Public Disclosures and Information Asymmetries: A Theory of the Mosaic*

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Abstract

We develop a model to analyze the impact of public disclosure on price efficiency and information asymmetry. We provide support for the positive association between public disclosure and information asymmetry documented by many empirical papers. Our model allows us to identify settings in which public disclosure policies can enhance the mosaic of information, allowing informed traders to improve the precision of their private information while simultaneously increasing the amount of information impounded in price. Depending on the weights managers place on share price versus private benefits, they may optimally choose no disclosure, partial disclosure, or full disclosure. Interestingly, partial disclosure is never sustainable if managers care only about private benefits. If informed traders are limited to private information that is non-material only (i.e., information that does not change their current valuation of the firm), non-disclosure is never optimal. Finally, when managers have a duty to disclose negative news, overall disclosure may either increase or decrease.

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1 Introduction

Standard arguments in favor of disclosure regulation assert that increasing disclosure decreases information asymmetry. As such, one might expect traders with private information to dislike all subsequent public disclosures, as they would diminish opportunities to exploit their information advantage. However, considerable empirical evidence exists suggesting that information asymmetry *increases* following both earnings announcements and management forecasts (e.g., Lee, Mucklow, and Ready (1993); Coller and Yohn (1997)). By allowing information gathering only after earnings announcements, Kim and Verrecchia (1994) provide a model consistent with that empirical finding and which predicts an increase in trading volume at the earnings announcement due to the increase in information asymmetry.[1] In contrast, our paper assumes informed traders have private information prior to the announcement and examines the alleviation or exacerbation of information asymmetries that result from public disclosures. Public disclosures can permit partially informed traders to complete the mosaic by complementing their private information with public disclosure. They are better off but at the same time, prices are more efficient.

We begin with non-strategic disclosures (i.e., exogenous) and evaluate the conditions under which informed traders benefit from the public release of information. The distribution of the firm's cash flows is symmetric with three outcomes, high, medium and low. Informed traders gather noisy private information about the firm's cash flows prior to public disclosures by management. More public disclosure reduces the uncertainty about the cash flows and increases price efficiency. One might wrongly conclude that because prices are more informative, informed traders are worse off. We define partial disclosures as public disclosures of certain states of the world. We show that partial disclosures can increase the informed trader's overall expected profits when the informed trader does not have "decisive" directional information or the distribution of his signal violates the Monotone Likelihood Ratio Property (hereafter, MLRP).[2] Buy orders move price (in expectation) upwards. Therefore, informed traders, transacting on private information alone, will lose money whenever they buy and the true state is medium or low. If manager publicly discloses either 'high' or 'not high,' the skilled investor can piece together this mosaic of information (i.e., private and public) to generate a new

[1] Kim and Verrecchia (1994) seems descriptive of settings with few market participants conducting fundamental analyses based on publicly disclosed financial information. Since earnings are the most widely known and processed piece of information (Bloomfield and Libby 1996), it seems reasonable to expect an information advantage gained by superior interpretation of an earnings announcement alone would be quickly dissipated.

[2] In practice, violation of MLRP is likely to occur when the information's content ambiguously affects market price. Examples include increases in research and development spending and increases in inventory. Greater research and development (R&D) spending can indicate early successes that warrant further investment, but can also indicate a need to change course after existing projects have proven unpromising. An increase in inventory may represent a deliberate build up anticipating large growth in sales or a pile up of less desirable product (and an unwillingness to write off obsolete inventory).

signal. Although the insider forgoes profits when the true state is high since all market participants perfectly know the value, his superior information in the other two states can offset this lost revenue, and lead to greater expected profits than without the public disclosure. The net effect of partial disclosure (compared to no disclosure) increases the information asymmetry when the distribution of the private signal violates MLRP, the private information is not very precise, and the volatility of the firm's cash flows are low. This contrasts with conventional wisdom, linking lower information asymmetry to higher price efficiency.[3] It is consistent, however, with the large body of empirical papers (e.g., Lee, Mucklow, and Ready (1993), Yohn (1998), Krinsky and Lee (1996), Affleck-Graves, Callahan, and Chipalkatti (2002), Barron, Byard, and Kim (2002) and Barron, Harris, and Stanford (2005)) showing increased disclosure leading to increased information asymmetry. Without assuming that disclosures themselves lead to increased opportunities for information gathering, we believe this is the first paper to provide a theoretical explanation for the simultaneous increase in information asymmetry and decrease of price volatility.[4]

Then, we allow managers to choose a disclosure policy optimally by incorporating his preferences into the model's structure. The manager's utility is an increasing function of stock price and private benefits (which are increasing in the trading profits of informed traders).[5] Implicitly, we are assuming that the informed traders are the source of the private benefits, and the higher their profits, the greater the perquisites for the manager.[6] The manager subsequently learns the realization of firm value and selects, but cannot commit to, a disclosure policy to maximize his utility. Since the actual disclosure is made after the manager becomes informed, we consider only those disclosure policies that are ex post incentive compatible. To maximize stock price alone, managers would disclose whenever price is less than the realized state, consistent with the standard unraveling argument in Milgrom (1981). However, the manager's preferences for both private benefits and stock price generate frictions that can prevent full disclosure. Information asymmetry can be highest when the manager partially discloses information; the positive association between public disclosure and information asymmetry is not guaranteed. Although a partial disclosure strategy can increase trading profits (relative to no dis-

[3]For example, the linkage principle in auction theory indicates that a seller possessing private information about an appraisal of an asset to be auctioned should disclose this information to bidders to reduce information asymmetry (e.g., Milgrom and Weber (1982)).

[4]Lundholm (1988) focuses on how the correlation between public and private signals relates to the signal's impact on stock prices but does not address the issue of information asymmetry or price efficiency.

[5]An approach similar in spirit to ours is Zou (1994) and dates back as far as Kurz (1968), where an individual's utility is modeled as a weighted average of consumption and wealth accumulation. The emphasis on two opposing interests leads to differences in savings rates and growth rates. Closer to our setting with double audiences, Newman and Sansing (1993) model the disclosure choice when it impacts multiple users whose interests do not align.

[6]Levine and Smith (2003) consider a manager who has preferences that depend on the profits of other informed traders (affiliates); the manager affects informed trading profits by regulating how much information he provides to the (otherwise uninformed) affiliates. In our model, the manager affects the informed trader's profits and information asymmetry through public disclosures.

closure) in expectation, it is never sustainable if the manager places too much weight on his private benefits. Utility skewed in the favor of private benefits would encourage the manager to withhold his information when the true state is the state he initially planned to disclose and the only possible policy is one of no disclosure. When the manager cares predominantly about stock price, he fully discloses to avoid lower than deserved market valuations that arise when better states are pooled with worse states. When his preferences over stock price and private benefits are more evenly balanced, the manager follows a partial disclosure policy where he partially discloses good news, whenever possible.

Next, we consider the consequences of Reg FD, which prohibits the sharing of material, non-public information selectively. We assume that the rules are followed in the broadest sense: the informed trader receives a private signal that, when received, does not materially change his expectations about firm value. Regardless of the original source of the information, if optimal disclosure decisions are made under the assumption that only non-material information has been shared traders can generate material signals using the mosaic of public and private signals. Information may be non-material at the time it is disclosed to the informed trader, but can be rendered material if the manager follows a policy of partial disclosure. Following the partial disclosure, the informed trader knows the state perfectly, for every realization, whereas the rest of the market only has perfect information for one of the states. Therefore, by selecting a partial disclosure policy, Reg FD is subverted. If the source of the information is the manager, then Reg FD allows the manager to take advantage of the imprecise definition of materiality and provide benefits to select traders (and himself). If informed traders have other sources, the combination of private and public information seems like a legitimate use of mosaic theory, described in the CFA Institute Handbook as:

> *A financial analyst gathers and interprets large quantities of information from many sources. The analyst may use significant conclusions derived from the analysis of public and nonmaterial nonpublic information as the basis for investment recommendations and decisions even if those conclusions would have been material inside information had they been communicated directly to the analyst by a company. Under the "mosaic theory," financial analysts are free to act on this collection, or mosaic, of information without risking violation (CFA Institute 2010).*

The vague definition of materiality may allow managers (or other corporate insiders) to follow the letter of the law, but not its spirit. [7]

[7] Solomon and Soltes (2013) find that investors in the post-Reg FD period make more profitable trading decisions following one-on-one meetings between managers and investors, but this is not prima facie evidence that managers are sharing *material* information and is consistent with the implications of our model. Arya, Glover, Mittendorf, and Ye (2005) argue that replacing public disclosures with selective disclosures encourages analysts to herd, leading the mangers to prefer

Finally, we examine the effects of the duty to disclose poor performance on managers' optimal disclosure policies. In the absence of duty to disclose regulation, the manager has incentives, ex post, to withhold his information when the low state occurs, making partial disclosure of the low state not credible. The only sustainable equilibrium may be full disclosure, which eliminates all private benefits. If the manager is required to disclose low outcomes following a duty to disclose, partial disclosure becomes recoverable when private benefits are high. Because he can now credibly commit to disclosing the low state whenever it occurs, it reduces the settings in which he could not prevent himself from full disclosure in absence of the duty to disclose. While mandating disclosure can lead to more disclosure in some settings, it leads to less disclosure in others, and "different" but not better disclosure in still others. An unintended consequence of mandatory regulation is a decrease in price efficiency or an increase in information asymmetry relative to that which would result from unconstrained, optimal voluntary disclosure when the manager mainly cares about private benefits.

Completing the mosaic is the process of combining information gathered and analyzed by the informed trader with public disclosures leading to a more valuable signal. The expected value of this information is higher if the distribution of the non-public information violates MLRP and the cash flows of the firm are not too volatile. In auction theory, the linkage principle is a strong result that shows that a seller possessing private information about an appraisal of an asset to be auctioned should disclose this information to bidders to reduce information asymmetry and thus improve the selling price, but relies on the assumption that MLRP is satisfied.[8] When MLRP is violated, we cannot necessarily apply the intuition from these results. There are some parallels to the finding in Lundholm (1988) where public signals may have the "wrong" expected impact on price, due to the correlation structure between public and private signals. Einhorn (2005) further generalizes this framework to a voluntary disclosure setting. In our setting, if the market learns publicly which states have not occurred, there may be an overreaction (i.e., price is too low) relative to the informed trader's private information, allowing the privately informed trader to exploit his private information. Alles and Lundholm (1993) also highlight how public disclosures interact with costly private information acquisition. The cost of being relatively less informed may be lower than the cost of increased endowment risk, leading uninformed traders to prefer settings with private information acquisition over public disclosure, in some cases. Our model eliminates the endowment (signal) risk in Alles and Lundholm (1993) but finds a cost to uninformed traders of public disclosure nonetheless. Allowing for pre-existing information asymmetries allow us to better understand the underpinnings of mosaic theory; public disclosures help some individuals more than others, regardless of their initial endowments.

no disclosure of material information.

[8] Milgrom and Weber (1982) find that among policies for revealing information (including censored, noisy, or partial revelation), fully and publicly announcing all information a seller possesses is an expected-revenue-maximizing policy.

An important assumption in our model is the pre-existing information asymmetry.[9] Because the informed trader has an information advantage before the public disclosure, the disclosure can subsume or complement their information advantage.[10] Although our informed trader's information may come from superior processing of existing information in the public domain, the focus of this paper is how subsequent public disclosures then affect that trading environment. Our paper does not provide an explanation of the additional increase in trading volume as in Kim and Verrecchia (1994) but admits a genuine form of mosaic trading at the time of public disclosure announcements, allowing us to generate the result that public disclosures and information asymmetry are positively associated.

