses on the range of q that satisfies $\mu_0(q) \geq \mu$, i.e., for which a partially informative recommendation of the form in Proposition 4 is optimal. The parameters are $N = 25$ and $\mu = 0.9$. The relevant derivations are in Section B of the Online Appendix.

Implication 5: Informed shareholders and proxy advisors' recommendations. The empirical literature highlights that asset managers differ in their incentives to become informed and, as a consequence, in the extent to which they rely on proxy advisors' recommendations (e.g., Iliev and Lowry (2015)). Such a difference also arises in our model: shareholders with larger concerns about the proposal are more likely to base their votes on the analysis in the research reports, rather than entirely on the recommendations.

[23] This relationship is reversed when q is close to zero. In this case, without public recommendations, non-subscribing shareholders vote primarily according to their priors, i.e., against the proposal with probability close to one. Since the fraction of non-subscribers is close to one, lopsided votes occur with probability close to one. In contrast, under strategic recommendation design, the "controversial" recommendation induces a high probability of a close vote, and hence the probability of a lopsided vote is bounded away from one.

Moreover, our model highlights that there is another direction of causality as well. Not only proxy advisors' recommendations have differential effects on shareholders with different concerns about voting outcomes, but the opposite is also true: the extent to which shareholders care about voting and have incentives to become informed will influence the recommendations. This implication follows from Propositions 6, 10, and 11: shareholders' concerns about voting (distribution H) and how informed shareholders are (fraction q and/or χ) determine the advisor's costs and benefits of creating more controversy via recommendations. A specific prediction, which follows from Corollary 1 (see Section B.3 in the Online Appendix) and could be tested using time-series or cross-sectional variation, is the following:

Prediction: If the fraction of informed shareholders increases: (1) the proxy advisor's recommendations against the a priori more likely alternative become more frequent; (2) the probability of a close vote upon such recommendations declines.

5.3 One-size-fits-all approach

Proxy advisors have been criticized for issuing recommendations according to pre-specified guidelines that do not take into account the individual circumstances of the company.[24] One-size-fits-all guidelines naturally arise in the context of our model, as a way to implement recommendations with controversy. The "overboarded director" policy discussed in the introduction is one example: both ISS and Glass Lewis 2022 guidelines state that they will recommend against directors who sit "on more than five public company boards." Such a guideline creates controversy around negative recommendations: without reading the report, shareholders observing a negative recommendation cannot infer whether it reflects low director quality or purely his six board positions, and hence are unsure how to vote. In this sense, guidelines that commit the advisor to give a negative recommendation if a certain condition is satisfied help implement the information policy described in Part 2 of Proposition 4. Proxy advisors use a similar approach for other issues, such as executive compensation.[25]

[24]See, e.g., the discussion at the SEC's 2013 Proxy Advisory Services Roundtable and the field study evidence in Hayne and Vance (2019). Other aspects of the criticism of a "one-size-fits-all" approach include that the advice would not adjust to reflect other portfolio holdings of the investors (which would be particularly relevant in a merger and acquisition situation) or the tax circumstances of the investor. Since the preferences of shareholders in our model are aligned, we do not capture these other aspects of the criticism.

[25]For example, in addition to various qualitative and quantitative characteristics of the compensation plan that ISS takes into account, there are several "overriding negative factors" that will lead to a negative recommendation regardless of how the company scores on other factors. One such overriding factor is concentration of pay at the top executive level: ISS will always recommend against if the grant's concentration ratio exceeds 30% for the CEO (see "Equity Compensation Plans Frequently Asked Questions," ISS, December 17, 2021).

Implication 6: **Sensitivity to recommendations based on one-size-fits-all guidelines**. Given the logic above, our model predicts differential sensitivity of voting outcomes to proxy advisors' recommendations, depending on whether these recommendations originated from the one-size-fits-all guidelines or not. For example, if we consider all recommendations against directors, some of them are due to the director in question being overboarded, while others are due to the underlying characteristics of the director not specified in the guidelines. We expect shareholder voting outcomes to be more sensitive to the second type of negative recommendations. This is because the information in the research report is likely to be more consistent with the recommendation for the second type of recommendations, so informed subscribers are more likely to vote against in the second case than in the first. This prediction can be tested using proxy advisors' research reports, which explain the reasons for the recommendations. While these reports are not publicly available, they can be purchased from proxy advisors.

In addition, across shareholders, we expect larger asset managers (who are likely to have larger v_i and hence subscribe to the reports) to be more likely to deviate from recommendations originating from the one-size-fits-all guidelines. By reading and internalizing the information in the reports, these shareholders would be accounting for firm-specific factors and often coming to different conclusions from proxy advisors. This prediction is consistent with the analysis of one-size-fits-all recommendations in Section 5.1 of Iliev and Lowry (2015), who conclude: "There are important issues on which ISS is predisposed to recommending against management, and active voter mutual funds frequently come to a different conclusion than ISS on these issues."

A specific type of one-size-fits-all recommendations is when proxy advisors always recommend against or in favor of certain proposals. For example, the 2022 ISS guidelines contain a "general recommendation" to "vote for proposals to repeal classified boards" and Glass Lewis 2022 guidelines state that "Glass Lewis favors the repeal of staggered boards." Our analysis predicts the emergence of such recommendations as well. In particular, Proposition 5 shows that when prior beliefs are sufficiently uncertain, the proxy advisor optimally designs completely uninformative recommendations, and always recommending in favor of a certain type of proposals (or always against) is one way to implement this. Assuming that there is sufficient ex-ante uncertainty about the value of shareholder proposals on board declassification, uninformative (always "for") recommendations become optimal.

5.4 Information content of research reports and recommendations

Our model predicts that the proxy advisor's research reports will be accurate and unbiased ("fully informative") but that its recommendations will be partially informative and biased.[26] Consistent with this, many asset managers that invest in stewardship point out that their prime interest in the feedback from proxy advisors is the detailed data and reports they generate, rather than specific recommendations. For example, Michelle Edkins, Global Head of BlackRock's Investment Stewardship team, has praised the information content of proxy advisors' research reports stating "we get to read a lot [of proxy statements], and it can be very hard to find the pertinent information ... so having that information synthesized and accessible is hugely important to us being able to take an informed decision," but emphasized that they rely less on specific recommendations: "we take our decisions on a case-by-case basis."[27] This view is consistent with a broader view of other large asset managers, as evidenced by the survey evidence in Bew and Fields (2012) cited in the introduction.

Relatedly, Ertimur, Ferri, and Oesch (2013, 2018) examine the information content of ISS and Glass Lewis research reports and its relation to shareholders' voting on say-on-pay proposals and director elections, respectively. Their evidence is consistent with shareholders utilizing the contents of the research reports beyond the information contained in the recommendations. In particular, they show that shareholders' tendency to vote against the company's executive compensation policies and its directors is stronger if the research report identifies multiple, rather than a single, reasons for concern, and if the severity of these concerns is higher.

The quotes from institutional investors presented above also highlight that proxy advisors' information is not necessarily private: on more routine issues, such as say-on-pay or uncontested director elections, their reports aggregate and synthesize the public information in proxy statements. Yet, this information is valuable and not freely available to asset managers because of the large number of firms in their portfolio, the increasingly lengthy and complex proxy statements, and the concentration of shareholder meetings in a short period of time between April and May. Of course, on more important issues, such as proxy contests and controversial M&A deals, proxy advisors often gather private information as well: in proxy contests, for

[26] The difference between proxy advisors' recommendations and reports resembles the difference between equity analysts' buy/hold/sell recommendations and actual reports or the difference between academic reviewers' recommendations and referee reports. In all three examples, the report is typically more nuanced and detailed than the recommendation, and its tone and strength of the arguments contain valuable information.

[27] See the transcript of the SEC's Proxy Advisory Services Roundtable in 2013 at https://www.sec.gov/spotlight/proxy-advisory-services/proxy-advisory-services-transcript.txt.

example, they often have multiple meetings with both dissidents and management.

Importantly, even though the advisor's recommendations in our model are biased and less informative than its reports, they nevertheless contain valuable information. Is there evidence that proxy advisors' recommendations are indeed informative about the value of the proposal? Alexander et al. (2010) examine the price impact to ISS recommendations in proxy contests and conclude that the answer is yes. Their analysis suggests that the price impact contains both a "prediction" component (recommendations affect prices by changing the beliefs about who will win the proxy contest) and a "certification" component (recommendations are informative about the value that the dissident or incumbent team would create for the firm); the latter component suggests that recommendations are at least partially informative.

5.5 Discussion

In this subsection, we discuss some institutional features of the proxy advisory process in the context of our model and assumptions.

Multiple proposals and firms. Our baseline model features one firm and one proposal. In practice, proxy advisors sell their research as a bundle: a subscribing shareholder receives research reports for all companies in the shareholder's portfolio, and the research report for each company contains information on all proposals on the company's agenda. Such bundling does not change the conclusions of our model, in that the incentives to create controversy arise in this case as well. To see this, suppose that there is still one firm, but it has K proposals on the agenda. The proxy advisor combines its research on all K proposals in one report, so that shareholders choose whether to purchase the report with information on all K proposals, or not purchase any information at all. Importantly, for any of these proposals, the value of the report to the shareholders increases in the probability that the vote on this proposal is close. Hence, for a given fee, the problem of the proxy advisor can be considered as K separate problems, and the incentives to create controversy will arise on any of those K proposals, as long as its prior probability of being value-increasing is sufficiently asymmetric. (Of course, the fee that the advisor will charge for its report will now depend on the combined value of these proposals to the shareholders.) A similar argument applies to a subscription that bundles research across multiple firms in the subscribing shareholders' portfolios.

Communication among shareholders. In our setting, since all shareholders' interests are perfectly aligned, it would be in the ex-post interest of the subscribing shareholders to disclose the information they learn from the advisor's report to other shareholders, and such communication would be credible. For example, even though sharing the report itself is likely not possible given contractual restrictions, the subscribers could nevertheless disclose how they are going to vote. We assume that this does not happen, because in practice, the extent of such disclosure is often limited for several reasons. One reason is that communication with other shareholders could be considered as "forming a group," which may trigger a poison pill or require filing form 13D.[28] Another reason is that, based on anecdotal evidence, publicly disclosing one's vote against management is viewed by management much more negatively than a negative (but private) vote per se, so institutional investors are usually reluctant to disclose such votes to avoid managerial retaliation.

6 Conclusion

This paper analyzes the information design problem of a proxy advisor that aims to maximize its profits from information sale to voters. The advisor designs two signals: a research report, which is only available to the advisor's subscribers for a fee, and a voting recommendation, which is available to all shareholders for free.

We show that the advisor has incentives to issue recommendations that are biased against the a priori more likely alternative. By "creating controversy" in this way, the advisor generates uncertainty around the vote, thereby increasing shareholders' incentives to become informed and subscribe to its research report, which it makes fully informative and unbiased. These results help rationalize the one-size-fits-all approach to recommendations, which proxy advisors frequently use and are criticized for. Moreover, they highlight that proxy advisors' recommendations may not be a suitable benchmark for evaluating the votes of asset managers. Hence, the paper offers a reinterpretation of the empirical evidence that large institutional investors are often more supportive of management relative to proxy advisors. Such voting could be a way to correct for the anti-management controversy bias in proxy advisors' recommendations, rather than reflect a shareholder's passivity or bias towards management.

Our paper focuses on a monopolistic proxy advisor, whereas in practice, the proxy advisory industry is a duopoly. Analyzing the joint information design problem of two advisors competing with each other is an interesting direction that we leave for future research.

[28]For example, the 2011 report by Dechert LLP states that "shareholder concern about unintentionally forming a group has chilled communications among large holders of shares in U.S. public companies." See https://www.jdsupra.com/legalnews/us-court-clarifies-shareholders-actin-14535/.

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Appendix: Proofs

Proof of Proposition 1. Proven in the main text. ■

Proof of Proposition 2. Since we focus on weakly undominated strategies, subscribing shareholders vote according to the state: $a_i = \theta$.

Next, plugging (2)-(3) into (1) and simplifying, we obtain

$$\Pr(\theta = 1|s, Piv) = \frac{\left((q + (1-q)\pi)(1-\pi)\right)^{\frac{N-1}{2}} \mu_s}{\left((q + (1-q)\pi)(1-\pi)\right)^{\frac{N-1}{2}} \mu_s + \left(\pi(1-\pi(1-q))\right)^{\frac{N-1}{2}} (1-\mu_s)}. \tag{14}$$

First, consider a candidate equilibrium in which $\pi = 1$, i.e., each non-subscribing shareholder votes in favor. Then $\Pr(Piv|\theta = 1, s) = 0$ (since all subscribers then also vote in favor) and $\Pr(Piv|\theta = 0, s) > 0$ (since $q \in (0,1)$ and all subscribers then vote against). These imply that if $\mu_s < 1$, then (1) shareholder i is pivotal with a strictly positive probability and (2) $\Pr(\theta = 1|s, Piv) = 0$, which together imply that the shareholder finds it strictly optimal to deviate and vote against, so this equilibrium cannot exist. If $\mu_s = 1$, then $\theta = 1$ with certainty, so both subscribing and non-subscribing shareholders vote in favor. Since the shareholder is then never pivotal, he is indifferent between voting for and against, and in particular, voting in favor is indeed optimal. Hence, equilibrium with $\pi = 1$ exists if and only if $\mu_s = 1$.

Second, consider a candidate equilibrium in which $\pi = 0$, i.e., each non-subscribing shareholder votes against. Then $\Pr(Piv|\theta = 0, s) = 0$ (since all subscribers then also vote against) and $\Pr(Piv|\theta = 1, s) > 0$ (since $q \in (0,1)$ and all subscribers then vote in favor). These imply that if $\mu_s > 0$, then (1) shareholder i is pivotal with a strictly positive probability and (2) $\Pr(\theta = 1|s, Piv) = 1$, which together imply that the shareholder finds it strictly optimal to deviate and vote in favor, so this equilibrium cannot exist. If $\mu_s = 0$, then $\theta = 0$ with certainty, so both subscribing and non-subscribing shareholders vote against. Since the shareholder is then never pivotal, he is indifferent between voting for and against, and in particular, voting against is indeed optimal. Hence, equilibrium with $\pi = 0$ exists if and only if $\mu_s = 0$.