Our paper extends the voluntary disclosure literature in several ways. First, we provide a setting in which information has a mosaic nature. Public disclosure of information does not necessarily reduce information asymmetry because there are traders with information, which taken in conjunction with the public disclosure, generate superior signals. Standard models, in which private and public information is represented as truth plus noise, will always result in reduced information asymmetry following public disclosures (barring further information gathering). Second, the tension in our paper that prevents unravelling or full disclosure is the manager's preference to maximize both market price and private benefits. In a similar vein, Arya, Frimor, and Mittendorf (2010) show that when firms' segments that have conflicting interests, the unraveling result stands in aggregate but not by segment. In a voluntary disclosure setting, Bertomeu, Beyer, and Dye (2011) provide conditions where more public disclosures can increase information asymmetries between the uninformed traders and a trader who does not know the value of the firm but knows whether the manager has received information. In contrast, our trader is directly informed (albeit imperfectly) about firm value. Because the manager values stock price and private benefits, we can characterize settings in which partial disclosure is sustainable in equilibrium. Finally, our model provides an explicit and rigorous way to evaluate special information and disclosure structures to better understand the implications of regulation on disclosures and information sharing.

Our paper proceeds as follows. Section 2 presents the model, taking the public disclosure decision as given. Section 3 endogenizes the public disclosure decision. Section 4 looks at the regulatory

[9] Kim and Verrecchia (1991) and McNichols and Trueman (1994) also model a pre-existing information symmetry prior to the public disclosure but the nature of their models do not allow for an analysis of the relation between information asymmetry and public disclosures. McNichols and Trueman (1994) demonstrate how the anticipation of public disclosure drives the private information gathering whereas in Kim and Verrecchia (1991), all investors are informed with different precisions and as a consequence, public disclosures affect differently their beliefs, which explains the abnormal trading volume at the time of the public announcement but cannot explain the documented increased information asymmetry among traders. Varian (1988), Harris and Raviv (1993) and Kandel and Pearson (1995) also examine how different perceptions of the same public information affect volume.

[10] We do want to suggest that information gathering in our model constitutes insider trading. The key is that the inference from the signals is material, even if each piece is obtained legally. This is consistent with the modeling in Hirshleifer and Teoh (2003) where investors have differential ability to attend to information, or Fischer and Verrecchia (1999) where investors have different processing powers.

interventions of Reg FD and the duty to disclose. Section 5 discusses empirical implications of our model and Section 6 concludes.

2 Model

In the model, there are four risk-neutral players: a manager, an informed trader, liquidity traders and a market maker. There is a single risky asset (the "firm") which generates cash flows θ_τ at the end of period τ. Within each period, there are four relevant dates. At date 1, the manager of the firm chooses a public disclosure policy. At date 2, the informed trader receives a private signal informative about the distribution of the cash flows of the firm. At date 3, the manager observes true cash flows and discloses or withholds his information according to the pre-specified disclosure policy. At date 4, traders (informed and liquidity) submit their orders, the market maker observes net order flow, and the market maker sets price. For details, see the timeline given in Figure 1.

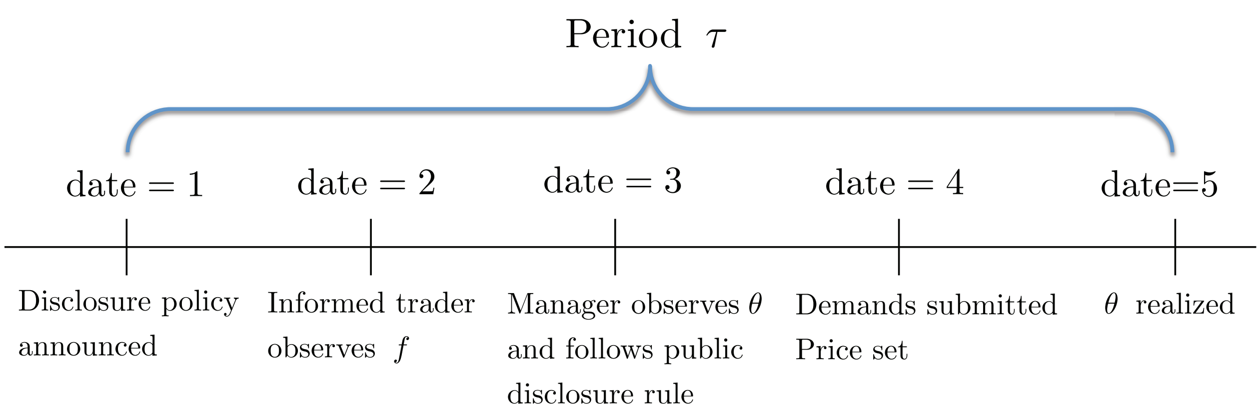


Figure 1: Timing of events within period τ

The cash flows θ_τ can take on three possible values, $\{L, M, H\}$, where

$$L = \mu - e; \qquad M = \mu; \qquad H = \mu + e;$$

with $0 < e < \mu$. State M occurs with probability $1-n$ and states L and H each occur with probability $n/2$, giving us a symmetric distribution. Further, we assume that $0 < n < 2/3$ to ensure the probability distribution is single peaked and that extreme outcomes are less likely than moderate ones.

The manager's disclosure policy specifies the states of the world he will disclose publicly at date 3. We denote the disclosure policy by d_P, where d_N indicates no disclosure, d_F indicates full disclosure, and d_θ indicates disclosure of state θ only. Specifically, if the manager's policy is d_H, whenever he observes $\theta = H$ he discloses and whenever he observes $\theta \in \{M, L\}$ he stays silent. Hereafter, we refer to disclosure policies in which only one state is perfectly revealed as partial disclosure.

The manager does not trade on his private information directly and does not disclose his private information selectively.[11] However, he cares about both stock price and private benefits. We assume that private benefits are provided by the informed trader and therefore model his utility as an increasing function of informed trading profits and stock price, computed prior to the realization of cash flows. For example, the informed trader might give the manager (or one of the manager's family members) a position on the board of his company, provide some favorable trading terms on corporate or personal goods, or other perquisites. If the informed trader is an institutional investor, the informed trader might approve a greater compensation package or demand less monitoring of a manager whose disclosure policies enhance his trading advantage.[12] Since stock price is part of the manager's utility, absent monitoring or a duty to disclose low valued information, partial disclosure of the low state is never ex post optimal.[13]

At date 2, the informed trader observes a binary signal $f \in \{0, 1\}$. When $\theta = i$, the informed trader observes signal $f = 1$ with probability t_i and $f = 0$ with probability $(1 - t_i)$.[14] We assume that $t_L < t_H$. That way, in the absence of additional public disclosure, the action associated with $f = 1$ is a purchase and with $f = 0$ is a sale, and each trade would make positive expected profits.[15] An example of this binary information might be: there is a greater than 50% chance that inventory turnover has increased ($f = 1$) or decreased ($f = 0$).

At date 3, the manager learns (privately) the realization of the firm's cash flows. The manager either discloses his information publicly, θ or withholds his information. We assume that when he discloses his information, he is required to report it truthfully.[16] We require the manager's disclosure decision to be ex post incentive compatible; in other words, his disclosure must be optimal after he

[11]In section 5, we suppress this latter assumption and allow the manager to engage in some selective disclosure.

[12]There is an extensive literature on private benefits of control that supports the argument that managers take advantage of their position within the company for their own benefit. See Barclay and Holderness (1989), Kandel and Pearson (1995), Affleck-Graves, Callahan, and Chipalkatti (2002), Eckbo and Thorburn (2003), Belen and Raphael (2006) and Doidge, Karolyi, Lins, and Stulz (2009).

[13]We consider the implications of a duty to disclose in Section 4.2.

[14]To introduce noisy information in a discrete model, the number of private signals needs to be lower than the number of states of the world.

[15]The ordering of t_H and t_L is without loss of generality. If we made the opposite assumption, the informed trader would simply sell whenever $f = 1$ and buy when $f = 0$. The key is that the signal is informative, not which signal has which designation.

[16]Truthfulness is a standard assumption in the voluntary disclosure literature (e.g., Verrecchia (1983), Dye (1985)). Stocken (2000) provides a setting where voluntary disclosures are truthful if we take into account that in a multi-period game, the cost of lying through severe punishment disciplines the manager's behavior. In the context of mandatory disclosures, Goex and Wagenhofer (2009) also assume that the manager reports truthfully the information generated by an information system. Beyer, Cohen, Lys, and Walther (2010) offers a nice survey and arguments to support truthful disclosures.

has observed the true θ. If the manager were interested in maximizing stock price alone, we would have the standard unraveling result and full disclosure. The joint interest in maximizing stock price and informed traders' profits prevents from unraveling.

At $t = 4$, liquidity traders post a demand $\tilde{\varepsilon}$ equal to either 1 or -1, each with probability $\frac{1}{2}$. The informed trader chooses a demand X conditional on his public information and the disclosure. In order to use liquidity trades as disguise, the informed trader will only buy or sell a single unit. The market clearing price of the firm is then established via a Kyle (1985)-type market maker, who sets the market price P equal to expected value based on observing the aggregate order flow $\tilde{Y} = \tilde{X} + \tilde{\varepsilon}$ and public disclosures.

2.1 Informed Trading Profits

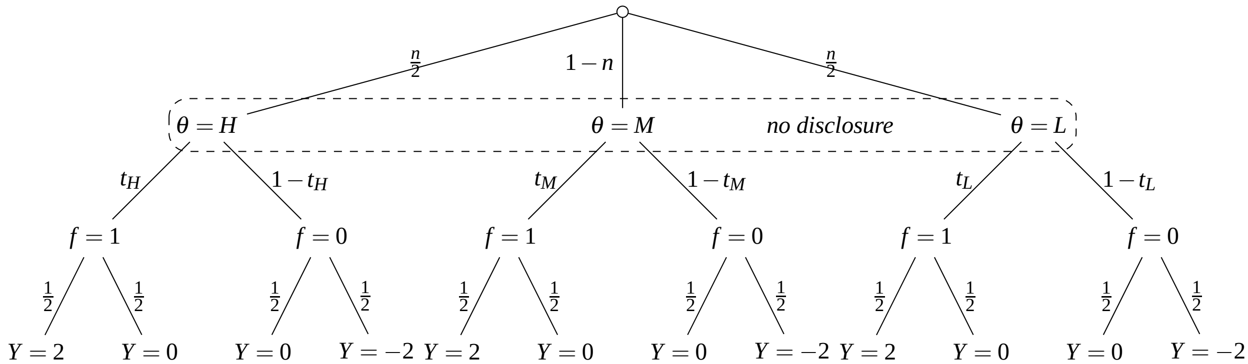
In this section we take the disclosure policy as fixed. We endogenize disclosure policies in a later section. Under a full disclosure policy, the firm value is disclosed in every state. The market maker sets price equal to value. There are no opportunities for the informed trader to benefit from his information, as it is wholly subsumed by the public disclosure. Under a no disclosure policy, market price can only update price based on order flow. Under a partial disclosure policy, the market maker sets price equal to value when the state is disclosed, and sets it to the conditional expectation based over the remaining undisclosed states and the order flow when the state is not revealed. We then measure the price efficiency and information asymmetries of each possible disclosure policy.

There are only three possible levels of order flow, $Y \in \{2, 0, -2\}$. In the case of no disclosure, the informed trader always trades since his information is noisy. An order flow of Y=2 (Y=-2) indicates to the marker maker that the informed trader submitted a buy (sell) order. An order flow $Y = 0$ provides no information about the direction of the informed trader's trade as it is completely disguised by the offsetting liquidity trade.

2.1.1 No Disclosure

Given the policy of no disclosure, the probability sub-tree is presented in Figure 2.

Figure 2: No disclosure



The market maker sets price equal to expected value given the disclosure policy, disclosure and order flow. In this case, the disclosure policy is d_N, or no disclosure, and therefore the disclosure itself is $\emptyset$.

$$\begin{aligned} P(2,\emptyset,d_N) = E(\theta|Y=2) &= \mu + \frac{en(t_H - t_L)}{n(t_H + t_L - 2t_M) + 2t_M} \\ P(0,\emptyset,d_N) = E(\theta|Y=0) &= \mu \\ P(-2,\emptyset,d_N) = E(\theta|Y=-2) &= \mu + \frac{en(t_H - t_L)}{n(t_H + t_L - 2t_M) - 2(1-t_M)} \end{aligned}$$

By inspection, we can see that $H > P(2,\emptyset) > P(0,\emptyset) = M > P(-2,\emptyset) > L$, or price is increasing in order flow, as it should. The informed trader earns profits, in expectation, because price does not fully adjust to order flow, since his information is noisy and when the market maker observes an order flow $Y=0$, he does not know the trading decision of the informed trader.

Calculating the informed trader's expected profits, when $f=1$ and the insider has purchased or $f=0$ and sold gives:

$$\Pi(f=1,\emptyset,d_N) = \frac{en(t_H - t_L)}{2(n(t_H + t_L - 2t_M) + 2t_M)} \tag{1}$$

$$\Pi(f=0,\emptyset,d_N) = \frac{en(t_H - t_L)}{2(2(1-t_M) - n(t_H + t_L - 2t_M))} \tag{2}$$

The expected profits are positive if the insider buys when $f=1$ and sells when $f=0$. Expected profits of trading (buying or selling) are increasing in t_H, e and n and decreasing in t_L and t_M. Trading profits increase if the informed trader buys and the information is more precise about the high event relative to the low event. Trading profits increase if the informed trader sells and the information is more precise about the low event relative to the high event.

Now, consider the *ex ante* profits for the no-disclosure case, the probability that the informed trader gets signals $f=1$ and $f=0$ are

$$\begin{aligned} Pr(f=1,\emptyset) &= \frac{n}{2}(t_H+t_L)+(1-n)t_M \quad (3)\\ Pr(f=0,\emptyset) &= \frac{n}{2}(2-t_H-t_L)+(1-n)(1-t_M) \quad (4) \end{aligned}$$

Applying these probabilities to the expected profit in each case (equations (1)-(2) respectively) yields

$$\Pi(d_N)=\frac{en(t_H-t_L)}{2} \quad (5)$$

We compute price informativeness as the conditional variance of value given prices, or

$$V(d_N)=\frac{e^2n\left(2n^2(t_H+t_L-2t_M)^2+n\left(t_H^2-2t_H(t_L-4t_M+2)+8(t_L+1)t_M+(t_L-4)t_L-16t_M^2\right)+8(t_M-1)t_M\right)}{2(n(t_H+t_L-2t_M)+2(t_M-1))(n(t_H+t_L-2t_M)+2t_M)} \quad (6)$$

The variance is increasing in e, n and t_L, decreasing in t_H . In other words, price becomes more informative if the informed trader's purchase (sell) decision is driven by a higher (lower) probability that the high event will occur or a lower (higher) probability of the low event. If the distribution of the signal satisfies MLRP, price efficiency is ambiguously affected by t_M because it adds noise to orders; if the informed trader knew the true state M perfectly, he would prefer not to trade. If the distribution of the signal violates MLRP and $t_M>t_H$, price efficiency increases in t_M because the precision of the private signal becomes more informative as the distance between t_M and t_H gets bigger. To summarize, in the absence of public disclosures, more precise private information increases information asymmetries but also improves price efficiency because more information is impounded into the price. The riskier the firm (higher the n), the greater the information asymmetries and the lower price efficiency.[17]

2.1.2 Partial Disclosure

Disclosure of M only (d_M)

The informed trader is indifferent between no disclosure and partial disclosure of M. While partial disclosure eliminates the unprofitable trades when the signal indicates buy or sell and the true value is M, the price sensitivity increases commensurately to reflect the greater precision of information. To

[17] The magnitude of the difference between H and L is another measure of riskiness, and here, the implications of a higher e are the same as the implications of a higher n.

see this,

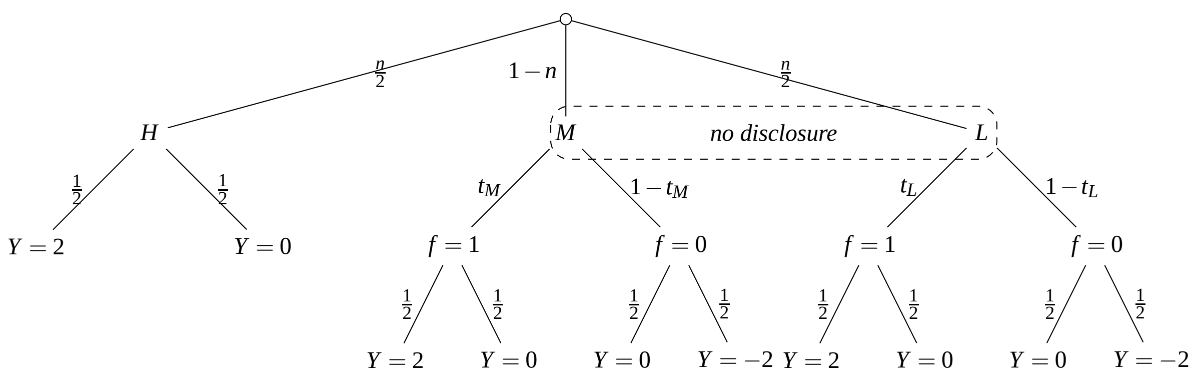
$$\begin{aligned}
P(2,\emptyset,d_M) &= \mu + \frac{e(t_H - t_L)}{(t_H + t_L)} \\
P(0,\emptyset,d_M) &= \mu \\
P(2,\emptyset,d_M) &= \mu - \frac{e(t_H - t_L)}{(2 - t_H - t_L)},
\end{aligned}$$

and expected profits, $\Pi(d_M)$ are $en(t_H - t_L)/2$, which is identical to the expected profits under no disclosure (Equation (5)).

Disclosure of H only (d_H)

When the disclosure policy is d_H, the market perfectly knows the value of the firm when H is disclosed and knows the value is either L or M when there is no disclosure. To simplify the exposition, we present here the case where $t_M > t_L$, or $f = 1$ is "good" news. The probability sub-tree, given a policy of disclosing H only is given by Figure 3.

Figure 3: Partial disclosure, d_H



The market maker then sets prices conditional on the observed disclosure (or lack of disclosure), given H will be disclosed whenever the manager has it, as:

$$\begin{aligned}
P(\cdot,H,d_H) &= \mu + e \\
P(2,\emptyset,d_H) &= \mu - \frac{ent_L}{2t_M(1-n) + nt_L} \\
P(0,\emptyset,d_H) &= \mu - \frac{en}{2-n} \\
P(-2,\emptyset,d_H) &= \mu - \frac{en(1-t_L)}{2(1-t_M) - n(1+t_L-2t_M)}
\end{aligned}$$

When H is disclosed, price is set equal to $H = \mu + e$ for all possible order flows.[18] Otherwise, price is in the range $(\mu - e, \mu)$. If $t_M > t_L$, when the informed trader observes $f = 1$, he prefers to buy and when $f = 0$ he prefers to sell. Prices adjust to the expected value of the asset whenever order flow unambiguously reveals the direction of the trade, and adjusts to the expected value (conditional only on the asset not having value H) whenever net order flow is zero. The informed trader earns zero expected profits in the cases where order flow is positive or negative, and expected positive profits otherwise.

Expected profits are then computed as:

$$\begin{aligned}
\Pi(H, d_H) &= 0 \\
\Pi(\emptyset, d_H) &= \frac{2en(1-n)(t_M - t_L)}{(2-n)^2}
\end{aligned}$$

Thus, the total *ex ante* profits of the partial disclosure regime, d_H is

$$\Pi(d_H) = \frac{n}{2}(0) + \left(1 - \frac{n}{2}\right)\frac{2en(1-n)(t_M - t_L)}{(2-n)^2} = \frac{en(1-n)(t_M - t_L)}{(2-n)} \tag{7}$$

Ex ante profits are increasing in t_M and e, and decreasing in t_L. The informed trading with partial public disclosure is valuable because the informed trader can better assess the likelihood of the medium state relative to the low state. However, the ex ante profits are maximized at $n^* = 2 - \sqrt{2}$. On one hand, if the spread between the occurrence of the medium and the low is large, the signal f is quite informative about the remaining undisclosed two states. On the other hand, more weight on the extreme events is associated with larger foregone profits (for the often realized high state).

Measuring price informativeness under d_H using the volatility of prices gives

$$V(d_H) = \frac{e^2(n-1)n\left(-2(n-1)t_M(nt_L + n - 2) + nt_L(n + t_L - 2) + (n-1)(3n-4)t_M^2\right)}{(n-2)(n(t_L - 2t_M) + 2t_M)(nt_L - 2nt_M + n + 2t_M - 2)} \tag{8}$$

The volatility is decreasing in t_M and increasing in t_L and e, and inversely U-shaped in n. When $t_M > t_L$, the informed trader submits a buy order if he does not observe any public disclosures and thus, when t_M is large or t_L is low, the informed trader's information is closer to the true cash flows of the firm which is impounded in the price. The higher the accuracy of the informed trader's information, the lower the volatility in prices. The unconditional probabilities play an important role in the expected volatility. Public disclosures of the high state removes any volatility in the high state but if the unconditional probability on the extremes is too high, the price is less informative in the

[18] The informed trader is indifferent between trading and not trading when the state is fully disclosed. In both cases, he earns zero profits

remaining non disclosed events.

Disclosure of L only (d_L)

When the disclosure policy is d_L, the market perfectly knows the value of the firm when L is disclosed and knows the value is either M or H when there is no disclosure. The analysis is parallel to the case with partial disclosure of H. Therefore, we can suppress the details of the analysis and directly derive the ex-ante expected profits and the price informativeness under d_L using the volatility of prices when $t_H > t_M$:[19]

$$\Pi(d_L) = \frac{n}{2}(0) + \left(1-\frac{n}{2}\right)\frac{2en(1-n)(t_H-t_M)}{(2-n)^2} = \frac{en(1-n)(t_H-t_M)}{(2-n)} \quad (9)$$

$$V(d_L) = \frac{e^2(n-1)n\left(-2(n-1)t_M(nt_H+n-2)+nt_H(n+t_H-2)+(n-1)(3n-4)t_M^2\right)}{(n-2)(n(t_H-2t_M)+2t_M)(nt_H-2nt_M+n+2t_M-2)} \quad (10)$$

Again, the informed trader benefits from a larger spread between t_H and t_M and simultaneously more information is impounded in the price reducing the volatility of prices. However, in contrast to an environment where no mandatory disclosures are in place, the more volatile firms would not necessarily have more information asymmetries if partial disclosures are mandated. Firms with moderate level of volatility in their cash flows would have the highest level of information asymmetries and simultaneously would have almost the lowest price efficiency.

2.2 Comparison of no-disclosure and partial disclosure: Informed trader's perpective

In a setting where the informed trader's information is perfect, public disclosure is bad for the informed trader because it reduces his opportunities for advantaged trade. Even without perfect information, when the signal satisfies the monotone likelihood ratio property (such that the probability of the good signal is increasing in the outcome), informed traders are made worse off by any public disclosures.

Proposition 1 *Suppose the probabilities of the informed trader's signal preserve the monotone likelihood ratio property, or $t_H > t_M > t_L$. Information asymmetry is highest and price efficiency is lowest under no disclosure when compared to partial and full disclosure.*

With MLRP met, the informed trader 'owns' directional information about the firm's cash flows. Partial disclosure policies d_L and d_H provide information about the extremes that overlap with the in-

[19] If $t_H < t_L$, the ex-ante profits and the volatility of prices are symmetric.

formed trader's directional information. Therefore, such disclosures decrease the informed trader's information advantage (i.e., information asymmetry is lower under partial disclosure than no-disclosure). At the same time, price efficiency is the lowest (or prices are most volatile) if no public disclosures are released. These results supports the conventional wisdom that more public disclosures decrease information asymmetries and improves price informativeness. Thus, a necessary condition to exacerbate information asymmetries with more public disclosures is to violate MLRP.