Third, consider a candidate equilibrium in which each non-subscribing shareholder votes $a_i = 1$ with probability $\pi \in (0,1)$. Since $q \in (0,1)$ and non-subscribing shareholders randomize, then for any $\mu_s \in [0,1]$, the shareholder is pivotal with a strictly positive probability. Hence, to find it optimal to randomize between voting for and against, he must be indifferent between proposal acceptance and rejection, which is the case if and only if $\Pr(\theta = 1|s, Piv) = \frac{1}{2}$, or equivalently, using (14),

$$\left((q + (1-q)\pi)(1-\pi)\right)^{\frac{N-1}{2}} \mu_s = \left(\pi(1-\pi(1-q))\right)^{\frac{N-1}{2}} (1-\mu_s). \tag{15}$$

In particular, this equilibrium can only exist if $\mu_s \in (0,1)$: if the state is known with certainty ($\mu_s = 0$ or 1), and since the shareholder is pivotal with a strictly positive probability, he is never indifferent between voting for and against.

Denoting $z_s \equiv \left(\frac{\mu_s}{1-\mu_s}\right)^{\frac{2}{N-1}}$, we can rewrite (15) as $w(\pi, z_s) = 0$, where

$$w(\pi, z_s) = (z_s - 1)(1-q)\pi^2 + (1 + (2q-1) z_s)\pi - z_s q. \tag{16}$$

Note that

$$\begin{aligned} w(0, z_s) &= -z_s q < 0 & (17) \\ w(1, z_s) &= (z_s - 1)(1-q) + 1 + (2q-1) z_s - z_s q = q > 0. & (18) \end{aligned}$$

First, if $z_s = 1$, it is equivalent to $q(2\pi - 1) = 0 \Leftrightarrow \pi = \frac{1}{2}$. Second, if $z_s > 1$, then (since $-z_s q < 0$) this quadratic equation has a unique positive root $\pi(q, \mu_s)$, which is the larger of the two roots and is given by (4). Given (17)-(18), this root lies between 0 and 1. Third, if $z_s < 1$, $w(\pi, z_s)$ is an inverted parabola, and given (17)-(18), it has a unique root in $(0, 1)$, which is the smaller of the two roots, and hence is also given by (4). Note that by l'Hopital's rule, $\lim_{\mu_s \to \frac{1}{2}} \pi(q, \mu_s) = \frac{1}{2}$, so the probability of voting in favor is continuous in μ_s.

Finally, we prove that $\pi(q, \mu_s)$ increases in μ_s. To see this, note that z_s increases in μ_s, and the derivative of $w(\pi, z_s)$ with respect to z_s is

$$\delta(\pi, q) \equiv (1-q)\pi^2 + (2q-1)\pi - q.$$

Since $\frac{\partial \delta(\pi,q)}{\partial q} = -\pi^2 + 2\pi - 1 < 0$ and $\delta(\pi, 0) = \pi^2 - \pi < 0$ for $\pi \in (0, 1)$, then $\delta(\pi, q) < 0$ for all $q \in (0, 1)$. Hence, $w(\pi, z_s)$ is decreasing in z_s in the neighborhood of its root $\pi(q, \mu_s)$. When $z_s > 1$, $w(\cdot, z_s)$ is a parabola and $\pi(q, \mu_s)$ is the larger of the two roots, so it increases in μ_s. When $z_s < 1$, $w(\cdot, z_s)$ is an inverted parabola and $\pi(q, \mu_s)$ is the smaller of the two roots, so it also increases in μ_s. ■

Proof of Proposition 3. The statement of the proposition is proven in the main text. Here, we provide an equivalent representation of $\Pr(Piv|q, \mu_s)$, which will be useful in subsequent proofs. We can rewrite the indifference condition $\Pr(\theta = 1|s, Piv) = \frac{1}{2}$ as

$$\Pr(Piv|\theta = 1, s)\mu_s = \Pr(Piv|\theta = 0, s)(1 - \mu_s). \tag{19}$$

Using (19) and (2)-(3), we get three equivalent formulas for $\Pr(Piv|q, \mu_s)$:

$$\begin{aligned} \Pr(Piv|q, \mu_s) &= \Pr(Piv|\theta = 1, s)\mu_s + \Pr(Piv|\theta = 0, s)(1 - \mu_s) & (20) \\ &= 2(1 - \mu_s) C_{N-1}^{\frac{N-1}{2}} (\varrho_0(q, \mu_s)(1 - \varrho_0(q, \mu_s)))^{\frac{N-1}{2}} & (21) \\ &= 2\mu_s C_{N-1}^{\frac{N-1}{2}} (\varrho_1(q, \mu_s)(1 - \varrho_1(q, \mu_s)))^{\frac{N-1}{2}}, & (22) \end{aligned}$$

where

$$\begin{aligned} \varrho_1(q, \mu_s) &= q + (1-q)\pi(q, \mu_s) & (23) \\ \varrho_0(q, \mu_s) &= (1-q)\pi(q, \mu_s) & (24) \end{aligned}$$

are the equilibrium probabilities that a shareholder votes "for" conditional on $\theta = 1$ and $\theta = 0$, respectively (and on s), and $\pi(q, \mu_s)$ is the equilibrium probability that a non-subscribing shareholder votes "for," given by (4). ■

Proof of Lemma 1. The proof is provided in Section A.1 of the Online Appendix. ■

Proof of Proposition 4. We start by noting that since the equilibrium in the voting subgame depends on the recommendation only via $\Pr(Piv|q, \mu_s)$, the advisor's problem (12) can be solved using concavification of the function $\Pr(Piv|q, \mu_s)$, which is a function of a single variable μ_s. The concave closure of this function is a combination of either the function itself or a linear function connecting two points. Hence, it is without loss of generality to restrict attention to binary recommendations that induce at most two different posteriors.

We first prove part 1 (applying to small μ). It is easy to show that $\Pr(Piv|q, \mu_s)$ is increasing in μ_s if and only if $\mu_s < \frac{1}{2}$. Consider $\mu < \underline{\mu}$ from Lemma 1. Since $\Pr(Piv|q, 0) = 0$ and $\Pr(Piv|q, \mu_s)$ is strictly increasing and strictly convex in μ_s for $\mu_s \in \left(0, \underline{\mu}\right)$, then the concave closure of $\Pr(Piv|q, \mu_s)$ is linear in the neighborhood of $\mu_s = 0$, and the optimal recommendation design for $\mu \in \left(0, \underline{\mu}\right)$ takes the following form: signal $s = 0$ induces belief $\mu_0 = 0$ and signal $s = 1$ induces belief μ_1, where μ_1 is one that maximizes

$$\Pr(Piv|q, \mu_1)\, \tau_1 + \Pr(Piv|q, 0)\, \tau_0$$

subject to the Bayes Plausibility constraint $\mu_1 \tau_1 + \mu_0 \tau_0 = \mu$. Since $\mu_0 = 0$, the latter implies $\tau_1 = \frac{\mu}{\mu_1}$, and since $\Pr(Piv|q, 0) = 0$, the average probability of a split vote is $\Pr(Piv|q, \mu_1)\frac{\mu}{\mu_1}$, and μ_1 solves

$$\mu_1 = \arg\max_m \frac{\Pr(Piv|q, m)}{m} \mu. \tag{25}$$

The point μ_1 is one where the linear function that starts at $(0, 0)$ is tangent to the function $\Pr(Piv|q, \cdot)$. To find μ_1, we substitute (22) into (25) and get

$$\frac{\Pr(Piv|q, m)}{m} \mu = 2C_{N-1}^{\frac{N-1}{2}} \left(\varrho_1(q, m)\left(1 - \varrho_1(q, m)\right)\right)^{\frac{N-1}{2}} \mu,$$

where $\varrho_1(q, m)$ is given by (23). Hence, μ_1 solves

$$\max_m \left(\varrho_1(q, m)\left(1 - \varrho_1(q, m)\right)\right)^{\frac{N-1}{2}}. \tag{26}$$

Thus, if feasible, the optimal posterior μ_1 is such that $\varrho_1(q, \mu_1) = \frac{1}{2}$. In other words, the probability of a shareholder voting "for" conditional on a controversial recommendation ($s = 1$) and the state being in line with the recommendation ($\theta = 1$), is 50%. If $m = \frac{1}{2}$, then $\pi(q, m) = \frac{1}{2}$, so $\varrho_1(q, m) = \frac{1+q}{2} > \frac{1}{2}$. Hence, $m = \frac{1}{2}$ does not solve (26) if there exists $m \in \left(\mu_1, \frac{1}{2}\right)$ that satisfies $\varrho_1(q, m) = \frac{1}{2}$. Consider $m \neq \frac{1}{2}$. Then, using (4), $\varrho_1(q, m) = \frac{1}{2}$ is equivalent to

$$(1-q)(1-\pi(q,m)) = \frac{1}{2} \Leftrightarrow \frac{1}{2} + \frac{2q - \sqrt{(z-1)^2 + 4q^2 z}}{2(z-1)} = \frac{1}{2} \tag{27}$$
$$\Leftrightarrow z^2 - 2\left(1-2q^2\right)z + 1 - 4q^2 = 0 \Leftrightarrow (z-1)\left(z-1+4q^2\right) = 0,$$

where $z \equiv \left(\frac{m}{1-m}\right)^{\frac{2}{N-1}}$. Since $m \neq \frac{1}{2}$, the only root is $z = 1 - 4q^2$. Equating it to $\left(\frac{\mu_1}{1-\mu_1}\right)^{\frac{2}{N-1}}$ implies that the optimal posterior is

$$\mu_1(q) \equiv \frac{\left(1-4q^2\right)^{\frac{N-1}{2}}}{1 + \left(1-4q^2\right)^{\frac{N-1}{2}}}. \tag{28}$$

Note that $\mu_1 < \frac{1}{2}$. It follows that the optimal recommendation induces beliefs $\mu_0 = 0$ and $\mu_1(q) \in \left(\mu, \frac{1}{2}\right)$ given by (28). The average probability of a shareholder being pivotal given this recommendation design is $2C_{N-1}^{\frac{N-1}{2}}\left(\frac{1}{4}\right)^{\frac{N-1}{2}}\mu = \mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. To show that this recommendation is indeed optimal for any $\mu \leq \mu_1(q)$, consider any other recommendation with posteriors $\mu_l < \mu_h$. The Bayes Plausibility constraint implies that $\mu_l < \mu < \mu_h$ and that μ_l and μ_h are induced with probability $\frac{\mu_h - \mu}{\mu_h - \mu_l}$ and $\frac{\mu - \mu_l}{\mu_h - \mu_l}$, respectively. Note that for any μ_s, (22) implies that

$$\Pr(Piv|q,\mu_s) = 2\mu_s C_{N-1}^{\frac{N-1}{2}}\left(\varrho_1(q,\mu_s)(1-\varrho_1(q,\mu_s))\right)^{\frac{N-1}{2}} \leq 2\mu_s C_{N-1}^{\frac{N-1}{2}}\left(\frac{1}{4}\right)^{\frac{N-1}{2}} = \mu_s C_{N-1}^{\frac{N-1}{2}} 2^{2-N}.$$

Hence, the average probability of being pivotal given (μ_l, μ_h) is

$$\begin{gathered} \tfrac{\mu_h - \mu}{\mu_h - \mu_l}\Pr(Piv|q,\mu_l) + \tfrac{\mu - \mu_l}{\mu_h - \mu_l}\Pr(Piv|q,\mu_h) \\ \leq \tfrac{\mu_h - \mu}{\mu_h - \mu_l}\mu_l C_{N-1}^{\frac{N-1}{2}} 2^{2-N} + \tfrac{\mu - \mu_l}{\mu_h - \mu_l}\mu_h C_{N-1}^{\frac{N-1}{2}} 2^{2-N} = \mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}, \end{gathered} \tag{29}$$

as required.
Next, we prove part 2 (applying to large μ). Given the symmetry of the problem in μ around $\mu = \frac{1}{2}$, this proof is almost identical to the proof of part 1 and hence is relegated to Section A.2 of the Online Appendix. In that proof, we show that for any $\mu \geq \mu_0(q)$, the optimal recommendation induces beliefs $\mu_0(q)$ and $\mu_1 = 1$, and that the average probability of a shareholder being pivotal given this recommendation design is $(1-\mu)C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. ■

Proof of Corollary 1. The statement directly follows from Proposition 4 and the fact that function $\mu_0(q)$ increases in q, $\lim_{q\to 0}\mu_0(q) = \frac{1}{2}$, and $\lim_{q\to\frac{1}{2}}\mu_0(q) = 1$, and function $\mu_1(q)$ decreases in q, $\lim_{q\to 0}\mu_1(q) = \frac{1}{2}$, and $\lim_{q\to\frac{1}{2}}\mu_1(q) = 0$. ■

Proof of Proposition 5. By Lemma 1, $\Pr(Piv|q,\mu_s)$ is strictly concave in $\left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$. Moreover, $\Pr(Piv|q,\mu_s)$ is increasing for $\mu_s < \frac{1}{2}$ and decreasing for $\mu_s > \frac{1}{2}$, and hence has a maximum at $\frac{1}{2}$. Therefore, the concavification approach implies that the optimal recommendation is uninformative. ■

Proof of Proposition 6. Since the equilibrium in the voting subgame depends on the recommendation only via $\Pr(Piv|q,\mu_s)$, we can without loss of generality focus on binary recommendations (for the same reasons as in the proof of Proposition 4).

To prove the first statement of the proposition, we first note that for any μ,

$$\begin{aligned}\max\nolimits_q \Pr(Piv|q,\mu) &= \max\nolimits_q 2\mu C_{N-1}^{\frac{N-1}{2}} \left(\varrho_1(q,\mu)\left(1-\varrho_1(q,\mu)\right)\right)^{\frac{N-1}{2}} \\ &\leq 2\mu C_{N-1}^{\frac{N-1}{2}} \max\nolimits_{\varrho_1\in[0,1]} \left(\varrho_1(1-\varrho_1)\right)^{\frac{N-1}{2}} = \mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N},\end{aligned} \tag{30}$$

where the first equality follows from (22). Now consider an arbitrary binary recommendation, such that signal $s=0$ induces belief μ_l and signal $s=1$ induces belief μ_h, and an arbitrary choice of q. The Bayes plausibility constraint requires that $\mu_l \leq \mu \leq \mu_h$, and for any $\mu_l \in [0,\mu]$ and $\mu_h \in [\mu,1]$, the Bayes plausibility constraint requires that $\tau_0 = \frac{\mu_h-\mu}{\mu_h-\mu_l}$ and $\tau_1 = \frac{\mu-\mu_l}{\mu_h-\mu_l}$. Then the expected profit of the advisor, $\Pi(q,\mu_l,\mu_h)$, satisfies

$$\Pi(q,\mu_l,\mu_h) \leq q^* H^{-1}(1-q^*)\,\mu C_{N-1}^{\frac{N-1}{2}} 2^{1-N} N. \tag{31}$$

To see this, note that

$$\frac{2}{N}\Pi(q,\mu_l,\mu_h) = qH^{-1}(1-q)\left(\frac{\mu_h-\mu}{\mu_h-\mu_l}\Pr(Piv|q,\mu_l) + \frac{\mu-\mu_l}{\mu_h-\mu_l}\Pr(Piv|q,\mu_h)\right) \tag{32}$$

$$\leq \max_{q,\mu_l,\mu_h}\left\{\left(qH^{-1}(1-q)\right)\left(\frac{\mu_h-\mu}{\mu_h-\mu_l}\Pr(Piv|q,\mu_l) + \frac{\mu-\mu_l}{\mu_h-\mu_l}\Pr(Piv|q,\mu_h)\right)\right\} \tag{33}$$

$$\leq \max_q\left\{qH^{-1}(1-q)\right\}\max_{\mu_l,\mu_h}\left\{\frac{\mu_h-\mu}{\mu_h-\mu_l}\max_q \Pr(Piv|q,\mu_l) + \frac{\mu-\mu_l}{\mu_h-\mu_l}\max_q \Pr(Piv|q,\mu_h)\right\} \tag{34}$$

$$\leq \max_q\left\{qH^{-1}(1-q)\right\}\max_{\mu_l,\mu_h}\left\{\frac{\mu_h-\mu}{\mu_h-\mu_l}\mu_l C_{N-1}^{\frac{N-1}{2}}2^{2-N} + \frac{\mu-\mu_l}{\mu_h-\mu_l}\mu_h C_{N-1}^{\frac{N-1}{2}}2^{2-N}\right\} \tag{35}$$

$$= \max_q\left\{qH^{-1}(1-q)\right\}\mu C_{N-1}^{\frac{N-1}{2}}2^{2-N} \leq q^* H^{-1}(1-q^*)\,\mu C_{N-1}^{\frac{N-1}{2}}2^{2-N}. \tag{36}$$

The first equality follows from (7) and (9). The first inequality is trivial. The second inequality follows from the fact that the maximum of a product cannot be higher than the product of the maxima. The third inequality follows from (30). The last equality follows algebraically. Finally, the last inequality follows from the fact that q^* maximizes $qH^{-1}(1-q)$.

Suppose that $\mu \leq \mu_1(q^*)$ and $q^* < \frac{1}{2}$. Then, the proof of Proposition 4 shows that choosing $q = q^*$ and the information policy with $\mu_l = 0$ and $\mu_h = \mu_1(q^*)$ yields the expected profit of $q^* H^{-1}(1-q^*)\,\mu C_{N-1}^{\frac{N-1}{2}}2^{1-N}N$. Hence, (31) implies that $(\mu_l,\mu_h,q) = (0,\mu_1(q^*),q^*)$ is optimal. The optimal fee that induces q^* then follows directly from (9).

By symmetry of $\Pr(Piv|q,\mu)$ around $\mu = \frac{1}{2}$, the proof of the second statement of the proposition is analogous. ■

Proof of Proposition 7. The statement holds for any distribution H for which $qH^{-1}(1-q)$ is increasing on $[0,q^*]$ and $q^* > \frac{1}{2}$. The assumption of an increasing hazard rate is sufficient but not necessary: as we show at the end of this proof, an increasing hazard rate of $H(\cdot)$

guarantees that $qH^{-1}(1-q)$ is increasing on $[0, q^*]$.

Consider any $\mu \in (0, \frac{1}{2}]$ (the proof for $\mu \geq \frac{1}{2}$ is similar). Note that $\mu_1(q)$ defined in Proposition 4 decreases in q, $\lim_{q \to 0} \mu_1(q) = \frac{1}{2}$, and $\lim_{q \to \frac{1}{2}} \mu_1(q) = 0$. Hence, for any $\mu \in (0, \frac{1}{2}]$, we can define $\hat{q} \equiv \mu_1^{-1}(\mu) \in [0, \frac{1}{2})$, which is the highest q for which $\mu_1(q) \geq \mu$. For any $q < \hat{q}$, Proposition 4 implies that the optimal recommendation is partially informative, with the probability of signal $s = 1$ being $\frac{\mu}{\mu_1(q)}$, and the average probability of being pivotal being $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. By continuity, when $q = \hat{q}$ and the recommendation becomes uninformative, the probability that a shareholder is pivotal is also $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. Since $\hat{q} < \frac{1}{2} \leq q^*$ and $qH^{-1}(1-q)$ is increasing on $[0, q^*]$, we have $qH^{-1}(1-q) < \hat{q}H^{-1}(1-\hat{q})$ for any $q < \hat{q}$. Since the advisor's expected profit is the average probability of being pivotal multiplied by $\frac{N}{2}H^{-1}(q)\, q$, the advisor is strictly better off picking an uninformative recommendation and the fee that induces $\hat{q}$, rather than picking some $q < \hat{q}$ and $\mu_0 = 0$, $\mu_1(q)$ (which is the optimal recommendation design for this q by Proposition 4), as required.

It remains to prove that an increasing hazard rate of $H(\cdot)$ implies that $\lambda(q) \equiv qH^{-1}(1-q)$ is increasing on $[0, q^*]$. Denoting $\eta(q) \equiv H^{-1}(1-q)$, we get

$$\lambda'(q) = H^{-1}(1-q) - \frac{q}{H'(H^{-1}(1-q))} = \eta(q) - \frac{1 - H(\eta(q))}{H'(\eta(q))}.$$

Since the hazard rate $\frac{H'(\eta)}{1-H(\eta)}$ is increasing in η, the function $\eta - \frac{1-H(\eta)}{H'(\eta)}$ is increasing in η as well. Since $\eta(q)$ is decreasing, $\lambda'(q)$ is decreasing in q. Finally, note that when $q = 0$, $\lambda'(0) > 0$. It follows that if q^* is the maximum of $\lambda(q)$, then $\lambda(q)$ increases on $[0, q^*]$. ■

Proof of Corollary 2. Consider $\mu \leq \frac{1}{2}$ (the case of $\mu \geq \frac{1}{2}$ is similar by symmetry). Notice that q^* only depends on the distribution of v_i and is independent of μ and N. When $q^* < \frac{1}{2}$, $(1 - 4q^{*2})^{\frac{N-1}{2}} \in (0, 1)$, is decreasing in N, and converges to zero as $N \to \infty$, and hence $\mu_1(q^*)$ from part 1 of Proposition 6 is also decreasing in N and converges to zero as $N \to \infty$. Thus, for any μ, there exists $\bar{N}(\mu)$ such that the recommendation design inducing posteriors 0 and $\mu_1(q^*)$ only arises for $N \leq \bar{N}(\mu)$. The properties of recommendation design follow directly from Proposition 6 and the comparative statics of $\mu_1(q^*)$ in N. ■

Proof of Proposition 8. The complete proof is provided in Section A.3 of the Online Appendix, and we only provide a sketch of the proof here.

Fix any realization of public signal s inducing posterior μ_s. Then, the set of signals R can be divided into three subsets: R_0, R_1, R_m, defined, respectively, as all signals that induce posteriors (conditional on r and s) strictly below $\frac{1}{2}$ (inducing subscribers to vote against); strictly above $\frac{1}{2}$ (inducing subscribers to vote in favor); and equal to $\frac{1}{2}$. Because shareholders are indifferent between voting for and against if their posterior belief is exactly $\frac{1}{2}$, the value of the report is zero conditional on the third set of signals, and hence it is optimal to not induce such beliefs. Moreover, because the probability of being pivotal is the same for all signals from R_0 (and for all signals from R_1), we can combine such signals into one. Thus, the problem reduces to a report with binary signals: $R = \{0, 1\}$. We next repeat the arguments in the proof of Proposition 2 and show that for any $\mu_s \in (0,1)$, non-subscribing shareholders vote for the proposal with probability $\pi(q, \tilde{z}_s) \in (0,1)$, where $\pi(q, \cdot)$ is given by (4) and

$$\tilde{z}_s = \left(\frac{p_1 - \frac{1}{2}}{p_1 - \mu_s} \frac{p_0 + \mu_s - 1}{p_0 - \frac{1}{2}} \right)^{\frac{2}{N-1}}, \tag{37}$$

where $p_0 \equiv \Pr(\theta = 0 | r = 0, \mu_s)$ and $p_1 \equiv \Pr(\theta = 1 | r = 1, \mu_s)$. Finally, we repeat the arguments in Section 3.2 and, using (37), derive the value of the report as a function of q, μ_s, p_0, and p_1. We complete the proof by showing that for any μ_s, this value is maximized if $p_0 = p_1 = 1$. ■

Other results: The proofs of the extensions in Section 4 are presented in the Online Appendix, which is available on the authors' websites.

Online Appendix for
"Creating Controversy in Proxy Voting Advice"

Andrey Malenko, Nadya Malenko, and Chester Spatt

The Online Appendix is organized as follows. Section A contains the additional proofs for the main results (Lemma 1, the second part of Proposition 4, Proposition 8, the extensions presented in Section 4), as well as a more in-depth derivation of the value from buying the report. Section B contains the additional derivations for the empirical implications that are presented in Section 5 of the paper.

A Additional proofs for the main results

A.1 Proof of Lemma 1

Plugging (4) from Proposition 2 into (22) and simplifying the expression,

$$\Pr\left(Piv|q,\mu_s\right) = 2C_{N-1}^{\frac{N-1}{2}} q^{\frac{N-1}{2}} \left(\frac{\sqrt{\left(z\left(\mu_s\right)-1\right)^2 + 4q^2 z\left(\mu_s\right)} - q\left(1+z\left(\mu_s\right)\right)}{\left(z\left(\mu_s\right)-1\right)^2}\right)^{\frac{N-1}{2}} \mu_s,$$

where $z\left(\mu_s\right) \equiv \left(\frac{\mu_s}{1-\mu_s}\right)^{\frac{2}{N-1}}$. Define $\Omega\left(\mu_s|q\right)$ as

$$\Omega\left(\mu_s|q\right) \equiv \left(\frac{\varphi\left(z\left(\mu_s\right)\right)}{\left(z\left(\mu_s\right)-1\right)^2}\right)^{\frac{N-1}{2}} \mu_s, \tag{38}$$

where

$$\varphi\left(z\right) \equiv \sqrt{\left(z-1\right)^2 + 4q^2 z} - q\left(1+z\right). \tag{39}$$

Then, $\Pr\left(Piv|q,\mu_s\right) = 2C_{N-1}^{\frac{N-1}{2}} q^{\frac{N-1}{2}} \Omega\left(\mu_s|q\right)$, so $\Pr\left(Piv|q,\mu_s\right)$ is increasing (decreasing) and concave (convex) in μ_s at some $\left(\mu_s, q\right)$ if and only if $\Omega\left(\mu_s|q\right)$ is increasing (decreasing) and concave (convex) in μ_s at this $\left(\mu_s, q\right)$. Taking the first and second derivative of $z\left(\mu_s\right)$ and the first four derivatives of $\varphi(z)$, and dropping the subscript s in μ_s for ease of notation, we get:

$$\begin{aligned}
z'(\mu) &= \frac{2}{N-1}\frac{z(\mu)}{\mu(1-\mu)}, && (40)\\
z''(\mu) &= \frac{2}{N-1}\frac{\frac{2}{N-1}-(1-2\mu)}{\mu^2(1-\mu)^2}z(\mu), && (41)\\
\varphi'(z) &= \frac{z-1+2q^2}{\sqrt{(z-1)^2+4q^2z}}-q, && (42)\\
\varphi''(z) &= \frac{4q^2(1-q^2)}{\left((z-1)^2+4q^2z\right)^{\frac{3}{2}}} && (43)\\
\varphi'''(z) &= \frac{-12q^2(1-q^2)(z-1+2q^2)}{\left((z-1)^2+4q^2z\right)^{\frac{5}{2}}} && (44)\\
\varphi''''(z) &= \frac{60q^2(1-q^2)(z-1+2q^2)^2}{\left((z-1)^2+4q^2z\right)^{\frac{7}{2}}}-\frac{12q^2(1-q^2)}{\left((z-1)^2+4q^2z\right)^{\frac{5}{2}}} && (45)
\end{aligned}$$