Proposition 2 *Suppose the probabilities of the informed trader's signal violate the monotone likelihood ratio property. Let*

$$\begin{aligned} t_1 &= \tfrac{2(1-n)t_M+nt_L}{2-n} & t_2 &= \tfrac{(2-n)t_L-2(1-n)t_M}{n}, \\ t_3 &= \tfrac{2(1-n)t_M+(2-n)t_L}{4-3n} & t_4 &= \tfrac{(4-3n)t_L-2(1-n)t_M}{2-n}. \end{aligned}$$

Then, the following orderings hold for information asymmetry and price efficiency.

Parameters	*Information asymmetry*	*Price efficiency*
$t_M > t_H > t_1 > t_L$	$IA(d_N) > IA(d_H) > IA(d_L)$	$PE(d_H) > PE(d_L) > PE(d_N)$
$t_M > t_1 > t_H > t_3 > t_L$	$IA(d_H) > IA(d_N) > IA(d_L)$	$PE(d_H) > PE(d_L) > PE(d_N)$
$t_M > t_3 > t_H > t_L$	$IA(d_H) > IA(d_L) > IA(d_N)$	$PE(d_H) > PE(d_L) > PE(d_N)$
$t_H > t_2 > t_L > t_M$	$IA(d_N) > IA(d_L) > IA(d_H)$	$PE(d_L) > PE(d_H) > PE(d_N)$
$t_4 > t_H > t_L > t_M$	$IA(d_L) > IA(d_H) > IA(d_N)$	$PE(d_L) > PE(d_H) > PE(d_N)$
$t_2 > t_H > t_4 > t_L > t_M$	$IA(d_L) > IA(d_N) > IA(d_H)$	$PE(d_L) > PE(d_H) > PE(d_N)$

Partial disclosure policies can be beneficial because they enhance the quality of the informed trader's signal in the undisclosed states. Effectively, the disclosure can exacerbate the information asymmetry by eliminating some of the noise in the informed trader's signal. To understand why, note that when the information structure violates MLRP (i.e., $t_M \geq t_H \geq t_L$ or $t_H \geq t_L \geq t_M$), the informed trader does not have "decisive" directional information, although trading based on his signal is profitable in expectation. If the manager discloses the high state only or the low state, the informed trader can now order the remaining states more reliably using f. The disclosure comes at a cost since the informed trader forgoes his potential profits whenever the state disclosed occurs. It follows that disclosure is more valuable for the informed trader if the unconditional probabilities on the extreme cash flows are low. When $t_M \geq t_L$, the most desirable disclosure for the informed trader is partial disclosure of H while when $t_L \geq t_M$, partial disclosure of L is the optimal policy for the informed trader. The most desirable outcome for the informed trader when MLRP is violated is associated with more price informativeness. This positive relation between information asymmetries and price informativeness suggests some caution in confounding information asymmetries and price informativeness. These two measures capture the impact of public disclosures but with a different angle.

Figure 4 illustrates the information asymmetry results of Propositions 1 and 2. The region between the dotted line and the dashed line are the parameters for which MLRP is satisfied. There, no disclosure is always better for the informed trader than partial disclosure of L or H. A higher spread between the probability to receive a buy signal given that the true state is H, i.e., t_H and the probability to receive a buy signal given that the true state is t_L, leads to higher profits and with MLRP satisfied, the informed trader is more likely to have correct information about the extreme values. The section of the graph below the dashed line represents parameters for which $t_M < t_L < t_H$. The signal is still informative, but there is a high probability of trading losses stemming from incorrect trades when the signal indicates to sell and the true value is actually M. In that region, if these trading losses are too big if M occurs, information asymmetry is highest under a policy of d_L which allows the informed trader to more precisely order the states M and H, when L is disclosed. The region to the northwest of the dotted line is the MLRP violation where $t_L < t_H < t_M$. A policy of d_H reduces the informed traders profits when H is realized, but increases the profitability of trades on L and M since disclosure of H effectively recovers MLRP for the remaining (undisclosed) states. In all cases, partial disclosure leads to greater price efficiency than no disclosure; above the dashed line, the price efficiency is highest when H is disclosed and below it, price efficiency is highest when L is disclosed. In addition to getting perfect pricing for the disclosed state, by disclosing the state with the "middling" probability, it increases the precision of the remaining information the most.

Corollary 1 *Partial disclosure policies always increase price efficiency. The benefits to the insider of partial disclosure (i.e., increases to information asymmetry) are decreasing in the difference between* t_H *and* t_L*.*

When the expected losses avoided by partial disclosures outweigh the expected profits forgone, information asymmetries increase under disclosure. Still, price is more informative because prices benefit from both the disclosure and the more precise information implicit in demand. Therefore, it is possible through public disclosure to simultaneously improve the welfare of the informed trader and increase the amount of information impounded into price. This contradicts the conventional wisdom that more public disclosure increases price efficiency and decreases information asymmetries. In our setting, the informed trader has information which when combined with a disclosure policy forms a "mosaic," resulting in information that can better direct his trades. Absent any public disclosure and if true outcome is M, the informed trader is suffering larger losses with a buy decision if the distribution of his signal violates MLRP than if the distribution of his signal satisfies MLRP. Partial disclosure eliminates any of his profits if the state disclosed occurs but will now turn his losses into substantial

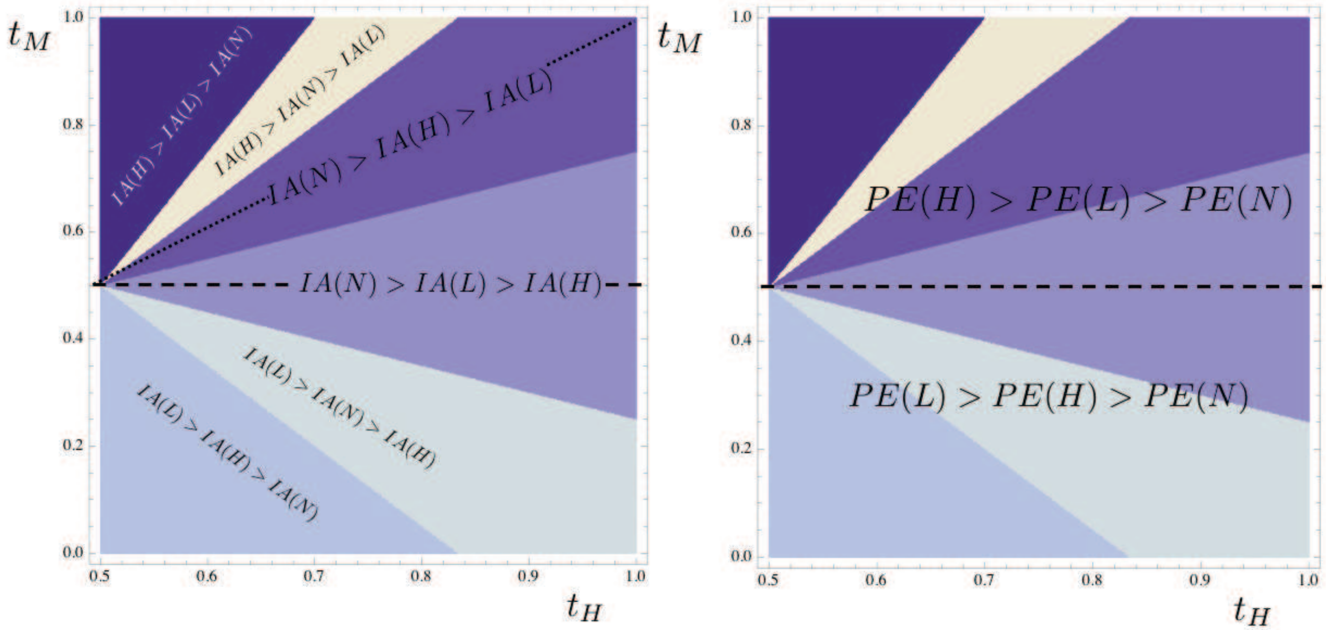


Figure 4: Information Asymmetry Orderings with $t_L = \frac{1}{2}$ and $n = \frac{1}{2}$

gains when the true state is M. In the extreme case where $t_H = t_L$, partial disclosure always increases information asymmetries because in absence of public disclosures, the informed trader cannot profitably trade. His otherwise non-material private information becomes valuable when he is able to piece together his information and material public information into a mosaic.

To summarize, more public disclosure always increases price efficiency but only decreases information asymmetries when either MLRP is satisfied or cash flows of the firm are not too volatile and/or the informed trader's information is low quality (i.e., does a poor job differentiating among states) .

3 Implementation of Public Disclosures

For simplicity, we assume that $t_L = 0$ for the remainder of the paper. This assumption means that the informed trader never observes a good signal if true cash flows are low. This assumption allows us to focus on only one possible violation of MLRP ($t_M > t_H > t_L$) and seems consistent with the general conservatism in reporting that would understate good outcomes but not overstate bad ones. Additionally, the expressions are more tractable yet results are qualitatively similar to settings in which $0 < t_L < t_H$.

We endogenize the disclosure policy, allowing the manager to select the policy that maximizes his personal utility, after observing the true realization θ. Most compensation packages provide CEOs with incentives to increase the firm's stock price. Were the manager to maximize market price alone, he would disclose all of his information.[20] This is the standard unraveling result, where the manager, when faced with a price below his signal, will always disclose his signal to increase the price, leaving only the lowest possible value undisclosed. In our model, the manager has a second component to his utility – the incentive to maximize informed trading profits. The benefits from trading profits need not be associated with illegal insider trading by the manager, but may arise instead from other informed traders (e.g., analysts, institutional investors, other influential investors) that provide the manager with private benefits (i.e., favorable recommendations, board memberships, exclusive memberships and other perks) based on their perceived level of investing success. Regulation FD was designed to 'level the playing field' for all investors by prohibiting selective disclosures from managers to analysts and institutional investors, thereby requiring firms to make public, within 24 hours, all disclosures of material information. Our modeling of the manager's utility supports the idea that managers would, if they could, share valuable information with specific traders. Specifically, the CEO's utility function is defined over both informed traders' benefits and stock price, as

$$\forall a \in [0,1], \;\; U(\Phi, d|\theta) = a\Pi(\Phi, d) + (1-a)P(\Phi, d|\theta)$$

where $\Phi \in \{\emptyset, \theta\}$ is the public information set, where the outcome $\Phi = \theta$ of the cash flows is disclosed or the information is withheld, i.e., $\Phi = \emptyset$. The manager's decision to disclose depends on his beliefs about the stock price, given his information, and his expected private benefits. The conditional stock price is not the same as the unconditional stock price since the manager has better information than the market maker about which order flow to expect. We assume that the informed trader offers the private benefits to the manager prior to the cash flow realization but after observing the manager's disclosure, and therefore $\Pi(\Phi, d)$ are based on the informed trader's expected profits given the disclosure policy and disclosure, not realized profits. Based on the results of the previous section, expected informed trading can be greater with disclosure than without it. Therefore, a manager who cares only about private benefits profits ($a = 1$) might have incentives to voluntarily disclose part of his information for some parameters. On the other hand, a manager who cares only about stock price only ($a = 0$) would have a policy of full disclosure.

Definition 1 *A value maximizing disclosure equilibrium is one in which*

$$\begin{aligned} d = argmax_D \quad & U(d) > U(d') \quad && where \;\; d, d' \in D = \{d_N, d_\theta, d_F\}, \\ subject\ to \quad & U(\Phi, d|\theta) > U(\Phi', d|\theta) \quad && where \;\; \Phi, \Phi' \in \{\emptyset, \theta\}. \end{aligned}$$

[20] There are no exogenous disclosure costs in this model.