We first prove the convexity/concavity properties of $\Omega(\mu|q)$. Differentiating $\Omega(\mu|q)$ twice with respect to μ and using z to denote $z(\mu)$ for ease of notation, we get

$$\begin{aligned}
\Omega''(\mu|q) &= \Omega'(\mu|q)\left(\frac{N-1}{2}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)z'(\mu)+\frac{1}{\mu}\right) && (46)\\
&\quad+\Omega(\mu|q)\left(\begin{array}{c}\frac{N-1}{2}\left(\left[\frac{\varphi'(z)}{\varphi(z)}\right]'+\frac{2}{(z-1)^2}\right)(z'(\mu))^2\\ +\frac{N-1}{2}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)z''(\mu)-\frac{1}{\mu^2}\end{array}\right).
\end{aligned}$$

Since $\mu\in(0,1)$, $q\in(0,1)$, and $\pi(q,\mu)\in(0,1)$, we have $\Pr(Piv|q,\mu)>0$ and hence $\Omega(\mu|q)>0$. Then, using (40)-(41) and simplifying,

$$\begin{aligned}
\frac{\Omega''(\mu|q)}{\Omega(\mu|q)}\mu^{\frac{2N-4}{N-1}} &= \left(\begin{array}{c}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)^2+\\ +\frac{2}{N-1}\left(\frac{\varphi''(z)\varphi(z)-(\varphi'(z))^2}{\varphi(z)^2}+\frac{2}{(z-1)^2}\right)\end{array}\right)\mu^{\frac{2}{N-1}}(1-\mu)^{-\frac{2(N+1)}{N-1}}\\
&+2\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)(1-\mu)^{-\frac{N+1}{N-1}}+\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)\left(\frac{2}{N-1}-(1-2\mu)\right)(1-\mu)^{-\frac{2}{N}}.
\end{aligned}$$

1) First, consider the limit case of $\mu\to 0$. When $\mu\to 0$, $\lim_{\mu\to 0}z(\mu)=0$, $\lim_{\mu\to 0}\varphi(z)=1-q$, $\lim_{\mu\to 0}\varphi'(z)=(q-1)(2q+1)$, and $\lim_{\mu\to 0}\varphi''(z)=4q^2(1-q^2)$. Therefore,

$$\lim_{\mu\to 0}\frac{\Omega''(\mu|q)\mu^{\frac{2N-4}{N-1}}}{\Omega(\mu|q)}=\frac{N+1}{N-1}(1-2q).$$

Since $\Omega(\mu|q) > 0$, we have $\lim_{\mu\to 0}\Omega''(\mu|q) > 0$ if and only if $q < \frac{1}{2}$. By continuity of the second derivative, there exists $\underline{\mu}$ such that: 1) if $q < \frac{1}{2}$, then $\Omega''(\mu|q) > 0$ for $\mu \in \left(0, \underline{\mu}\right)$ and 2) if $q > \frac{1}{2}$, then $\Omega''(\mu|q) < 0$ for $\mu \in \left(0, \underline{\mu}\right)$.

2) Second, consider the limit case of $\mu \to 1$. By symmetry of $\Omega(\mu|q)$ around $\mu = \frac{1}{2}$, this case is identical to $\mu \to 0$, so there exists $\bar{\mu}$ such that: 1) if $q < \frac{1}{2}$, then $\Omega''(\mu|q) > 0$ for $\mu \in (\bar{\mu}, 1)$ and 2) if $q > \frac{1}{2}$, then $\Omega''(\mu|q) < 0$ for $\mu \in (\bar{\mu}, 1)$.

3) Third, consider the limit case of $\mu \to \frac{1}{2}$. Using $\Omega'\left(\frac{1}{2}|q\right) = 0$ and the expressions (40)-(41), (46) at $\mu = \frac{1}{2}$ yields

$$\frac{\Omega''\left(\frac{1}{2}|q\right)}{\Omega\left(\frac{1}{2}|q\right)} = \frac{32}{N-1}\lim_{z\to 1}\left(\frac{\varphi''(z)\varphi(z) - (\varphi'(z))^2}{\varphi(z)^2} + \frac{2}{(z-1)^2}\right) + \frac{32}{N-1}\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)} - \frac{2}{z-1}\right) - 4.$$

Consider the second limit. Notice that $\varphi(1) = \varphi'(1) = 0$ and $\varphi''(1) \neq 0$. Applying l'Hopital's rule three times,

$$\begin{aligned}
\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)} - \frac{2}{z-1}\right) &= \lim_{z\to 1}\left(\frac{\varphi'(z)(z-1) - 2\varphi(z)}{\varphi(z)(z-1)}\right) \\
&= \lim_{z\to 1}\left(\frac{\varphi''''(z)(z-1) + \varphi'''(z)}{\varphi'''(z)(z-1) + 3\varphi''(z)}\right) = \frac{\varphi'''(1)}{3\varphi''(1)} = -\frac{1}{2},
\end{aligned}$$

where the last transition is from evaluating (43) and (44) at $z = 1$. Consider the first limit. Using $\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)} - \frac{2}{z-1}\right)^2 = \frac{1}{4}$ and applying l'Hopital's rule four times,

$$\begin{aligned}
\lim_{z\to 1}\left(\frac{\varphi''(z)\varphi(z) - (\varphi'(z))^2}{\varphi(z)^2} + \frac{2}{(z-1)^2}\right) &= \lim_{z\to 1}\left(\frac{\varphi''(z)(z-1)^2 + 6\varphi(z) - 4\varphi'(z)(z-1)}{\varphi(z)(z-1)^2}\right) - \frac{1}{4} \\
&= \lim_{z\to 1}\left(\frac{\varphi''''''(z)(z-1)^2 + 4(z-1)\varphi'''''(z) + 2\varphi''''(z)}{\varphi''''(z)(z-1)^2 + 8(z-1)\varphi'''(z) + 12\varphi''(z)}\right) - \frac{1}{4} \\
&= \frac{\varphi''''(1)}{6\varphi''(1)} - \frac{1}{4} = \frac{\frac{3\left(1-q^2\right)\left(5q^2-1\right)}{8q^3}}{6\frac{1-q^2}{2q}} - \frac{1}{4} = \frac{3q^2-1}{8q^2},
\end{aligned}$$

where the transition on the last line is from evaluating (43) and (45) at $z = 1$. Hence,

$$\lim_{z\to 1}\frac{\Omega''\left(\frac{1}{2}|q\right)}{\Omega\left(\frac{1}{2}|q\right)} = \frac{32}{N-1}\frac{3q^2-1}{8q^2} - \frac{32}{N-1}\frac{1}{2} - 4 = \frac{4}{N-1}\left(-1 - \frac{1}{q^2}\right) - 4 < 0.$$

Since $\Omega\left(\frac{1}{2}|q\right) > 0$, then for any q, $\Pr(Piv|q,\mu)$ is strictly concave at $\mu = \frac{1}{2}$ and, by continuity of the second derivative, there exists $\varepsilon > 0$ such that it is also strictly concave in $\mu \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$.

A.2 Additional analysis for the proof of Proposition 4

The main appendix contains the proof of part 1 of the proposition, which applies to small enough μ. Here, we present the proof of part 2 (applying to large enough μ), which is very similar to the proof of part 1 given the symmetry of the problem in μ around $\mu = \frac{1}{2}$.

Consider $\mu > \bar{\mu}$ from Lemma 1. Since $\Pr(Piv|q,1) = 0$ and $\Pr(Piv|q,\mu_s)$ is strictly decreasing and strictly convex in μ_s for $\mu_s \in (\bar{\mu}, 1)$, then the concave closure of $\Pr(Piv|q,\mu_s)$ is linear in the neighborhood of $\mu_s = 1$, and the optimal recommendation design for $\mu \in (\bar{\mu}, 1)$ takes the following form: signal $s = 0$ induces belief μ_0 and signal $s = 1$ induces belief $\mu_1 = 1$, where μ_0 is one that maximizes

$$\Pr(Piv|q,\mu_0)\,\tau_0 + \Pr(Piv|q,1)\,\tau_1$$

subject to the Bayes Plausibility constraint $\mu_1\tau_1 + \mu_0\tau_0 = \mu$. Since $\mu_1 = 1$ and $\tau_1 = 1 - \tau_0$, the latter implies $\tau_0 = \frac{1-\mu}{1-\mu_0}$, and since $\Pr(Piv|q,1) = 0$, the average probability of a split vote is $\Pr(Piv|q,\mu_0)\frac{1-\mu}{1-\mu_0}$, and μ_0 solves

$$\mu_0 = \arg\max_{m} \frac{\Pr(Piv|q,m)}{1-m}(1-\mu)\,. \tag{47}$$

The point μ_0 is one where the linear function that starts at $(1,0)$ is tangent to the function $\Pr(Piv|q,\cdot)$. To find μ_0, we substitute (21) into (47) and get

$$\frac{\Pr(Piv|q,m)}{1-m}(1-\mu_0) = 2C_{N-1}^{\frac{N-1}{2}}\left(\varrho_0(q,m)\left(1-\varrho_0(q,m)\right)\right)^{\frac{N-1}{2}}(1-\mu)\,,$$

where $\varrho_0(q,m)$ is given by (24). Hence, μ_0 solves

$$\max_{m}\left(\varrho_0(q,m)\left(1-\varrho_0(q,m)\right)\right)^{\frac{N-1}{2}}. \tag{48}$$

Therefore, if feasible, the optimal m is such that $\varrho_0(q,m) = \frac{1}{2}$. In other words, the probability of a shareholder voting in favor conditional on a controversial recommendation (i.e., $s = 0$) and the state being in line with the recommendation ($\theta = 0$), is exactly 50%. If $m = \frac{1}{2}$, then $\pi(q,m) = \frac{1}{2}$, so $\varrho_0(q,m) = \frac{1-q}{2} < \frac{1}{2}$. Hence, $m = \frac{1}{2}$ does not solve (48) if there exists $m \in \left(\frac{1}{2}, \mu\right)$ that satisfies $\varrho_0(q,m) = \frac{1}{2}$. Consider $m \neq \frac{1}{2}$. Then, using (4), $\varrho_0(q,m) = \frac{1}{2}$ is equivalent to

$$\frac{z(1-2q)-1+\sqrt{(z-1)^2+4q^2z}}{2(z-1)} = \frac{1}{2}$$
$$\Leftrightarrow z^2\left(1-4q^2\right) + z\left(4q^2-2\right) + 1 = 0 \Leftrightarrow (z-1)\left(z-\frac{1}{1-4q^2}\right) = 0,$$

where $z \equiv \left(\frac{m}{1-m}\right)^{\frac{2}{N-1}}$. Since $m \neq \frac{1}{2}$, the only root is $z = \frac{1}{1-4q^2}$. Equating it to $\left(\frac{\mu_0}{1-\mu_0}\right)^{\frac{2}{N-1}}$ implies that the optimal posterior is

$$\mu_0(q) = \frac{1}{1+\left(1-4q^2\right)^{\frac{N-1}{2}}}. \tag{49}$$

Note that $\mu_0 > \frac{1}{2}$. It follows that the optimal recommendation induces beliefs $\mu_0 \in \left(\frac{1}{2}, \mu\right)$ given by (49) and $\mu_1 = 1$. The average probability of a shareholder being pivotal given this recommendation design is $2C_{N-1}^{\frac{N-1}{2}}\left(\frac{1}{4}\right)^{\frac{N-1}{2}}(1-\mu) = (1-\mu)\,C_{N-1}^{\frac{N-1}{2}}2^{2-N}$. The proof that this recommendation is indeed optimal for any $\mu \geq \mu_0(q)$ is similar to the corresponding proof for $\mu \leq \mu_1(q)$: (21) implies that for any (μ_l, μ_h) such that $\mu_l < \mu < \mu_h$, we have

$$\frac{\mu_h - \mu}{\mu_h - \mu_l}\Pr\left(Piv|q, \mu_l\right) + \frac{\mu - \mu_l}{\mu_h - \mu_l}\Pr\left(Piv|q, \mu_h\right) \leq (1-\mu)\,C_{N-1}^{\frac{N-1}{2}}2^{2-N},$$

as required.

A.3 Proof of Proposition 8

Recall that we focus on symmetric equilibria, and also on equilibria in undominated strategies at the voting stage. Consider any public recommendation design $\mathcal{S}$. We will show that for any realization s inducing posterior μ_s and any expected fraction of subscribers q, a shareholder's willingness to pay for an imperfectly informative report is weakly lower than for a fully informative report. With a slight abuse of notation, we will use $\Pr(\cdot|\mu_s)$ to denote the conditioning based on the realization s and $\Pr(\cdot|\mu_s, q)$ to denote the conditioning based on the realization s when the probability of subscribing is q. We also denote

$$L(x) \equiv C_{N-1}^{\frac{N-1}{2}}\left(x(1-x)\right)^{\frac{N-1}{2}},$$

which captures the probability of a shareholder being pivotal if other shareholders vote for the proposal with probability x.

If $\mu_s = 0$ or 1, the value of the report is zero, regardless of its information content. Consider $\mu_s \in (0,1)$. The proof consists of the following steps.

1: It is sufficient to focus on binary signals, $R = \{0, 1\}$**.**

Consider an arbitrary report $\mathcal{R}$, and let $W(\mathcal{R}, \mu_s, q)$ denote the value of the report (divided by v_i) for shareholder i, conditional on s and given q. Note that when deciding how to vote, a subscribing shareholder does not learn any additional information from the event of being pivotal: this is because all shareholders' votes are based on r and/or s, and the subscribing shareholder knows both of them. Hence, the subscribers only condition on r and s when deciding how to vote.