The first part of the definition requires that disclosure policy d maximizes the manager's ex ante expected profits relative to d, under beliefs consistent with d and d' respectively. The second part of the definition requires that the disclosure policy is ex post incentive compatible. In other words, the manager follows his stated disclosure policy for all realizations of θ and has no incentives to disclose more (or less) than he committed to with his disclosure policy.[21,22]

Proposition 2 shows that there are net benefits to partial public disclosure when the distribution of the private signal violates MLRP and either $t_H < t_1 < t_M$ or $t_1 < t_H < t_3 < t_M$. The gains from improved information in the undisclosed states compensate for the lost profits of the disclosed state. However, when the disclosure decision is determined by a manager who cares about maximizing private benefits, he may renege on his stated disclosure policy to avoid the foregone profits if the true state is the one he agreed to disclose.

3.1 Manager's preferred disclosure policy

First, we compute the expected price under no disclosure, conditional on θ as

$$P_\theta(d_N) \quad = \quad \frac{1}{2}\left(t_\theta(P(2,\emptyset,d_N)+P(0,\emptyset,d_N))+(1-t_\theta)(P(0,\emptyset,d_N)+P(-2,\emptyset,d_N))\right)$$

The two incentive compatibility constraints to guarantee no disclosure is preferred to disclosing either H or M when the manager observes those states are:

$$\begin{aligned} a\Pi(d_\emptyset)+(1-a)P_H(d_N) &\geq (1-a)(\mu+e) \quad && (IC_{N,H}) \\ a\Pi(d_\emptyset)+(1-a)P_M(d_N) &\geq (1-a)\mu && (IC_{N,M}) \end{aligned}$$

Rewriting and solving for a, we have

$$\begin{aligned} a &\geq \frac{2+Gnt_H}{(G+1)nt_H+2} \equiv a_{N,H}, \text{ and} \\ a &\geq \frac{n(2t_M-t_H)}{n^2(2t_M-t_H)^2-4(1-t_M)t_M+n(2t_m-t_H)(3-4t_M)} \equiv a_{N,M} \\ \text{where} \quad & G = \frac{1-t_H}{n(2t_M-t_H)+2(1-t_M)} - \frac{t_H}{n(t_H-2t_M)+2t_M} \end{aligned}$$

When there is uncertainty about the state, price is always below $\mu+e$. Therefore, to satisfy $IC_{N,H}$, the weight on private benefits, a, needs to be sufficiently large (above $a_{N,H}$) so that the private benefits exceed the cost of a lower stock price when the true state is H. If the expected price, conditional on

[21] We do not allow the manager to falsely disclose.

[22] Kamenica and Gentzkow (2011) also focus on equilibria candidates that maximizes the sender's profits.

observing M is above μ then there is no benefit to disclosing M. If the expected price, conditional on observing M is below μ, then, similar to the condition for disclosing H, a has to be sufficiently large (larger than $a_{N,M}$) so that the cost of a lower stock price is outweighed by the private benefits.

The threshold $a_{N,H}$ is decreasing in n and t_M and increasing in t_H. The threshold $a_{N,M}$ is decreasing in t_H and increasing in t_M; if MLRP is violated, it is also increasing in n. When n is small, $a_{N,H} > a_{N,M}$. because the price (with no disclosure) is largely driven by the moderate outcome (since the probability M is realized is large). A disclosure of H will have a large impact on the price and therefore the value of the private benefits must be quite large to surpass the utility gains of increasing stock price. When n is large, trades on H are very profitable, but trades on M are unprofitable. By retaining M, the private benefits decrease, and thus for large n, it is possible that $a_{N,H} < a_{N,M}$. The lower the value of t_M, the more informative the private signal. By the previous analysis, we know that if $t_M < t_H$, disclosure reduces the information asymmetry, and therefore reduces the expected private benefits, pushing down the threshold above which the manager prefers no disclosure to H disclosure. If $t_M > t_H$, the lost private benefits when H is realized are offset by the significantly higher price, because price was not particularly sensitive to order flow when the signal was very noisy.

Revealing H when $a < a_{N,H}$ is preferred to following a no disclosure policy. However, that does not ensure that d_H is ex post incentive compatible. First, we need to confirm that a manager would indeed disclose H when it occurs, given market participants believe the manager is following the disclosure policy d_H. The informed trader earns profits (and provides private benefits to the manager) only when there is no disclosure. Therefore, if the manager's compensation heavily weights private benefits, the manager will have incentives to renege on his stated policy and mislead the market. Although the stock price will be lower, if private benefits are highly valued, he might prefer to withhold his information when $\theta = H$. Comparing disclosure with withholding when $\theta = H$ and non-disclosure is interpreted as $\theta \in \{L, M\}$ gives the following condition:

$$(1-a)(\mu+e) > a\Pi(\emptyset, d_H) + (1-a)P_H(\emptyset, d_H) \quad \rightarrow$$
$$a \leq \frac{(n-2)((n-2)(nt_H-4)-2(n-4)(n-1)t_M)}{(n-2)^2(nt_H-4)+8(n-1)^2nt_M^2-2(n-1)(3n-4)(n-2)t_M} \equiv a_{H,N}$$

Additionally, to guarantee a preference for partial disclosure over full disclosure, we must consider the manager's utility when the true state is M. The price of the firm when nothing is disclosed under d_H is always below μ. The manager prefers to (disclose M) withhold M whenever his private benefits from trading (do not) offset the lower price. The ex post incentive compatibility condition such that the manager prefers partial disclosure of H to full disclosure is

$$a\Pi(\emptyset, d_H) + (1-a)P_M(\emptyset, d_H) \geq (1-a)\mu \quad \rightarrow \quad a \geq \frac{(n-2)(n(3t_M-2)-4t_M+4)}{8(n-1)^2t_M^2 - n(n-2)t_M - 2(n-2)^2} \equiv a_{H,M}$$

Suppose d_M is planned. Then, whenever M is realized, the stock price is M and the private benefits are zero. Since stock price would be H for positive order flow and M when order flow is zero, the manager has incentives to renege and mislead the market. Therefore, d_M cannot be part of any equilibrium strategy. Partial disclosure of L cannot be part of an equilibrium disclosure strategy. If the manager were following d_L, the informed traders and market maker would have to believe that the true state is either M or H when nothing is disclosed. Under those beliefs, the manager would always withhold his information upon observing L since he can increase both his private benefits and the market price. Consequently, if there is partial disclosure, it will be partial disclosure of H only.

Full disclosure eliminates all private benefits. The more weight the manager places on stock price, the less costly is full disclosure. Moreover, if the market expects full disclosure, there is no advantage to withholding information. Beliefs when nothing is revealed would be that value is L. Price would be set accordingly and the informed traders would expect to earn zero profits (and therefore pass along no private benefits) because price is equal to value. When there is no other ex post incentive compatible disclosure policy, the manager will have to resort to full disclosure.

3.2 Optimal disclosure policies

Recall from Proposition 2, partial disclosure of increases information asymmetry (and the informed traders expected profits) whenever MLRP is violated and either $t_M > t_1 > t_H$ or $t_M > t_3 > t_H$. However, even under those conditions, the further condition that $a < a_{H,N}$ is required for partial disclosure to be incentive compatible. Therefore, when the manager cares predominantly about private benefits (a large), he will implement a policy of no disclosure. Although he might prefer partial disclosure, ex ante, he cannot sustain it ex post (i.e., in the absence of forced compliance with the stated disclosure policy, he would renege). If partial disclosure is ex post incentive compatible, the manager will select partial disclosure. In all other regions, the manager chooses full disclosure. When a is low, it is his preference to prioritize stock price over private benefits. When a is moderately high, he may not be able to convincingly follow a policy of partial disclosure.

To gain some intuition for how managers can effectively execute strategies ranging from no disclosure to full disclosure, consider the case of conference calls. No disclosure might be uninformative, boilerplate responses to analysts' questions in a conference call regardless of the state of the firm, partial disclosure could entail a detailed description of the environment when there is very good (or very bad) news, but a more vague description that does not allow the analysts to distinguish among the other two states. Full disclosure might be detailed, comprehensive responses to questions, regardless of the state.

Proposition 3

I: If $t_M > t_H > t_1 > t_L \rightarrow a_{H,N} > a_{N,H} > a_{N,M}$. *The manager's optimal disclosure policy is*

$$\textit{If } a > a_{N,H} \rightarrow \quad d_N; \quad \textit{If } a_{H,M} < a \leq a_{N,H} \rightarrow \quad d_H; \quad \textit{If } a \leq a_{H,M} \rightarrow \quad d_F.$$

II: Otherwise, $a_{H,N} < \max\{a_{N,H}, a_{N,M}\}$. *The manager's optimal disclosure policy is*

$$\textit{If } a > \max(a_{N,H}, a_{N,M}) \rightarrow d_N; \quad \textit{If } a_{H,N} < a \leq \max(a_{N,H}, a_{N,M}) \rightarrow d_F;$$
$$\textit{If } a_{H,M} < a \leq a_{H,N} \rightarrow d_H; \quad \textit{If } a \leq a_{H,M} \rightarrow d_F$$

No disclosure is credible and optimal for large a as the manager's interests are tied to the informed trader's profits and he would not deviate to any other disclosure to increase the market price. The reason that no disclosure is chosen over partial disclosure is that partial disclosure may enhance ex ante profits for the informed trader, but the manager is considering the impact on ex post profits. If the manager discloses H when he has it, the informed traders lose their full trading advantage, and the manager will receive no private benefits. When the manager's incentives place more emphasis on market price, he switches to partial disclosure of H. Finally, when partial disclosure of H is no longer credible, he fully discloses his information. The additional disclosure when moving from partial to full always increases price efficiency and decreases information asymmetry.

This provides an interesting and somewhat surprising conclusion. Informed traders' expected profits are highest when managers care (relatively) less about them, but not so little that there is full disclosure. As we have discussed, when a is very large (and managers care primarily about informed trading profits), they cannot credibly commit to a disclosure policy of d_H, although that is the policy preferred by informed traders, ex ante. This may explain why institutional investors, who provide private benefits, may also monitor the manager by tying his compensation to current share price (creating a moderate value of a rather than a very high one). Liquidity traders' expected losses are non monotonic in disclosure; they are lowest for full disclosure but highest for partial disclosure of H. The optimal disclosure as a function of a when MLRP is satisfied follows the same pattern although no disclosure is the informed traders preferred ex ante disclosure.

4 Regulatory Intervention

Implicitly, we are assuming the ability of informed traders to provide (at least some) private benefits to managers that creates a friction preventing full disclosure. The FASB, SEC, United States Congress and other bodies have imposed regulation on publicly traded firms to protect investors, particularly against non-disclosure and selective disclosure. Regulation Fair Disclosure (Reg FD) and the duty to disclose are broad disclosure rules that affect nearly all firms. Reg FD prohibits selective disclosure, or the practice of providing material, non-public information about a company to an analyst or other investor before disclosing it to the general public. The duty to disclose stems from the necessity of preventing managers from using their position to take unfair advantage of uninformed traders (i.e., keep prices artificially high to get short term gains). In the next two subsections, we consider Reg FD and the duty to disclose as imposing restrictions on our more general model. In the former case, the information set of the informed trader is restricted; in the latter, the optimal disclosure decision set is restricted.

4.1 Regulation FD

Thus far, we have not specified the source of the informed trader's signal, f. Generally speaking, signals may be based on the trader's expert analysis of publicly available information, based on new (non-public) information gathered from various sources, or a combination of both. In this section, we assume that the information comes from some corporate insider and therefore is restricted to the conditions of Regulation Fair Disclosure.

The SEC writes "[i]nformation is material if there is a substantial likelihood that a reasonable shareholder would consider it important in making an investment decision" (Securities & Exchange Commission 2000). Selective disclosure is prohibited by Reg FD; corporate insiders must promptly disclose to the public any material information he or she discloses privately. Recently, Raj Rajaratnam, the head of the Galleon Group, used mosaic theory as a defense against charges of insider trading. He argued that he did not have material private information, but instead combined non-material information, public information and his superior processing skills to generate a legally tradable signal (Henning 2011).