1.1: Breaking down R **into subsets.** Divide the set of signals R into three subsets:

$$R_0 \equiv \left\{r \in R : \Pr\left(\theta = 1|r, s\right) < \frac{1}{2}\right\}, \tag{50}$$

$$R_1 \equiv \left\{r \in R : \Pr\left(\theta = 1|r, s\right) > \frac{1}{2}\right\}, \tag{51}$$

$$R_m \equiv \left\{r \in R : \Pr\left(\theta = 1|r, s\right) = \frac{1}{2}\right\}. \tag{52}$$

Since we focus on equilibria in undominated strategies, R_0 (R_1) is the set of signals in the report that induce all subscribers to vote "against" ("for") with probability one, and R_m is the set of signals for which the subscribers are indifferent between voting "for" and "against." Note that the value of the report conditional on $r \in R_m$ is zero: because a subscriber is indifferent between voting for and against conditional on such r and being pivotal, he believes that each state is equally likely, so any vote brings the same value. It follows that all else equal, $W(\mathcal{R}, \mu_s, q)$ is higher if set R_m is empty. Hence, we can focus on reports where $\Pr(r \in R_0|\mu_s) + \Pr(r \in R_1 \mu_s) = 1$.

1.2: Non-subscribing shareholders mix in equilibrium. Suppose that non-subscribing shareholders vote for the proposal with probability π. We show that $\pi \in (0,1)$. Indeed, suppose that $\pi = 1$. Since $q \in (0,1)$, then $\Pr(Piv|r \in R_1, s) = 0$ and $\Pr(Piv|r \in R_0, s) > 0$. Hence, $\Pr(\theta = 1|Piv, s) = \Pr(\theta = 1|r \in R_0, s) < \frac{1}{2}$, where the inequality follows from (50). But then, since we focus on weakly undominated strategies, the shareholder must vote against, which contradicts $\pi = 1$. Similarly, suppose that $\pi = 0$. Then $\Pr(Piv|r \in R_0, s) = 0$ and $\Pr(Piv|r \in R_1, s) > 0$. Hence, $\Pr(\theta = 1|Piv, s) = \Pr(\theta = 1|r \in R_1, s) > \frac{1}{2}$, where the inequality follows from (51). But then, since we focus on weakly undominated strategies, the shareholder must vote in favor, which contradicts $\pi = 0$. Thus, indeed, $\pi \in (0,1)$, and we find π below.

1.3: Value from the report. Since $q \in (0,1)$ and non-subscribing shareholders randomize, then for any $\mu_s \in (0,1)$, a non-subscribing shareholder is pivotal with a strictly positive probability. Hence, to find it optimal to randomize, he must be indifferent between voting for and against conditional on s and being pivotal, i.e., $\Pr(\theta = 1|\mu_s, Piv) = \frac{1}{2}$. Then, $\Pr(d = \theta|\mu_s, Piv) = \frac{1}{2}$. Therefore, repeating the derivations in Section A.8 of the Online Appendix,

$$\begin{aligned}
W(\mathcal{R}, \mu_s, q) &= \Pr(Piv|q, \mu_s)\left[\Pr(d = \theta|\mathcal{R}, \mu_s, Piv) - \Pr(d = \theta|\mu_s, Piv)\right] \\
&= \Pr(Piv|q, \mu_s)\left[\Pr(d = \theta|\mathcal{R}, \mu_s, Piv) - \frac{1}{2}\right] \\
&= \sum_{r \in R} \Pr(r|\mu_s)\Pr(Piv|q, r, \mu_s)\left(\Pr(d = \theta|r, \mu_s, Piv) - \frac{1}{2}\right) \\
&= \sum_{r \in R_0} \Pr(r|\mu_s)\Pr(Piv|q, r, \mu_s)\left(\Pr(\theta = 0|r, \mu_s) - \frac{1}{2}\right) \\
&\quad + \sum_{r \in R_1} \Pr(r|\mu_s)\Pr(Piv|q, r, \mu_s)\left(\Pr(\theta = 1|r, \mu_s) - \frac{1}{2}\right),
\end{aligned}$$

where the last equality uses the fact that R_m is empty and the fact that $\Pr(\theta = 1|r, \mu_s, Piv) = \Pr(\theta = 1|r, \mu_s)$ because, as discussed above, there is no added information from the event of being pivotal conditional on observing r.

1.4: Focusing on binary signals. Since we focus on symmetric equilibria, then for any $r \in R_0$ the voting strategy of subscribers is the same, and hence $\Pr(Piv|q, r_1, \mu_s) = \Pr(Piv|q, r_2, \mu_s)$

for any $r_1, r_2 \in R_0$. Similarly, $\Pr(Piv|q, r_1, \mu_s) = \Pr(Piv|q, r_2, \mu_s)$ for any $r_1, r_2 \in R_1$. Thus,

$$\begin{aligned} W(\mathcal{R}, \mu_s, q) \;=\;& \Pr(Piv|q, r \in R_0, \mu_s) \Pr(r \in R_0|\mu_s) \left(\Pr(\theta = 0|r \in R_0, \mu_s) - \frac{1}{2} \right) \\ &+ \Pr(Piv|q, r \in R_1, \mu_s) \Pr(r \in R_1|\mu_s) \left(\Pr(\theta = 1|r \in R_1, \mu_s) - \frac{1}{2} \right). \end{aligned} \tag{53}$$

Notice that it is without loss of generality to combine all $r \in R_0$ into one signal, denoted $r = 0$, and all $r \in R_1$ into the other signal, denoted $r = 1$. Thus, we can focus on binary signals, as required.

2: An informative report is optimal.

2.1: Finding the strategy of non-subscribers.

Denote

$$\begin{aligned} p_0 &\equiv \Pr(\theta = 0|r = 0, \mu_s), \\ p_1 &\equiv \Pr(\theta = 1|r = 1, \mu_s). \end{aligned}$$

Then (50) and (51) imply

$$p_0 > \frac{1}{2} \text{ and } p_1 > \frac{1}{2}. \tag{54}$$

Since R_m is empty, we have:

$$\begin{gathered} p_1 \Pr(r = 1|\mu_s) + (1 - p_0) \Pr(r = 0|\mu_s) = \mu_s, \\ \Pr(r = 1|\mu_s) + \Pr(r = 0|\mu_s) = 1, \end{gathered} \tag{55}$$

where the top equation follows from Bayes' rule. Solving (55), we get:

$$\begin{gathered} \Pr(r = 1|\mu_s) = \tfrac{\mu_s - (1 - p_0)}{p_1 + p_0 - 1}, \\ \Pr(r = 0|\mu_s) = \tfrac{p_1 - \mu_s}{p_1 + p_0 - 1}. \end{gathered} \tag{56}$$

We next find π. Using Bayes rule,

$$\Pr(\theta = 1|s, Piv) = \frac{\Pr(Piv|\theta = 1, s)\, \mu_s}{\Pr(Piv|\theta = 1, s)\, \mu_s + \Pr(Piv|\theta = 0, s)(1 - \mu_s)}. \tag{57}$$

Recall that non-subscribing shareholders must be indifferent between voting for and against, i.e., $\Pr(\theta = 1|s, Piv) = \frac{1}{2}$. Together with (57), this implies

$$\Pr(Piv|\theta = 1, s)\, \mu_s = \Pr(Piv|\theta = 0, s)(1 - \mu_s),$$

or equivalently,

$$
\begin{aligned}
&\begin{pmatrix} \Pr\left(Piv|\theta=1,s,r=1\right)\Pr\left(r=1|\theta=1,s\right) \\ +\Pr\left(Piv|\theta=1,s,r=0\right)\Pr\left(r=0|\theta=1,s\right) \end{pmatrix}\mu_s \\
=&\begin{pmatrix} \Pr\left(Piv|\theta=0,s,r=1\right)\Pr\left(r=1|\theta=0,s\right) \\ +\Pr\left(Piv|\theta=0,s,r=0\right)\Pr\left(r=0|\theta=0,s\right) \end{pmatrix}\left(1-\mu_s\right).
\end{aligned} \tag{58}
$$

Using Bayes' rule and (56),

$$
\begin{aligned}
\Pr\left(r=1|\theta=1,s\right) &= \frac{\Pr\left(\theta=1|r=1,s\right)\Pr\left(r=1|s\right)}{\Pr\left(\theta=1|s\right)} = \frac{p_1}{\mu_s}\Pr\left(r=1|s\right) = \frac{p_1}{\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1}, \\
\Pr\left(r=0|\theta=1,s\right) &= \frac{\Pr\left(\theta=1|r=0,s\right)\Pr\left(r=0|s\right)}{\Pr\left(\theta=1|\mu_s\right)} = \frac{1-p_0}{\mu_s}\Pr\left(r=0|s\right) = \frac{1-p_0}{\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}, \\
\Pr\left(r=1|\theta=0,s\right) &= \frac{\Pr\left(\theta=0|r=1,s\right)\Pr\left(r=1|s\right)}{\Pr\left(\theta=0|\mu_s\right)} = \frac{1-p_1}{1-\mu_s}\Pr\left(r=1|s\right) = \frac{1-p_1}{1-\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1}, \\
\Pr\left(r=0|\theta=0,s\right) &= \frac{\Pr\left(\theta=0|r=0,s\right)\Pr\left(r=0|s\right)}{\Pr\left(\theta=0|s\right)} = \frac{p_0}{1-\mu_s}\Pr\left(r=0|s\right) = \frac{p_0}{1-\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}.
\end{aligned}
$$

Plugging this into (58), we get:

$$
\begin{aligned}
&\left(L\left(q+(1-q)\pi\right)\frac{p_1}{\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1}+L\left((1-q)\pi\right)\frac{1-p_0}{\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}\right)\mu_s \\
=&\left(L\left(q+(1-q)\pi\right)\frac{1-p_1}{1-\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1}+L\left((1-q)\pi\right)\frac{p_0}{1-\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}\right)\left(1-\mu_s\right),
\end{aligned}
$$

or equivalently,

$$
\begin{aligned}
& L\left(q+(1-q)\pi\right)\left[p_1\frac{\mu_s-(1-p_0)}{p_1+p_0-1}-(1-p_1)\frac{\mu_s-(1-p_0)}{p_1+p_0-1}\right] \\
=\;& L\left((1-q)\pi\right)\left[p_0\frac{p_1-\mu_s}{p_1+p_0-1}-(1-p_0)\frac{p_1-\mu_s}{p_1+p_0-1}\right]\Leftrightarrow \\
& L\left(q+(1-q)\pi\right)\left[(2p_1-1)\mu_s+p_0(2p_1-1)+1-2p_1\right] \\
=\;& L\left((1-q)\pi\right)\left[(2p_0-1)p_1+\mu_s(1-2p_0)\right],
\end{aligned}
$$

which gives:

$$
L\left(q+(1-q)\pi\right)\left(p_1-\frac{1}{2}\right)\left(\mu_s-(1-p_0)\right)=L\left((1-q)\pi\right)\left(p_0-\frac{1}{2}\right)\left(p_1-\mu_s\right). \tag{59}
$$

Next, (59) is equivalent to

$$\frac{L\left(q+(1-q)\pi\right)}{L\left((1-q)\pi\right)} = \frac{p_0-\frac{1}{2}}{p_1-\frac{1}{2}}\frac{p_1-\mu_s}{\mu_s-(1-p_0)} = \frac{p_1-\mu_s}{p_1-\frac{1}{2}}\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1} \Leftrightarrow \tag{60}$$

$$\frac{\pi\left(1-(1-q)\pi\right)}{\left(q+(1-q)\pi\right)(1-\pi)} = \left(\frac{p_1-\frac{1}{2}}{p_1-\mu_s}\frac{p_0+\mu_s-1}{p_0-\frac{1}{2}}\right)^{\frac{2}{N-1}} \equiv \tilde{z}_s \tag{61}$$

Denoting the right-hand side of (61) by $\tilde{z}_s$, we note that (61) gives exactly the same equation on π as in the proof of Proposition 2, i.e., $w\left(\pi,\tilde{z}_s\right)=0$, where $w\left(\pi,\cdot\right)$ is given by (16). The proof of Proposition 2 shows that for any $\tilde{z}_s$, there is a unique root in $(0,1)$, which equals $\frac{1}{2}$ if $\tilde{z}_s=1$, and is given by

$$\pi\left(q,\tilde{z}_s\right) = \frac{\tilde{z}_s(1-2q)-1+\sqrt{\left(\tilde{z}_s-1\right)^2+4q^2\tilde{z}_s}}{2\left(\tilde{z}_s-1\right)(1-q)}$$

if $\tilde{z}_s\neq 1$, where $\pi\left(q,\tilde{z}_s\right)$ increases in $\tilde{z}_s$ and $\pi\left(q,\tilde{z}_s\right)>\frac{1}{2}$ if and only if $\tilde{z}_s>1$. Note that $\tilde{z}_s>1$ if and only if $\mu_s>\frac{1}{2}$, and

$$\begin{aligned}\frac{d}{dp_1}\left(\frac{p_1-\mu_s}{p_1-\frac{1}{2}}\right) &= \frac{\mu_s-\frac{1}{2}}{\left(p_1-\frac{1}{2}\right)^2},\\ \frac{d}{dp_0}\left(\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}\right) &= \frac{\mu_s-\frac{1}{2}}{\left(p_0+\mu_s-1\right)^2}.\end{aligned} \tag{62}$$

2.2: Showing that $p_0=p_1=1$. Using (53) and (56),

$$W\left(\mathcal{R},\mu_s,q\right)=$$

$$= L\left((1-q)\pi\right)\Pr\left(r=0|\mu_s\right)\left(p_0-\frac{1}{2}\right)+L\left(q+(1-q)\pi\right)\Pr\left(r=1|\mu_s\right)\left(p_1-\frac{1}{2}\right) \tag{63}$$

$$= L\left((1-q)\pi\right)\frac{p_1-\mu_s}{p_1+p_0-1}\left(p_0-\frac{1}{2}\right)+L\left(q+(1-q)\pi\right)\frac{\mu_s-(1-p_0)}{p_1+p_0-1}\left(p_1-\frac{1}{2}\right) \tag{64}$$

where $\pi\in(0,1)$ is the probability with which each non-subscriber votes for the proposal found above (we omit its dependence on μ_s and q for brevity). Plugging (59) into (64), we get

$$\begin{aligned} W\left(\mathcal{R},\mu_s,q\right) &= L\left((1-q)\pi\right)\frac{p_1-\mu_s}{p_1+p_0-1}\left(2p_0-1\right) &(65)\\ &= L\left(q+(1-q)\pi\right)\frac{\mu_s-(1-p_0)}{p_1+p_0-1}\left(2p_1-1\right). &(66)\end{aligned}$$

Note that the function $L(x)$ increases in x for $x\in\left(0,\frac{1}{2}\right)$ and decreases for $x\in\left(\frac{1}{2},1\right)$.

2.2.1: Case of $\mu_s>\frac{1}{2}$. If $\mu_s>\frac{1}{2}$, then $\tilde{z}_s>1$, so $\pi>\frac{1}{2}$. Hence, $q+(1-q)\pi>q+(1-q)\frac{1}{2}>\frac{1}{2}$, and hence $L\left(q+(1-q)\pi\right)$ decreases in π in this range. Consider an increase in p_1. Then (62) implies that $\frac{p_1-\mu_s}{p_1-\frac{1}{2}}$ increases, so $\tilde{z}_s$ decreases, and hence π decreases. A reduction in $\pi>\frac{1}{2}$

increases $L\left(q+(1-q)\pi\right)$. Moreover, $\frac{d}{dp_1}\left[\frac{2p_1-1}{p_1+p_0-1}\right]=\frac{2p_0-1}{(p_1+p_0-1)^2}>0$ by (54). Hence, (66) implies that $W\left(\mathcal{R},\mu_s,q\right)$ increases. Hence, $p_1=1$ is optimal.

Similarly, consider an increase in p_0. Then (62) implies that $\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}$ increases, so $\tilde{z}_s$ decreases, and hence π decreases. A reduction in $\pi>\frac{1}{2}$ increases $L\left(q+(1-q)\pi\right)$. Moreover $\frac{d}{dp_0}\left[\frac{\mu_s-(1-p_0)}{p_1+p_0-1}\right]=\frac{p_1-\mu_s}{(p_1+p_0-1)^2}$. Given that $p_1=1$ is optimal, this derivative is positive, and hence (66) implies that $W\left(\mathcal{R},\mu_s,q\right)$ increases. Thus, $p_0=1$ is also optimal.

2.2.2: Case of $\mu_s<\frac{1}{2}$. If $\mu_s<\frac{1}{2}$, then $\tilde{z}_s<1$, so $\pi<\frac{1}{2}$. Hence, $(1-q)\pi<\frac{1-q}{2}<\frac{1}{2}$, and hence $L\left((1-q)\pi\right)$ increases in π in this range. Consider an increase in p_0. Then (62) implies that $\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}$ decreases, so $\tilde{z}_s$ increases, and hence π increases. An increase in $\pi<\frac{1}{2}$ increases $L\left((1-q)\pi\right)$. Moreover, $\frac{d}{dp_0}\left[\frac{2p_0-1}{p_1+p_0-1}\right]=\frac{2p_1-1}{(p_1+p_0-1)^2}>0$ by (54). Hence, (65) implies that $W\left(\mathcal{R},\mu_s,q\right)$ increases. Hence, $p_0=1$ is optimal.

Similarly, consider an increase in p_1. Then (62) implies that $\frac{p_1-\mu_s}{p_1-\frac{1}{2}}$ decreases, so $\tilde{z}_s$ increases, and hence π increases. An increase in $\pi<\frac{1}{2}$ increases $L\left((1-q)\pi\right)$. Moreover $\frac{d}{dp_1}\left[\frac{p_1-\mu_s}{p_1+p_0-1}\right]=\frac{p_0-1+\mu_s}{(p_1+p_0-1)^2}$. Given that $p_0=1$ is optimal, this derivative is positive, and hence (65) implies that $W\left(\mathcal{R},\mu_s,q\right)$ increases. Thus, $p_1=1$ is also optimal.

2.2.3: Case of $\mu_s=\frac{1}{2}$. If $\mu_s=\frac{1}{2}$, then $\tilde{z}_s=\frac{1}{2}$ and $\pi=\frac{1}{2}$, so

$$W\left(\mathcal{R},\mu_s,q\right)=\frac{1}{2}L\left((1-q)\frac{1}{2}\right)\frac{(2p_1-1)(2p_0-1)}{p_1+p_0-1}.$$

Here $\frac{d}{dp_1}\left[\frac{2p_1-1}{p_1+p_0-1}\right]=\frac{2p_0-1}{(p_1+p_0-1)^2}>0$, so $p_1=1$ is optimal, and $\frac{d}{dp_0}\left[\frac{2p_0-1}{p_1+p_0-1}\right]=\frac{2p_1-1}{(p_1+p_0-1)^2}>0$, so $p_0=1$ is also optimal.

Hence, for all μ_s, it is optimal to have $p_0=p_1=1$, i.e., a fully informative report.

A.4 Proof of Proposition 9

To prove part (i), we use the insight in Theorem 2 of McLennan (1998) that in a pure common value environment, a symmetric mixed strategy profile that maximizes the players' expected utility in the class of symmetric mixed strategy profiles must be a Nash equilibrium. Although our game is not a pure common value environment since shareholders differ in the extent to which they care about the proposal, v_i, the voting subgame is equivalent to a pure common value environment in which all shareholders care about the probability of a correct decision and learn the state with an exogenous probability. Formally, consider a voting game of N shareholders with the following ingredients. With probability q^*, a shareholder learns the state and votes $v_i=\theta$. With probability $1-q^*$, a shareholder does not learn the state and only learns a public signal realization s, induced by policy $\mathcal{S}^*$. Each shareholder maximizes the probability of a correct decision. This is a symmetric voting environment in which all voters have common interests. By Proposition 2, this voting game has a unique equilibrium in the class of symmetric mixed strategy equilibria, and in this equilibrium, non-subscribing shareholders vote "for" with probability $\pi(q^*, \mu_s)$, given by (4). Combining this result with Theorem 2 in McLennan (1998) implies that this equilibrium must maximize the expected probability of a correct decision, $\Pr(d=\theta)$, in the class of all symmetric mixed strategies, including those that do not depend on the public signal s. In other words, $\Pr(d=\theta)$ is higher under $\pi^*(q^*, \mu, s) \equiv \pi(q^*, \mu_s)$ than under $\pi(q^*, \mu)$, which is the equilibrium mixed strategy profile when the public signal is uninformative. Since $\mathcal{S}^*$ is not fully uninformative, i.e., $\Pr(\mu_s \neq \mu) > 0$, then $\Pr(\pi^*(q^*, \mu, s) \neq \pi(q^*, \mu_s)) > 0$, and hence the comparison is strict.

We next prove part (ii). Recall that we assume that the conditions of Proposition 6 are satisfied, i.e., $\mu < \mu_1(q^*)$ or $\mu > \mu_0(q^*)$. Consider $\mu < \mu_1(q^*)$ (the case of $\mu > \mu_0(q^*)$ is analogous). Consider the optimal q_{uninf}. Since the probability of a shareholder being pivotal under an uninformative recommendation is $\Pr(Piv|q, \mu)$,

$$q_{uninf} = \arg\max_q qH^{-1}(1-q)\Pr(Piv|q,\mu). \tag{67}$$

The derivative of the objective function is positive if and only if

$$\left[qH^{-1}(1-q)\right]' + qH^{-1}(1-q)\frac{\frac{\partial \Pr(Piv|q,\mu)}{\partial q}}{\Pr(Piv|q,\mu)} > 0. \tag{68}$$

The proof of Proposition 7 shows that the assumption of an increasing hazard rate of $H(\cdot)$ guarantees that

$$\left[qH^{-1}(1-q)\right]' \geq 0 \text{ for all } q \leq q^*, \tag{69}$$

where $q^* = \arg\max_q qH^{-1}(1-q)$. Next, we prove that $\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0$ for all $q \leq q^*$. For any q and $\mu \in (0,1)$, using (22), we have:

$$\Pr(Piv|q,\mu) = 2\mu C_{N-1}^{\frac{N-1}{2}}\left(\varrho_1(q,\mu)(1-\varrho_1(q,\mu))\right)^{\frac{N-1}{2}},$$

where

$$\varrho_1(q,\mu) = q + (1-q)\pi(q,\mu) = q + \frac{z(1-2q) - 1 + \sqrt{(z-1)^2 + 4q^2 z}}{2(z-1)}.$$

Hence, $\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0$ if and only if

$$\frac{\partial}{\partial q}\left[\varrho_1(q,\mu)(1-\varrho_1(q,\mu))\right] > 0 \Leftrightarrow \frac{\partial \varrho_1(q,\mu)}{\partial q}(1 - 2\varrho_1(q,\mu)) > 0,$$

where

$$\frac{\partial \varrho_1(q,\mu)}{\partial q} = 1 + \frac{-2z + \frac{8qz}{2\sqrt{(z-1)^2 + 4q^2 z}}}{2(z-1)} = \frac{\sqrt{(z-1)^2 + 4q^2 z} - \sqrt{4q^2 z^2}}{(1-z)\sqrt{(z-1)^2 + 4q^2 z}} > 0$$

since $z > 1$ (which, in turn, follows from $\mu < \mu_1(q^*) < \frac{1}{2}$). Hence, if $\mu < \mu_1(q^*)$, then

$\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0$ if and only if $\varrho_1(q,\mu) = q + (1-q)\pi(q,\mu) < \frac{1}{2}$. As shown in the proof of Proposition 4, at the optimal controversial recommendation, we have $\varrho_1(q,\mu_1(q)) = \frac{1}{2}$. Since $\pi(q,\mu)$ is increasing in μ, then $\varrho_1(q,\mu)$ is also increasing in μ. Then, for any $q \le q^*$, we have $\mu < \mu_1(q^*) \le \mu_1(q)$, and hence $\varrho_1(q,\mu) < \varrho_1(q,\mu_1(q)) = \frac{1}{2}$. Combined, we have

$$\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0 \text{ for all } q \le q^*. \tag{70}$$

Combining (68), (69), and (70), we conclude that the derivative of the advisor's objective function in (67) is strictly positive for all $q \le q^*$. Hence, $q_{uninf} > q^*$.

A.5 Derivations for the example in Section 4.1

Suppose that $v_i \in \{v_L, v_H\}$ with $v_H > v_L$ and $\Pr(v_i = v_H) = p$. All the arguments in the basic model continue to hold for this degenerate distribution. For this distribution, we have:

$$H(v) = \begin{cases} 0, & v < v_L \\ 1-p, & v \in [v_L, v_H) \\ 1, & v \ge v_H \end{cases}$$

Suppose μ is sufficiently small (the case of large μ, which is discussed in the paper, is equivalent by symmetry of the problem in μ around $\frac{1}{2}$).

No ban on public recommendations. First, consider the case in which there is no ban on public recommendations. As the arguments in the basic model show, in the range of (q,μ) for which a partially informative recommendation of the form in Proposition 4 is optimal, the average probability of a shareholder being pivotal is the same and equals $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$ (denote it ζ). Hence, in this range, the advisor's optimal q maximizes $qH^{-1}(1-q)$, which, in the two-type case, means choosing between: 1) selling only to high types, in which case the fee is $\frac{v_H}{2}\zeta$, and profits are $p\frac{v_H}{2}\zeta$ and 2) selling also to some low types, for a total fraction q, in which case the fee is $\frac{v_L}{2}\zeta$, and profits are $q\frac{v_L}{2}\zeta$. If v_H is sufficiently high, then $pv_H > qv_L$, and hence it is optimal for the advisor to only sell the report to high types ($q = p$). In particular, recall from Proposition 7 that a partially informative recommendation of the form in Proposition 4 is optimal only if $q < 0.5$. Hence, for any $v_H \ge 5$, we have $pv_H \ge 0.5 > qv_L$ for any such q.

We conclude that without a ban, if $v_H \ge 5$, then $q = p$, i.e., only high types subscribe to the research report. The optimal recommendation design for a fixed probability of subscribing is given by Proposition 4. Hence, if $\mu < \mu_1(p)$, where $\mu_1(\cdot)$ is defined in Proposition 4, then the optimal recommendation induces posterior beliefs 0 and $\mu_1(p)$, and the average probability that a shareholder is pivotal is $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$ (as shown in the proof of Proposition 4). Using (9), the price that the seller charges is

$$f = v_H \frac{1}{2}\left[\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}\right] = v_H \mu C_{N-1}^{\frac{N-1}{2}} 2^{1-N},$$

and the expected profits of the advisor are

$$\Pi_{no\ ban} = Nfp = Nv_H \mu C_{N-1}^{\frac{N-1}{2}} 2^{1-N} p.$$

The probability of a correct decision is:

$$V_{no\ ban} = \Pr\left(d = \theta | p, \mathcal{S}^*\right) = \mu \Pr\left(d = 1 | p, \mathcal{S}^*, \theta = 1\right) + (1-\mu) \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0\right),$$

where $\Pr\left(d = 1 | p, \mathcal{S}^*, \theta = 1\right) = \sum_{k=\frac{N+1}{2}}^{N} C_N^k \left(\frac{1}{2}\right)^N$ because, as shown in the proof of Proposition 4, $\varrho_1\left(p, \mu_1\left(p\right)\right) = \frac{1}{2}$. Next,

$$\begin{aligned}\Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0\right) &= \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 0\right) \Pr\left(s = 0 | \theta = 0\right) \\ &\quad + \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 1\right) \Pr\left(s = 1 | \theta = 0\right).\end{aligned}$$

Consider the first term. Since $s = 0$ induces belief $\mu_0 = 0$ and is given with probability $1 - \frac{\mu}{\mu_1(p)}$, we have

$$\begin{aligned}\Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 0\right) \Pr\left(s = 0 | \theta = 0\right) &= \Pr\left(s = 0 | \theta = 0\right) \\ = \frac{\Pr\left(\theta = 0 | s = 0\right) \Pr\left(s = 0\right)}{\Pr\left(\theta = 0\right)} &= \frac{1 - \frac{\mu}{\mu_1(p)}}{1-\mu}.\end{aligned}$$