Even under Reg FD, managers and other insiders are not prohibited from engaging in one-on-one meetings with analysts or investors without disclosing their conversations. In 2005, a district court judge dismissed an SEC claim that Siebel Systems violated Reg FD, noting that "Regulation FD does not require that corporate officials only utter verbatim statements that were previously publicly made." Moreover, the SEC was criticized for excessive scrutiny of general (presumably non-material) comments, which would in turn discourage disclosure of material information.

Thus, if a necessary condition of legal information sharing is that the information is not material, or *would not* change the market maker's expectation of the firm value *at the time of its release*, is it also a sufficient condition? "[M]ateriality has become one of the most unpredictable and elusive concepts of the federal securities laws." If the information will never materially impact the market price, it seems straightforward to conclude that sharing it is wholly within the bounds of Reg FD. However, if the information, in conjunction with other subsequent public disclosures *would* materially impact market price it is less clear. Since there is no explicit time-frame of when the information must change beliefs in the definition of materiality, the latter might follow the letter of Reg FD, but possibly not the spirit.

If an insider provides a private signal to the informed trader where $t_H \neq t_L$, he clearly violates Reg FD, since the informed trader would update his priors based on the private signal. In contrast, if $t_H = t_L$=0 and t_M=1, the information structure is one in which the informed trader learns only that the state is M, or not M. Since the priors on L and H are symmetric, the informed trader cannot update his priors at the time of the private signal's disclosure regardless of the new information. However, this private signal could be valuable for the informed trader if he can collect other pieces of information to disentangle the low from the high state.

Because the ex ante value (and price) of the asset is M, one could argue that learning f would not be "important in making investment decisions." On one hand, if f =1, the price is correct, and knowledge of f would not indicate a profitable trade. On the other hand, if f =0, the informed trader could not profitably trade because H and L are symmetric around M and equally likely. If, however, subsequent public disclosure moves price away from M, then the initial information, when combined with the public disclosure *is* rendered important for trading profitably because the informed trader has perfect information. However, from a law perspective, the private disclosure could have occurred months ago and given that the informed trader makes profits only if the manager does not make any disclosures, the SEC would face difficulties in finding evidence of insider trading. Further, if the private information comes in the first place from the manager, the manager would be subverting Reg FD by providing information that will knowingly become material when taken in conjunction with his subsequent planned disclosures. Could the SEC take action on selective disclosure of this type of information? Would it be successful, or even possible to disentangle who knew what and when?[23]

[23]Richard Moore, an investment banker at Canadian Imperial Bank of Commerce, was charged with insider trading on material, non-public information acquired in the course of his employment. Moore's colleague, the Managing Director of the Canada Pension Plan Investment Board (CPPIB) appeared very busy and mentioned that he was working on something "interesting and active" but did not disclose the parties of the deal. Through their friendship, Moore learned that the Managing Director was making frequent trips to London. Upon spotting the Managing Director chatting with the CEO of Tomkins at a charity event, Moore pieced together the players in the acquisition and purchased shares of Tompkins ADRs prior to the deal's announcement and when the merger was disclosed, the share price increased 27% . The successful disgorgement of profits and penalties imposed on Moore suggests that there may be an implicitly higher standard on

In the case, where the signal reveals only M or 'not M', the manager gets no utility from a policy of non-disclosure. The informed trader cannot earn profits based on f alone and the lack of disclosure will not affect (i.e., improve) prices. Instead, the manager will always disclose H publicly. It has the effect of increasing price (which has some weight in his utility function) when the true state is H and the effect of allowing for profitable trading when the states are M or L. When M or L are realized, the informed trader knows perfectly the state either because he learned M directly, or learned $(\tilde{M} \cup \tilde{H})$. Whether the manager also prefers to disclose M publicly depends on his preference for informed trading profits relative to stock price. As before, the threshold $a_{H,M}$ determines whether the manager prefers to disclose only the high state or to disclose fully.

Corollary 2 *If $t_H = t_L = 0$ and $t_M = 1$, which represents information sharing under Reg FD, there is always some public disclosure by managers. When $a < a_{H,M}$ or $a > a_{H,N}$, the manager fully discloses. Otherwise the manager partially discloses.*

If we view these parameters as representing the information environment following the implementation of Reg FD, we would observe greater public disclosure than with selective disclosure of material information, in the absence of other private signals.[24] Bailey, Li, Mao, and Zhon (2003) concludes that Reg FD increases the quantity of information available to the public, consistent with the prediction of our model.

Whenever a is high and the true state is M, what may have been immaterial at the point of selective disclosure will become material. That is, when the true state is M, and the manager following d_H makes no further disclosure, price will adjust downwards to its new expected value (between L and M). Therefore, taken together, selective disclosure of M and a disclosure policy of d_H is actually material information, and the informed trader's ability to profit on such information is fully known by the manager before the selective disclosure is made.[25] It seems that an informed trader could argue, somewhat correctly, that the information he or she received was not material at the time, despite the fact that the selective disclosure would surely help them sometime in the future. The courts will have a difficult problem disentangling the value of the information provided selectively if the only guidance they have is "materiality."

selective disclosure than materiality at the time of release (United States District Court Southern District Of New York 2013).

[24]For brevity, we have omitted from the body of the paper the case where insiders can selectively disclose information perfectly. If an insider can provide the informed trader a perfect signal, the is a larger range of non-disclosure because trading profits (and thus private benefits) are higher with perfect underlying information.

[25]Consistent with this prediction, Bushee, Jung, and Miller (2013) show that investors benefit from selective access to management even in the post-Reg FD period.

4.2 Duty to Disclose

In the previous sections, disclosure of the low state was never part of an equilibrium strategy. Recognizing this to be generally true, regulators have imposed rules specifically to elicit bad news (e.g., impairment rules, duty to disclose rules). Therefore, we consider a disclosure environment where public disclosure of L is mandatory (and enforceable). This imposes a minimum disclosure requirement, but not a maximum disclosure requirement on the manager. That is, the manager can effectively choose full disclosure if disclosing H, when it is observed, or can stay silent. The manager compares disclosing L only over revealing H when he has it, generating a threshold above which he partially discloses, and below which he fully discloses.

$$a > \frac{\phi}{\phi + \frac{4(1-n)n(t_M - t_H)}{(2-n)^2}} \equiv a_{L,F}$$

$$\text{where} \qquad \phi = n\left(-\frac{t_H^2}{nt_H - 2nt_M + 2t_M} + \frac{(1-t_H)^2}{nt_H - 2nt_M + n + 2t_M - 2} + \frac{1}{n-2}\right) + 2$$

Proposition 4 *When a duty to disclose L exists, if $a > a_{L,F}$, the manager follows policy d_L. If instead $a < a_{L,F}$, the manager chooses a full disclosure policy, or d_F.*

Depending on the location of $a_{L,F}$ and the other parameters, the duty to disclose can increase disclosures, decrease disclosures or simply change the disclosure policy from d_H to d_L. There are several cases, but they can be easily illustrated in Figure 5. We plot Cases I and II from Proposition 3, below the top and bottom lines segmented by thresholds on a respectively. In Case I and no duty to disclose, there was a region of full disclosure (low a), a region of partial disclosure (moderate a) and a region of no disclosure (high a). The duty to disclose pushes up the full disclosure threshold and replaces no disclosure with partial disclosure, creating overall more disclosure in those regions. If $a_{L,F}$ is below $a_{N,H}$, there is a region in which partial disclosure of L replaces partial disclosure of H. Case II has a surprising twist. When $a_{L,F} < \max\{a_{N,H}, a_{N,M}\}$, mandatory disclosure of L will lead to less disclosure if MLRP is violated. There was a region of full disclosure above $a_{H,N}$ because of the inability of the manager to credibly commit to a partial disclosure policy. Mandatory disclosure introduces an effective commitment to partial disclosure, allowing the manager to avoid full disclosure. The other characteristics of mandatory disclosure are similar to Case I; the full disclosure region expands and the no disclosure region is eliminated and replaced by partial disclosure. As before, L may replace H, but only for some parameters. To summarize, when the manager places a relatively high weight on private benefits, the duty to disclose L grants him the ability to credibly commit to partial disclosure, increasing his private benefits and increasing the information asymmetry in the market, an unintended consequence of the duty to disclose.

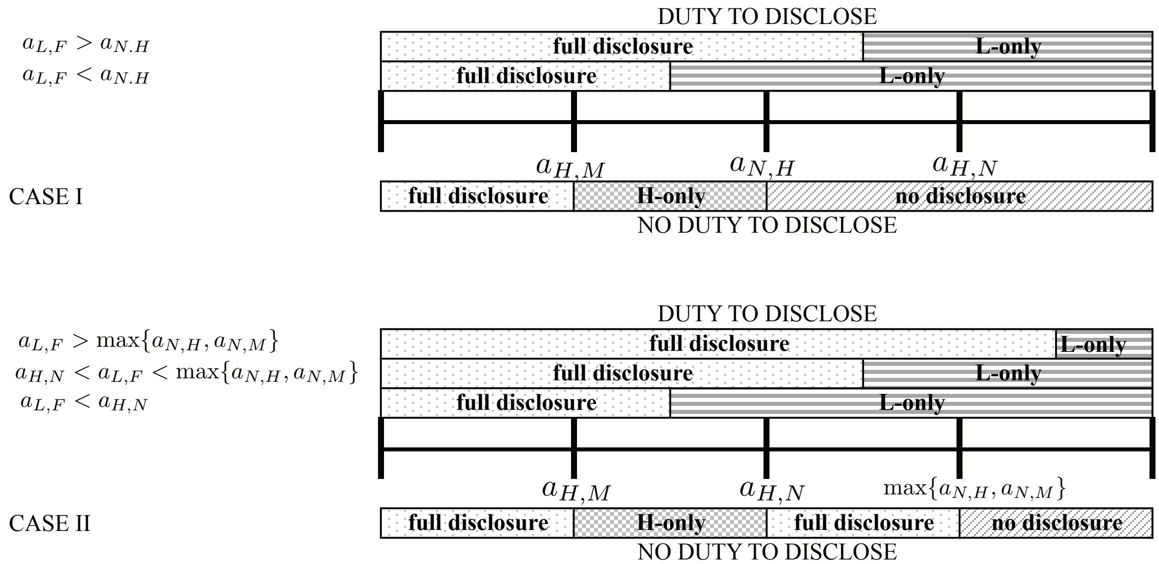


Figure 5: Cases with Duty to Disclose

The value of $a_{L,F}$, which partitions the line into L-only and full disclosure is increasing in the probability of a good signal when the true state is high, t_H, and decreasing in the probability of a good signal when the true state is medium, t_M. When MLRP is satisfied, $a_{L,F}$ is increasing in the prior probability of the middle state, n, and when violated, it is decreasing in n. Finally, $a_{N,H}$ is decreasing in t_H; therefore if t_H is large (small), it is more likely that $a_{L,F}$ is greater than (less than) $a_{N,H}$.

The closer t_θ is to 1/2, the less informative the signals are with respect to state θ. Fixing $t_L = 0$ and $t_M = 1$, we can consider two extremes; one where $t_H = 1$, the other where $t_H = 0$. In the former case, the informed trader knows L perfectly. When the total market demand is negative, the market maker knows for sure that the value of the firm is L, sets prices accordingly, and the informed trader earns zero profits. When total market demand is zero, the informed trader always trades profitably because his information was perfect and price cannot adjust. In the case where $t_H = 0$, the informed trader perfectly knows M, but without subsequent disclosure, his information cannot lead to profitable trade absent subsequent public disclosure. Therefore, it is essential that regulators consider the signal characteristics in conjunction with disclosure rules. Imposing a duty to disclose if $t_H = 0$ is advantageous to the informed trader by increasing the value of his information in the other two states whereas imposing a duty to disclose if $t_H = 1$ completely eliminates his trading advantage.

5 Empirical Implications

Although our model uses a simple discrete distribution, we believe it provides some interesting, testable predictions.[26] Gow, Taylor, and Verrecchia (2012) suggest that more precise earnings are associated with a larger increase in information asymmetry at the time of the earnings announcement. In contrast, our model suggests a non-monotonicity between disclosure precision and information asymmetry. When disclosures are very precise such that they perfectly differentiate between states (equivalently, full disclosure), or when disclosures are very imprecise (equivalently, no disclosure) information asymmetry is low. In between, when there is partial disclosure, information asymmetry can be higher (if signals violate MLRP and n is sufficiently low and/or the private information is not very precise). Cross-sectionally, for firms with a positive association between disclosure and information asymmetry, the degree of information asymmetry is similarly non-monotonic in the volatility of cash flows.

There are many instances in which information about a firm may be difficult to interpret. Is a large order backlog indicative of higher product demand or inefficient operations? Are increasing receivables the result of greater sales or poor collections? If the informed trader can differentiate perfectly between M and not M, but cannot differentiate between L and H, we can represent this in our model as $t_M = 1$ and $t_H = t_L = 0$. Based on our analysis in Section 4.1, when private information is noisy and provides limited ability to update priors because the same signal can be interpreted as both good or bad news, we would observe more public disclosures than in an environment where posteriors are directionally updated based on information.

Interpreting a greater weight on stock price in our manager's utility as greater equity incentives, our model predicts that there will be more voluntary disclosure the greater the manager's stock based incentives. This is consistent with several empirical findings and the conclusion in Core's (2001) review that we would expect a positive association between equity incentives and voluntary disclosure. If institutional investors provide the private benefits, our model would suggest that greater institutional holdings is associated with less voluntary disclosure, which is consistent with the empirical findings in Tasker (1998) and Bushee, Matsumoto, and Miller (2003). Shareholder activism or board independence may measure the degree of monitoring or success at severing the ties between managers and investors who provide private benefits. If the compensation committee can get the weight on private benefits low enough, they may be able to decrease information asymmetry and increase disclosures. However, if the weight on private benefits isn't sufficiently low, they may increase the disclosure while actually increasing the information asymmetry. Therefore, simply reducing the weight is not

[26] It would be possible to estimate the more continuous distribution of information with a three point distribution (symmetric around the mean) and test whether the restrictions are rejected.

enough; the reduction must be substantial and very effective. Finally, existing stock based compensation and expected option awards may influence the disclosure decision of managers. We predict that managers which plan to offload their shares in the short run have a greater incentives for disclosure and managers which are going to receive large awards have lesser incentives for disclosure.[27]

6 Conclusion

Our paper helps to put the empirical findings that information asymmetry increases around public disclosures into context, without the assumption that there are greater information gathering opportunities generated by the public disclosure. Private information is available with or without the disclosure, but the disclosure can improve the precision of the information. This is the essence of the mosaic, where individuals use information from many different sources combined with public disclosures to generate superior signals. Depending on the initial quality of the information, disclosure can increase or decrease expected informed trading profits.

In a model with optimal disclosure decisions selected by a manager who cares about both stock prices and informed trading profits, our paper provides insights into how public disclosures affect information asymmetry and the price formation process. Informed traders in our model have a noisy signal about firm value and managers can use public disclosure policies to implicitly reveal information about outcomes. That is, if the manager follows a policy where he discloses good news, no disclosure can be interpreted as "not good news." These partial disclosure policies can increase expected informed trading profits by improving the precision of the informed trader's signal. Because the information, if disclosed, can decrease informed trading profits for a particular realization, the optimal ex ante and ex post disclosure policies are not necessarily the same. Since our manager chooses optimally based on his private information and his compensation, providing the manager with joint incentives to maximize stock price and informed trading profits, rather than focusing on only informed profits, allows *expected* trading profits to increase.

We also focus on two special cases, Regulation Fair Disclosure and the SEC's duty to disclose. In the first, the informed trader has non-material private information. That is, his information does not change the expected value of the stock relative to current prices. Depending on the disclosure decision of the manager, however, his private signal can be transformed into a material one. That is, if the public disclosure eliminates some states of the world, the informed trader's remaining information is superior to the information of the market. Therefore, if an informed trader is supplied information by an insider that is temporarily non-material, it may be subject to subsequent scrutiny if later disclosures

[27] Aboody and Kasznik (2000) suggest that managers withhold good news and release bad news prior to option awards. In our setting, withholding news is equivalent to "not maximizing" stock price and is therefore similar in spirit.

allow for profitable trading. The second special case is where the firm faces a duty to disclose bad news. If the firm is required to partially disclose (but not optimally), it will change the optimal public disclosures. On one hand, there is more disclosure in regions in which there would have been no disclosure without the mandatory policy (the intended consequence). On the other, mandatory regulation also allows for commitment to an ex ante disclosure policy which may actually improve the expected profits of the informed traders (the unintended consequence).

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Proofs

Proof of Proposition 1:

$$\begin{aligned}\Pi(d_N) &= \frac{en(t_H-t_L)}{2}\\ \Pi(d_H) &= \frac{en(1-n)(t_M-t_L)}{(2-n)}\\ \Pi(d_L) &= \frac{en(1-n)(t_H-t_M)}{(2-n)}\end{aligned}$$

The derivative of $\frac{(1-n)}{(2-n)}$ in n is $-\frac{1}{(2-n)^2}<0$. Thus, $\frac{(1-n)}{(2-n)}<\frac{1}{2}$. Further $t_M-t_L \leq t_H-t_L$ and $t_H-t_M \leq t_H-t_L$ because $t_H \geq t_M \geq t_L$. We conclude that $\Pi(d_N) \geq \Pi(d_H)$ and $\Pi(d_N) \geq \Pi(d_L)$. Further, $\Pi(d_H) \geq \Pi(d_L)$ if $t_H \leq 2t_M - t_L$. Otherwise $\Pi(d_H) < \Pi(d_L)$.

We turn to the analysis of the variance of value given prices: $\forall i \in \{H,L,N\}, V(\theta - P|d_i) = V(d_i)$.

$$\begin{aligned}V(d_i) = {} & \frac{1}{2}nt_H\left(\frac{1}{2}(e-P(2,\emptyset,d_i)+\mu)^2+\frac{1}{2}(e-P(0,\emptyset,d_i)+\mu)^2\right)+\\ & \frac{1}{2}n(1-t_H)\left(\frac{1}{2}(e-P(-2,\emptyset,d_i)+\mu)^2+\frac{1}{2}(e-P(0,\emptyset,d_i)+\mu)^2\right)\\ & +\frac{1}{2}nt_L\left(\frac{1}{2}(-e-P(2,\emptyset,d_i)+\mu)^2+\frac{1}{2}(-e-P(0,\emptyset,d_i)+\mu)^2\right)\\ & +\frac{1}{2}n(1-t_L)\left(\frac{1}{2}(-e-P(-2,\emptyset,d_i)+\mu)^2+\frac{1}{2}(-e-P(0,\emptyset,d_i)+\mu)^2\right)\\ & +(1-n)t_M\left(\frac{1}{2}(\mu-P(2,\emptyset,d_i))^2+\frac{1}{2}(\mu-P(0,\emptyset,d_i))^2\right)\\ & +(1-n)(1-t_M)\left(\frac{1}{2}(\mu-P(-2,\emptyset,d_i))^2+\frac{1}{2}(\mu-P(0,\emptyset,d_i))^2\right)\end{aligned}$$

After rearranging the terms, we obtain expressions 6, 8 and 10.

We want to show that $\frac{\partial V(d_N)}{\partial t_H} = \frac{e^2n^2(t_H-t_L)\left(2n^2(t_L-t_M)(t_H+t_L-2t_M)+n(2t_M-1)(t_H+3t_L-4t_M)-4(1-t_M)t_M\right)}{(n(t_H+t_L-2t_M)+2(t_M-1))^2(n(t_H+t_L-2t_M)+2t_M)^2} < 0$. To prove that $V(d_N)$ is decreasing in t_H, we need to sign

$$\Gamma(n) = \left(2n^2(t_L-t_M)(t_H+t_L-2t_M)+n(2t_M-1)(t_H+3t_L-4t_M)-4(1-t_M)t_M\right),$$

which a quadratic polynomial expression in n. Let a_1, b_1 and c_1 be the following coefficients of the quadratic polynomial expression:

$$\begin{aligned} a_1 &= 2(t_L - t_M)(t_H + t_L - 2t_M) \\ b_1 &= (2t_M - 1)(t_H + 3t_L - 4t_M) \\ c_1 &= -4(1 - t_M)t_M < 0 \end{aligned}$$

If $\Gamma(n)$ admits two real roots then we have three scenarios to consider.

1. If $a_1 > 0$, then the two roots have opposite signs.
2. If $a_1 < 0$ and $b_1 > 0$ then the roots are positive.
3. If $a_1 < 0$ and $b_1 < 0$ both roots are negative.

For case 1., we know that $\Gamma(0) = c_1 < 0$. So if $\Gamma(1) < 0$, $\forall n \in [0,1], \Gamma(n) < 0$. Let us show that $\Gamma(1) < 0$:

$$\Gamma(1) = 2t_L^2 + (2t_H - 3)t_L - t_H \tag{11}$$

This expression is a quadratic polynomial expression in t_L where his coefficients are $2 > 0$ and $-t_H < 0$ and $(2t_H - 3)$, which sign is indeterminate. If expression 11 admits two real roots, they are of opposite signs. At $t_L = 0$, expression 11 is equal to $-t_H < 0$ and at $t_L = 1$, it is equal to $-(1 - t_H) < 0$. Thus, $\Gamma(1) < 0$. Further, notice that the discriminant of expression 11 cannot be negative.
For case 2., we further need to prove that the root of $\Gamma(n)$, i.e., $\frac{-b_1 + \sqrt{\Delta}}{2a_1} > 1$ to guarantee that $\Gamma(n) < 0$. Rearranging the terms, we need to show that $a_1 + b_1 + c_1 < 0$. Taking the coefficients a_1, b_1 and c_1 of $\Gamma(n)$, $a_1 + b_1 + c_1 = 2t_L^2 + (2t_H - 3)t_L - t_H = \Gamma(1) < 0$. Thus, $\Gamma(n) < 0$.
For case 3., it is immediate to see that $\Gamma(n) < 0$.
If $\Gamma(n)$ has a negative discriminant and given that $\Gamma(0) < 0$, $a_1 < 0$. Thus, $\forall n \in [0,1], \Gamma(n) < 0$.

At $t_H = 1$, we need to show that:

$$V(d_N) - V(d_H) = \frac{e^2 n \left(2n^3 (t_L - t_M)^2 - n^2 \left(5t_L^2 - 16t_L t_M + t_L + 2t_M(6t_M - 1)\right) + 2nt_M(-6t_L + 9t_M - 1) - 8t_M^2\right)}{2(n-2)(n(t_L - 2t_M) + 2t_M)(n(t_L - 2t_M + 1) + 2t_M)} > 0 \tag{12}$$

Collecting the terms in t_M of the numerator of expression 12, it yields:

$$\underbrace{(-8 + 18n - 12n^2 + 2n^3)}_{<0} t_M^2 + \underbrace{(-2n + 2n^2 - 12nt_L + 16n^2 t_L - 4n^3 t_L)}_{<0} t_M \underbrace{-n^2 t_L - 5n^2 t_L^2 + 2n^3 t_L^2}_{<0} < 0 \tag{13}$$

Collecting the terms in t_M of the denominator of expression 12, it yields:

$$\underbrace{-2(2-n)4(1 - n(2-n))}_{<0} t_M^2 \underbrace{-2(2-n)(2n(1-n) + 4nt_L(1-n))}_{<0} t_M \underbrace{-2(2-n)(n^2 t_L + n^2 t_L^2)}_{<0} < 0 \tag{14}$$

Therefore, $V(d_N) > V(d_H)$.
The reasoning is symmetric to prove that $\frac{\partial V(d_N)}{\partial t_L} > 0$ and at $t_L = 0$, $V(d_N) - V(d_L) > 0$. Therefore, $V(d_N) > V(d_L)$.
Moreover, at $t_H = t_L$, $V(d_H) = V(d_L)$. If $t_H > t_L$, $V(d_H) < V(d_L)$ otherwise $V(d_H) > V(d_L)$.