Consider the second term. Recall from the proof of Proposition that $\varrho_0\left(p, \mu_1\left(p\right)\right) = (1-p)\,\pi\left(p, \mu_1\left(p\right)\right)$, where $\mu_1\left(p\right)$ satisfies $(1-p)\left(1 - \pi\left(p, \mu_1\left(p\right)\right)\right) = \frac{1}{2}$. Hence,

$$1 - \varrho_0\left(p, \mu_1\left(p\right)\right) = 1 - (1-p)\,\pi\left(p, \mu_1\left(p\right)\right) = p + (1-p)\left(1 - \pi\left(p, \mu_1\left(p\right)\right)\right) = p + \frac{1}{2}$$

and $\varrho_0\left(p, \mu_1\left(p\right)\right) = \frac{1}{2} - p$. Since $s = 1$ induces belief $\mu_1\left(p\right)$ and is given with probability $\frac{\mu}{\mu_1(p)}$, we have

$$\begin{aligned}&\Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 1\right) \Pr\left(s = 1 | \theta = 0\right) \\ &= \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\varrho_0\left(p, \mu_1\left(p\right)\right)\right)^k \left(1 - \varrho_0\left(p, \mu_1\left(p\right)\right)\right)^{N-k}\right) \frac{\Pr\left(\theta = 0 | s = 1\right) \Pr\left(s = 1\right)}{\Pr\left(\theta = 0\right)} \\ &= \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\frac{1}{2} - p\right)^k \left(p + \frac{1}{2}\right)^{N-k}\right) \frac{\left(1 - \mu_1\left(p\right)\right) \frac{\mu}{\mu_1(p)}}{1-\mu}.\end{aligned}$$

Combining, we get:

$$\begin{aligned}V_{no\ ban} &= \mu \sum_{k=\frac{N+1}{2}}^{N} C_N^k \left(\tfrac{1}{2}\right)^N + (1-\mu)\left(\frac{1 - \frac{\mu}{\mu_1(p)}}{1-\mu} + \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\tfrac{1}{2} - p\right)^k \left(p + \tfrac{1}{2}\right)^{N-k}\right) \frac{\left(1 - \mu_1(p)\right) \frac{\mu}{\mu_1(p)}}{1-\mu}\right) \\ &= \mu \tfrac{1}{2} + 1 - \tfrac{\mu}{\mu_1(p)} + \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\tfrac{1}{2} - p\right)^k \left(p + \tfrac{1}{2}\right)^{N-k}\right) \left(1 - \mu_1\left(p\right)\right) \tfrac{\mu}{\mu_1(p)}.\end{aligned}$$

Ban on public recommendations. Next, consider the case with a ban on public recommendations. There are two possibilities: either the advisor continues to sell to only high types, or it also sells to a fraction of low types. First, consider the case in which the seller sells to some of the low types too. If the optimal recommendation is uninformative, then using (22), a shareholder is pivotal with probability

$$\Pr\left(Piv|q,\mu\right) = 2\mu C_{N-1}^{\frac{N-1}{2}}\left(\left(q+\left(1-q\right)\pi\left(q,\mu\right)\right)\left(1-q\right)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}}.$$

Then, the shareholder of a low type is willing to pay $\frac{v_L}{2}\Pr\left(Piv|q,\mu\right)$, and hence the optimal q solves:

$$q^{**} = \arg\max_q q\left(\left(q+\left(1-q\right)\pi\left(q,\mu\right)\right)\left(1-q\right)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}}, \tag{71}$$

where

$$\begin{gathered}\pi\left(q,\mu\right) = \tfrac{z(1-2q)-1+\sqrt{(z-1)^2+4q^2z}}{2(z-1)(1-q)} \Leftrightarrow \\ q+\left(1-q\right)\pi\left(q,\mu\right) = \tfrac{1}{2}+\tfrac{\sqrt{(z-1)^2+4q^2z}-2q}{2(z-1)},\end{gathered}$$

and $z = \left(\frac{\mu}{1-\mu}\right)^{\frac{2}{N-1}}$. Plugging this into (71), we can find q^{**} by solving:

$$q^{**} = \arg\max_q q\left(1-\frac{\left(\sqrt{\left(z-1\right)^2+4q^2z}-2q\right)^2}{\left(z-1\right)^2}\right)^{\frac{N-1}{2}}.$$

To induce such an expected fraction of subscribers, the proxy advisor needs to charge fee

$$f = \frac{v_L}{2}\Pr\left(Piv|q^{**},\mu\right) = \frac{v_L}{2}C_{N-1}^{\frac{N-1}{2}}\left(\left(q^{**}+\left(1-q^{**}\right)\pi\left(q^{**},\mu\right)\right)\left(1-q^{**}\right)\left(1-\pi\left(q^{**},\mu\right)\right)\right)^{\frac{N-1}{2}},$$

and the corresponding expected profits are:

$$\Pi_{ban,both\ types} = Nfq^{**} = N\frac{v_L}{2}C_{N-1}^{\frac{N-1}{2}}\left(\left(q^{**}+\left(1-q^{**}\right)\pi\left(q^{**},\mu\right)\right)\left(1-q^{**}\right)\left(1-\pi\left(q^{**},\mu\right)\right)\right)^{\frac{N-1}{2}}q^{**}.$$

The probability of the correct decision is:

$$V_{ban,both\ types} = \Pr(d=\theta|q^{**},\mu) = \mu \Pr(d=1|q^{**},\mu,\theta=1) + (1-\mu)\Pr(d=0|q^{**},\mu,\theta=0) \tag{72}$$

$$= \mu\left(\sum_{k=\frac{N+1}{2}}^{N} C_N^k \left(q^{**}+(1-q^{**})\pi(q^{**},\mu)\right)^k \left((1-q^{**})(1-\pi(q^{**},\mu))\right)^{N-k}\right) \tag{73}$$

$$+(1-\mu)\left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left((1-q^{**})\pi(q^{**},\mu)\right)^k \left(1-(1-q^{**})\pi(q^{**},\mu)\right)^{N-k}\right) \tag{74}$$

$$= \mu\left(\sum_{K=0}^{\frac{N-1}{2}} C_N^K \left(q^{**}+(1-q^{**})\pi(q^{**},\mu)\right)^{N-K} \left((1-q^{**})(1-\pi(q^{**},\mu))\right)^{K}\right)$$

$$+(1-\mu)\left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left((1-q^{**})\pi(q^{**},\mu)\right)^k \left(1-(1-q^{**})\pi(q^{**},\mu)\right)^{N-k}\right),$$

where the last equality uses the notation $K = N - k$ in the first term.

Second, consider the case where even with the ban the seller wants to sell to high types only. In this case, the same fraction p of shareholders observe the report (and the state), but all other shareholders are now informed. Part (i) of Proposition 9 then directly implies that the probability of the correct decision, $V_{ban,high\ type}$, is strictly lower than without a ban, $V_{no\ ban}$. The profits of the seller in this case are

$$\Pi_{ban,high\ type} = Nfp = N\frac{v_H}{2}C_{N-1}^{\frac{N-1}{2}}\left((p+(1-p)\pi(p,\mu))(1-p)(1-\pi(p,\mu))\right)^{\frac{N-1}{2}} p,$$

and the probability of the correct decision is given by the sum of (73) and (74) but with q^{**} replaced by p.

Numerically, we find that if $N = 25$ and $\mu = 0.1$ (or $\mu = 0.9$), then $q^{**} = 0.257$. If $v_H = 5$, then under the ban, the advisor's profit from selling to both high and low types is $\Pi_{ban,both\ types} = 0.0927$, whereas its profit from selling only to high types is $\Pi_{ban,high\ type} = 0.0732$. Hence, it is optimal to sell to both types and induce $q^{**} = 0.257$, which results in the probability of a correct decision of $V_{ban,both\ types} = 0.9565$. On the other hand, if $v_H = 7$, then under the ban, the advisor's profit from selling to both high and low types is $\Pi_{ban,both\ types} = 0.0927$, whereas its profit from selling only to high types is $\Pi_{ban,high\ type} = 0.1025$. Hence, it is optimal to sell to only the high types and induce fraction of subscribers $p = 0.1$, which results in the probability of a correct decision of $V_{ban,both\ types} = 0.9031$.

A.6 Proof of Proposition 10

The same arguments as in the proof of Proposition 8 imply that the optimal research report is fully informative about the state. The solution of this variation of the model is similar to the solution of the baseline model with the following modifications. A shareholder whose

$v_i > \hat{v}$ does not subscribe to the report. Let σ denote the probability with which a shareholder subscribes to the report conditional on $v_i < \hat{v}$. Then the probability with which a shareholder learns the state, which we denote by q, is

$$q = \Pr(v_i > \hat{v}) + \sigma \Pr(v_i < \hat{v}) = \chi + (1-\chi)\sigma,$$

or equivalently, $\sigma = \frac{q-\chi}{1-\chi}$ and $1-\sigma = \frac{1-q}{1-\chi}$.

The voting subgame and the information acquisition decisions of shareholders who are not exogenously informed depend on q, the overall probability of other shareholders becoming informed, in exactly the same way as before. In particular, the equilibrium probability that an uninformed shareholder votes for the proposal is $\pi(q, \mu_s)$ given by (4), and the value from the report for a shareholder whose v_i is below $\hat{v}$ is $v_i V(q, \mathcal{S})$, where $V(q, \mathcal{S})$ is given by (7). Therefore,

$$\sigma = \Pr(f/V(q,\mathcal{S}) \leq v_i < \hat{v} | v_i < \hat{v}) = \frac{H(\hat{v}) - H(f/V(q,\mathcal{S}))}{H(\hat{v})} = \frac{1 - H(f/V(q,\mathcal{S})) - \chi}{1-\chi}.$$

Hence, we can find the fee that induces expected fraction q of informed shareholders from

$$H(f/V(q,\mathcal{S})) = (1-\chi)(1-\sigma) = 1-q \Leftrightarrow f = V(q,\mathcal{S}) H^{-1}(1-q),$$

as in the basic model. The expected fraction of shareholders subscribing to the report is $\Pr(f/V(q,\mathcal{S}) \leq v_i < \hat{v}) = \sigma \Pr(v_i < \hat{v}) = q - \chi$, and thus the expected profit of the advisor is $N(q-\chi) V(q,\mathcal{S}) H^{-1}(1-q)$. Hence, the advisor's problem is now

$$\begin{gathered} \max_{q,S,\{\mu_s,\tau_s\}} (q-\chi) H^{-1}(1-q) \textstyle\sum_{s\in S} \Pr(Piv|q,\mu_s)\tau_s \\ s.to \ \textstyle\sum_{s\in S} \mu_s \tau_s = \mu. \end{gathered}$$

As in the basic model, we can decompose this problem into two steps: first, find the optimal recommendation design for a given q, and then, find the optimal q and fee. For a given q, the public recommendation design problem is exactly the same as in the basic model. Thus, the analysis of Section 3.3.1 and Proposition 4 are unchanged. The part that is different is the pricing of information by the advisor. As in the basic model, the average probability of being pivotal is the same for all pairs of (q, μ) for which the optimal recommendation is partially informative and of the form described by Proposition 4. Hence, it is optimal for the advisor to choose

$$q_e^*(\chi) = \arg\max_q (q-\chi) H^{-1}(1-q), \tag{75}$$

as long as $q_e^*(\chi) < \frac{1}{2}$ and priors are sufficiently asymmetric, as in Proposition 6. The cross-partial derivative of the objective function $(q-\chi) H^{-1}(1-q)$ in q and χ is $\frac{\partial}{\partial q}[-H^{-1}(1-q)]$, which is positive because $H^{-1}(\cdot)$ is an increasing function. Hence, by Topkis's theorem, $q_e^*(\chi)$ is increasing in χ. Therefore, the comparative statics of the recommendation design in χ is the same as the comparative statics of the optimal recommendation design in q in the basic model, which is given by Corollary 1. In addition, it follows that $q_e^*(\chi) > q_e^*(0) = q^*$, the equilibrium expected fraction of informed shareholders in the basic model.

A.7 Proof of Proposition 11

For shareholder i, the value of subscribing to the proxy advisor's services is

$$V_i(q, \mathcal{S}) + \omega = v_i \cdot V(q, \mathcal{S}) + \omega.$$

Given fee $f \geq \omega$, the shareholder becomes the subscriber if and only if $v_i \geq \frac{f-\omega}{V(q,\mathcal{S})}$, which implies

$$f = V(q, \mathcal{S}) H^{-1}(1-q) + \omega = H^{-1}(1-q) \left(\frac{1}{2} \sum_{s \in S} \Pr(Piv|q, \mu_s) \tau_s \right) + \omega.$$

Since the proxy advisor's expected revenue is Nqf, its problem is to choose q and $\mathcal{S}$ to solve:

$$\max_{q,\mathcal{S}} \left\{ qH^{-1}(1-q) \left(\sum_{s \in S} \Pr(Piv|q, \mu_s) \tau_s \right) + 2q\omega \right\}.$$

For a fixed q, the optimization problem of the proxy advisor over $\mathcal{S}$ is the same as in the basic model. Recall that its solution gives the maximum average probability of a shareholder being pivotal of $P(q, \mu)$. Therefore, the only aspect that changes compared to the basic model is the q targeted by the advisor. The optimization problem becomes

$$\max_{q} \left\{ qH^{-1}(1-q) P(q, \mu) + 2q\omega \right\}. \tag{76}$$

Let $q^*(\omega)$ be the solution to this problem. By Topkis's theorem, the sign of $\frac{\partial q^*(\omega)}{\partial \omega}$ coincides with the sign of

$$\frac{\partial^2}{\partial q \partial \omega} \left[qH^{-1}(1-q) P(q, \mu) + 2q\omega \right] |_{q=\tilde{q}(\omega)} = 2.$$

Hence, $q^*(\omega)$ increases in ω. Given an equilibrium increase in the expected fraction of subscribers, the effect on the recommendation design follows from Corollary 1.

A.8 Value of buying the report

We show that for an arbitrary research report $\mathcal{R}$ and recommendation policy $\mathcal{S}$, the value $W_i(\mathcal{R}, \mathcal{S})$ of buying the report for shareholder i is $v_i \cdot W(\mathcal{R}, \mathcal{S})$, where

$$\begin{aligned} W(\mathcal{R}, \mathcal{S}) &= \Pr(Piv) \left[\Pr(d = \theta | Piv, \mathcal{R}) - \Pr(d = \theta | Piv, \mathcal{S}) \right] & (77) \\ &= \sum_{s \in S} \Pr(s) \Pr(Piv|s) \left[\Pr(d = \theta | Piv, \mathcal{R}) - \Pr(d = \theta | Piv, s) \right], & (78) \end{aligned}$$

and Piv denotes the event that the shareholder is pivotal.

We first show that $W(\mathcal{R}, \mathcal{S}) = \Pr(d = \theta | \mathcal{R}) - \Pr(d = \theta | \mathcal{S})$. Indeed, let $U(\mathcal{R})$ and $U(\mathcal{S})$ denote the shareholder's expected utility (divided by v_i) from knowing the report and from knowing only the recommendation, respectively. Then

$$W(\mathcal{R},\mathcal{S}) = U(\mathcal{R}) - U(\mathcal{S}) = \sum_{\theta\in\{0,1\}} \left[u(\theta,\theta)\Pr(d=\theta|\mathcal{R}) + u(1-\theta,\theta)\Pr(d\neq\theta|\mathcal{R})\right]$$
$$- \sum_{\theta\in\{0,1\}} \left[u(\theta,\theta)\Pr(d=\theta|\mathcal{S}) + u(1-\theta,\theta)\Pr(d\neq\theta|\mathcal{S})\right]$$
$$= \sum_{\theta\in\{0,1\}} u(\theta,\theta)\left[\Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S})\right] + \sum_{\theta\in\{0,1\}} u(1-\theta,\theta)\left[\Pr(d\neq\theta|\mathcal{R}) - \Pr(d\neq\theta|\mathcal{S})\right]$$
$$= \sum_{\theta\in\{0,1\}} u(\theta,\theta)\left[\Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S})\right] + \sum_{\theta\in\{0,1\}} u(1-\theta,\theta)\left[-\Pr(d=\theta|\mathcal{R}) + \Pr(d=\theta|\mathcal{S})\right]$$
$$= \sum_{\theta\in\{0,1\}} \left[u(\theta,\theta) - u(1-\theta,\theta)\right]\left[\Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S})\right] = \Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S}),$$

where the last equality uses $u(1,1) - u(0,1) = u(0,0) - u(1,0) = 1$.

Next, let T denote the vote tally among the remaining $N-1$ shareholders. The shareholder's vote only changes the decision d if the votes of other shareholders are split, and hence $\Pr(d=\theta|T,\mathcal{R}) - \Pr(d=\theta|T,\mathcal{S}) = 0$ if $T \neq \frac{N-1}{2}$. Hence,

$$W(\mathcal{R},\mathcal{S}) = \sum_{T=0}^{N-1} \Pr(T)\left[\Pr(d=\theta|T,\mathcal{R}) - \Pr(d=\theta|T,\mathcal{S})\right]$$
$$= \Pr\left(T = \frac{N-1}{2}\right)\left[\Pr\left(d=\theta|T=\frac{N-1}{2},\mathcal{R}\right) - \Pr\left(d=\theta|T=\frac{N-1}{2},\mathcal{S}\right)\right],$$

which gives (77), as required. Expression (77) follows from Bayes rule.

Special case when the research report is fully informative. If the research report is fully informative, then $\Pr(d=\theta|Piv,\mathcal{R}) = 1$, and

$$W(\mathcal{R},\mathcal{S}) = \sum_{s\in S} \Pr(s)\Pr(Piv|s)\left[1 - \Pr(d=\theta|Piv,s)\right].$$

We can split the set S into two subsets depending on the posterior μ_s. If μ_s is 0 or 1, then all shareholders know the state with certainty, and given the focus on undominated strategies, they all vote according to the state, so $\Pr(Piv|s) = 0$. If $\mu_s \in (0,1)$, then, as shown in the proof of Proposition 2, $\Pr(d=\theta|Piv,s) = \frac{1}{2}$. Hence,

$$W(\mathcal{R},\mathcal{S}) = \sum_{s\in S} \Pr(s)\Pr(Piv|s)\frac{1}{2},$$

i.e., we get expression (7), as required.

B Implications for empirical voting patterns

In this section, we present additional derivations for the empirical implications in Section 5. In Sections B.1 and B.2, we derive the expressions used in the numerical example in Figure 4. In Section B.3, we prove an additional statement used in Section 5.2.

B.1 Probability of a close vote

We focus on the case of $q^* < \frac{1}{2}$ and $\mu \geq \mu_0(q^*) = (1+(1-4q^{*2})^{\frac{N-1}{2}})^{-1}$. The case of $\mu \leq \mu_1(q^*)$ is similar by symmetry. Denote T the voting tally, i.e., the random variable that stands for the number of votes in favor of the proposal.

We first provide the comparison for the probability of a close vote, defined as:

$$\Pr(\textit{close vote}) \equiv \Pr\left(T = \frac{N-1}{2}\right) + \Pr\left(T = \frac{N+1}{2}\right).$$

As we show next, the probability of a close vote defined this way is proportional to the probability of a shareholder being pivotal (i.e., the probability of a split vote among $N-1$ shareholders). Hence, this probability is maximized under the optimal recommendation design $\mathcal{S}^*$ and is higher than under uninformative recommendations. Formally, if the recommendation takes the form described in Part 2 of Proposition 6, then a close vote never occurs upon $s=1$, and hence

$$\Pr(\textit{close vote}|\mathcal{S}^*) = \Pr(\textit{close vote}|s=0)\Pr(s=0),$$

where

$$\Pr(\textit{close vote}|s=0) = \Pr(\textit{close vote}|\theta=1, s=0)\,\mu_0(q^*) + \Pr(\textit{close vote}|\theta=0, s=0)\,(1-\mu_0(q^*)).$$

Note that

$$\Pr(\textit{close vote}|\theta=1, s=0) = C_N^{\frac{N-1}{2}}\left[\varrho_1^*\right]^{\frac{N-1}{2}}\left[1-\varrho_1^*\right]^{\frac{N+1}{2}} + C_N^{\frac{N+1}{2}}\left[\varrho_1^*\right]^{\frac{N+1}{2}}\left[1-\varrho_1^*\right]^{\frac{N-1}{2}},$$

where using (23)-(24)

$$\begin{aligned} \varrho_1^* &\equiv \varrho_1(q, \mu_0(q^*)) = q + (1-q)\,\pi(q, \mu_0(q^*)), && (79)\\ \varrho_0^* &\equiv \varrho_0(q, \mu_0(q^*)) = (1-q)\,\pi(q, \mu_0(q^*)), && (80) \end{aligned}$$

and $\pi(q, \cdot)$ is given by (4). Using $C_N^{\frac{N-1}{2}} = C_N^{\frac{N+1}{2}}$, we get

$$\Pr(\textit{close vote}|\theta=1, s=0) = C_N^{\frac{N-1}{2}}\left[\varrho_1^*\right]^{\frac{N-1}{2}}\left[1-\varrho_1^*\right]^{\frac{N-1}{2}}$$

$$= C_{N-1}^{\frac{N-1}{2}}\left[\varrho_1^*\right]^{\frac{N-1}{2}}\left[1-\varrho_1^*\right]^{\frac{N-1}{2}}\left(C_N^{\frac{N-1}{2}}/C_{N-1}^{\frac{N-1}{2}}\right) = \Pr(Piv|\theta=1, s=0)\,\frac{2N}{N+1},$$

where $\Pr(Piv|\cdot)$ is the probability of a shareholder being pivotal, i.e., the probability of a split vote among $N-1$ shareholders. Using the same argument for $\Pr(\textit{close vote}|\theta=0, s=0)$, we can rewrite

$$\Pr\left(close\ vote|\mathcal{S}^*\right)=\Pr\left(Piv|s=0\right)\Pr\left(s=0\right)\frac{2N}{N+1}=P\left(q^*,\mu\right)\frac{2N}{N+1},$$

where $P\left(q^*,\mu\right)$ is the average probability of a shareholder being pivotal under recommendation policy $\mathcal{S}^*$. By exactly the same arguments, for an uninformative recommendation, $\Pr\left(close\ vote|\mathcal{S}_{uninf}\right)=\Pr\left(Piv|q^*,\mu\right)\frac{2N}{N+1}$, where $\Pr\left(Piv|q^*,\mu\right)$ is given by (22). Since $P\left(q^*,\mu\right)$ is the maximum possible average probability of a shareholder being pivotal, we automatically have $\Pr\left(close\ vote|\mathcal{S}^*\right)>\Pr\left(close\ vote|\mathcal{S}_{uninf}\right)$. The two coincide when $\mu=\mu_0\left(q^*\right)$.

B.2 Probability of a lopsided vote

We next derive the probability of a lopsided vote, defined as:

$$\Pr\left(lopsided\ vote\right)\equiv\Pr\left(T=0\right)+\Pr\left(T=N\right).$$

For the optimal recommendation design $\mathcal{S}^*$, a lopsided vote occurs with probability one upon $s=1$, and hence

$$\Pr\left(lopsided\ vote|\mathcal{S}^*\right)=\frac{\mu-\mu_0\left(q^*\right)}{1-\mu_0\left(q^*\right)}+\Pr\left(lopsided\ vote|s=0\right)\frac{1-\mu}{1-\mu_0\left(q^*\right)},$$

where

$$\begin{aligned}\Pr\left(lopsided\ vote|s=0\right) &= \Pr\left(lopsided\ vote|\theta=1,s=0\right)\mu_0\left(q^*\right)\\ &\quad+\Pr\left(lopsided\ vote|\theta=0,s=0\right)\left(1-\mu_0\left(q^*\right)\right)\\ &= \left(\left[\varrho_1^*\right]^N+\left[1-\varrho_1^*\right]^N\right)\mu_0\left(q^*\right)+\left(\left[\varrho_0^*\right]^N+\left[1-\varrho_0^*\right]^N\right)\left(1-\mu_0\left(q^*\right)\right),\end{aligned}$$

and ϱ_1^*,ϱ_0^* are given by (79)-(80). Similary, for an uninformative recommendation,

$$\begin{aligned}\Pr\left(lopsided\ vote|\mathcal{S}_{uninf}\right) &= \Pr\left(lopsided\ vote|\theta=1\right)\mu+\Pr\left(lopsided\ vote|\theta=0\right)\left(1-\mu\right)\\ &= \left(\left[\varrho_1\right]^N+\left[1-\varrho_1\right]^N\right)\mu+\left(\left[\varrho_0\right]^N+\left[1-\varrho_0\right]^N\right)\left(1-\mu\right),\end{aligned}$$

and using (23)-(24)

$$\varrho_1 \equiv \varrho_1\left(q,\mu\right)=q+\left(1-q\right)\pi\left(q,\mu\right), \tag{81}$$

$$\varrho_0 \equiv \varrho_0\left(q,\mu\right)=\left(1-q\right)\pi\left(q,\mu\right), \tag{82}$$

and $\pi\left(q,\cdot\right)$ is given by (4).

For any N, as $q\to 0$, we have $\varrho_1\to 1$ and $\varrho_0\to 1$, so $\Pr\left(lopsided\ vote|\mathcal{S}_{uninf}\right)\to 1$. For the optimal recommendation $\mathcal{S}^*$, $\lim_{q\to 0}\mu_0\left(q\right)\to\frac{1}{2}$, so $\pi\left(q,\mu_0\left(q^*\right)\right)\to\frac{1}{2}$, and hence $\varrho_1^*\to\frac{1}{2}$, $\varrho_0^*\to\frac{1}{2}$, so $\Pr\left(lopsided\ vote|\mathcal{S}^*\right)\to\left(2\mu-1\right)\left(1-2^{1-N}\right)$, which is bounded away from 1.

B.3 Effect of the fraction of informed shareholders on recommendation design

We next prove an additional statement used in the empirical implications of Section 5.2. Let q be the expected fraction of shareholders who observe the state, where q could correspond to q^* (the equilibrium expected fraction of subscribers) in the main model or to q_e^* (the equilibrium expected fraction of subscribers plus the fraction χ of exogenously informed shareholders) in the extension of Section 4.2. We prove the statement used in Implication 5, that the probability of a close vote conditional on the "controversial" recommendation decreases in q.

To prove this, consider the case of $\mu \leq \mu_1(q)$, when the "controversial" recommendation is $s = 1$ (the case of $\mu \geq \mu_0(q)$ is analogous). We first show that the probability of a shareholder being pivotal conditional on $s = 1$ decreases in q. Indeed, according to the proof of Proposition 4, the average probability of a shareholder being pivotal given the optimal recommendation design is $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. Since a shareholder is never pivotal upon $s = 0$ and since $\Pr(s = 1) = \frac{\mu}{\mu_1(q)}$, it follows that

$$\Pr(Piv|s = 1) \frac{\mu}{\mu_1(q)} = \mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}.$$

Since $\mu_1(q)$ decreases in q, $\Pr(Piv|s = 1)$ decreases in q as well. Finally, as shown in Section B.1 of the Online Appendix above, the probability of a close vote (i.e., a vote tally of $\frac{N-1}{2}$ or $\frac{N+1}{2}$) is $\frac{2N}{N+1}$ times the probability of a shareholder being pivotal, and hence the probability of a close vote conditional on $s = 1$ also decreases in q.

References

[1] McLennan, Andrew, 1998. Consequences of the Condorcet jury theorem for beneficial information aggregation by rational agents, *American Political Science Review* 92, 413–418.