Proof of Proposition 2: If $t_M \geq t_H \geq t_L$, $\Pi(d_H) > \Pi(d_L)$.

i. At $t_M = t_1 > t_L$, $\Pi(d_H) = \Pi(d_N)$ and if $t_M > t_1$, $\Pi(d_H) < \Pi(d_N)$.

ii. At $t_H = t_3 > t_L$, $\Pi(d_L) = \Pi(d_N)$ and if $t_H > t_3$, $\Pi(d_L) < \Pi(d_N)$.

iii. By inspection, $t_1 > t_3$.

If $t_M \geq t_L \geq t_H$, $\Pi(d_L) > \Pi(d_H)$.

i. At $t_H = t_2 > t_L$, $\Pi(d_L) = \Pi(d_N)$ and if $t_H > t_2$, $\Pi(d_L) < \Pi(d_N)$.

ii. At $t_H = t_4 > t_L$, $\Pi(d_L) = \Pi(d_N)$ and if $t_H > t_4$, $\Pi(d_L) < \Pi(d_N)$.

iii. By inspection, $t_2 > t_4$.

Proof of Proposition 3:

Conditions to implement no disclosure:

i. If H occurs, the profits by following the no disclosure policy are:

$$\begin{aligned} & \underbrace{a\frac{1}{2}ent_H}_{private\ benefits} + \underbrace{(1-a)\left(\frac{t_H(P(2,\emptyset,d_N)+P(0,\emptyset,d_N))}{2} + \frac{(1-t_H)(P(-2,\emptyset,d_N)+P(0,\emptyset,d_N))}{2}\right)}_{price} \\ = \quad & a\frac{1}{2}ent_H + (1-a)\left(\frac{ent_H^2}{2n(t_H-2t_M)+4t_M} - \frac{en(t_H-1)t_H}{2(n(t_H-2t_M)+2(t_M-1))} + \mu\right) \end{aligned} \tag{15}$$

The profits in 15 needs to be greater than disclosing publicly H and getting $(1-a)(\mu+e)$. Simplifying and rearranging $a \geq a_{N,H}$.

ii. If M occurs, the profits by following the no disclosure policy are:

$$\begin{aligned} & \underbrace{a\frac{1}{2}ent_H}_{private\ benefits} + \underbrace{(1-a)\left(\frac{t_M(P(2,\emptyset,d_N)+P(0,\emptyset,d_N))}{2} + \frac{(1-t_M)(P(-2,\emptyset,d_N)+P(0,\emptyset,d_N))}{2}\right)}_{price} \\ = \quad & a\frac{1}{2}ent_H + (1-a)\left(\frac{(1-a)\left(en^2t_H(t_H-2t_M)+2\mu(n(t_H-2t_M)+2(t_M-1))(n(t_H-2t_M)+2t_M)\right)}{2(n(t_H-2t_M)+2(t_M-1))(n(t_H-2t_M)+2t_M)}\right) \end{aligned} \tag{16}$$

The profits in 16 needs to be greater than disclosing publicly H and getting $(1-a)\mu$. Simplifying and rearranging $a \geq a_{M,H}$.

Therefore no disclosure is implementable if $a \geq \max(a_{N,H}, a_{N,M})$.

<u>Conditions to implement partial disclosure of H:</u>

i. If H occurs, the profits of hiding H are:

$$\begin{aligned}
& \underbrace{a\frac{2e(1-n)nt_M}{(n-2)^2}}_{private\ benefits} + \underbrace{(1-a)\left(\frac{t_H(P(2,\emptyset,d_H)+P(0,\emptyset,d_H))}{2} + \frac{(1-t_H)(P(-2,\emptyset,d_H)+P(0,\emptyset,d_H))}{2}\right)}_{price} \\
= \quad & a\frac{2e(1-n)nt_M}{(n-2)^2} + (1-a)\left(en\left(\frac{1}{n-2} - \frac{t_H-1}{-2nt_M+n+2t_M-2}\right) + 2\mu\right) \qquad (17)
\end{aligned}$$

The profits in 17 needs to be lower than disclosing publicly H and getting $(1-a)(\mu+e)$. Simplifying and rearranging $a \leq a_{H,N}$.

ii. If M occurs, the profits by following partial disclosure of H are:

$$\begin{aligned}
& \underbrace{a\frac{2e(1-n)nt_M}{(n-2)^2}}_{private\ benefits} + \underbrace{(1-a)\left(\frac{tm(P(2,\emptyset,d_H)+P(0,\emptyset,d_H))}{2} + \frac{(1-tm)(P(-2,\emptyset,d_H)+P(0,\emptyset,d_H))}{2}\right)}_{price} \\
& = a\frac{2e(1-n)nt_M}{(n-2)^2} \\
& +(1-a)\left(\frac{en\left((n-1)(3n-4)t_M^2 - 2(n-2)(n-1)t_M\right) + \mu(n-2)(2t_M-2nt_M)(-2nt_M+n+2t_M-2)}{(n-2)(2t_M-2nt_M)(-2nt_M+n+2t_M-2)}\right) \qquad (18)
\end{aligned}$$

The profits in 18 needs to be greater than disclosing publicly M and getting $(1-a)\mu$. Simplifying and rearranging $a \geq a_{H,M}$.

Therefore, partial disclosure of H is implementable if $a \geq a_{H,M}$ and $a \leq a_{H,N}$.

<u>Conditions to implement partial disclosure of M:</u>

i. If H occurs, the profits by following partial disclosure of M are:

$$\underbrace{a\frac{1}{2}et_H}_{private\ benefits} + \underbrace{(1-a)\left(\frac{t_H(P(2,\emptyset,d_M)+P(0,\emptyset,d_M))}{2} + \frac{(1-t_H)(P(-2,\emptyset,d_M)+P(0,\emptyset,d_M))}{2}\right)}_{price} \qquad (19)$$

$$= \quad a\frac{1}{2}et_H + (1-a)\left(\frac{(1-a)\left(2\mu(t_H-2)t_H - et_H^2\right)}{2(t_H-2)t_H}\right) \qquad (20)$$

The profits in 20 needs to be greater than disclosing publicly H and getting $(1-a)(\mu+e)$. Simplifying and rearranging $a \geq a_{M,H} \equiv \frac{3t_H-4}{t_H^2+t_H-4}$.

ii. If M occurs, the profits of hiding M are:

$$\begin{aligned}
& \underbrace{a\frac{1}{2}et_H}_{private\ benefits} + \underbrace{(1-a)\left(\frac{t_M(P(2,\emptyset,d_M)+P(0,\emptyset,d_M))}{2} + \frac{(1-t_M)(P(-2,\emptyset,d_M)+P(0,\emptyset,d_M))}{2}\right)}_{price} \\
= \quad & a\frac{1}{2}et_H + (1-a)\left(\frac{e(t_H-2t_M)+2\mu(t_H-2)}{2(t_H-2)}\right) \qquad (21)
\end{aligned}$$

The profits in 21 needs to be lower than disclosing publicly M and getting $(1-a)\mu$. Simplifying and rearranging $a < a_{M,N} \equiv \frac{2t_M - t_H}{(t_H-3)t_H + 2t_M}$, which needs to be lower than 1.

Therefore, partial disclosure of M is implementable if $a \geq a_{M,H}$ and $a \leq a_{M,N}$.

We show that partial disclosure of M is never implementable.

If $t_H < \frac{1}{2}\left(3-\sqrt{9-8t_M}\right)$, $a_{M,N} > 1$. From expression 21, at $a=1$, it is immediate to see that the manager would always withhold disclosure of M if it occurs. If $t_H \geq \frac{1}{2}\left(3-\sqrt{9-8t_M}\right)$, $a_{M,H} > a_{M,N}$.

Ordering of the thresholds.

Step 1. $a_{H,N}$ **and** $a_{H,M}$**:**

$$\frac{\partial a_{H,N}}{\partial t_H} = \frac{4(n-2)^2(1-n)n^2 t_M(2(1-n)t_M + n - 2)}{\left((n-2)^2(nt_H-4) + 8(n-1)^2 n t_M^2 - 2(n-1)(3n-4)(n-2)t_M\right)^2} < 0 \tag{22}$$

The difference $a_{H,N} - a_{H,M}$ is decreasing in t_H.

$$At\ t_H = 1, a_{H,N} - a_{H,M} = \frac{4(n-2)^2(1-n)t_M(n(5t_M-3) + 4(1-t_M))}{\left(8(n-1)^2 t_M^2 - n(n-2)t_M - 2(n-2)^2\right)\left(n(4(n-1)t_M - n + 6) - 8\right)} > 0 \tag{23}$$

Thus, $a_{H,M} < a_{H,N}$ and partial disclosure of H is always implementable.

Step 2. $a_{N,H}$ **and** $a_{H,N}$**:**

$$\begin{aligned} \frac{\partial a_{N,H}}{\partial t_H} &= \frac{n\left(nt_H^2 G'(t_H) - 2\right)}{(nt_H G(t_H) + nt_H + 2)^2} \\ \text{where } G'(t_H) &= \frac{(n-2)n^2 t_H^2 - 4(n-1)t_M\left(n^2 t_H - 2\right) + 4(n-1)^2(n+2)t_M^2}{(n(t_H - 2t_M) + 2t_M)^2(nt_H - 2nt_M + 2t_M - 2)^2} \end{aligned}$$

We show that $G'(t_H) < 0$. To do that, we only need to sign the numerator, which is a quadratic polynomial expression in t_M.

$$\underbrace{-(2-n)n^2 t_H^2}_{<0} \underbrace{-4(1-n)\left(2-n^2 t_H\right)}_{<0} t_M + \underbrace{4(1-n)^2(n+2)}_{>0} t_M^2,$$

which admits only one positive root. At $t_M = 0, -(2-n)n^2 t_H^2 < 0$
and at $t_M = 1, n(n(2-t_H)(-n(2-t_H) + 2t_H) - 4) < 0$. Thus, $G'(t_H) < 0$ and $a_{N,H}$ is decreasing in t_H.

$$\begin{aligned} &At\ t_H = 0,\ a_{N,H} - a_{H,N} = \frac{2(n-1)nt_M(n(2t_M-1) - 2t_M + 2)}{n^3 t_M(4t_M - 3) + n^2\left(-8t_M^2 + 13t_M - 2\right) + 2n\left(2t_M^2 - 9t_M + 4\right) + 8(t_M - 1)} > 0 \\ &At\ t_H = t_1 < t_M,\ a_{N,H} - a_{H,N} = 0 \end{aligned}$$

Thus, if $t_M \geq t_H \geq t_1$, $a_{N,H} < a_{H,N}$. Otherwise $a_{N,H} > a_{H,N}$.

Step 3. $a_{N,H}$ **and** $a_{N,M}$**:**

We only need to order them for $t_M > t_H > t_1$.

$$\frac{\partial a_{N,M}}{\partial t_H} = \frac{n\left(n^2(t_H-2t_M)^2+4(1-t_M)t_M\right)}{\left(n^2(t_H-2t_M)^2+n(4t_M-3)(t_H-2t_M)+4(t_M-1)t_M\right)^2} > 0 \tag{24}$$

The difference $a_{N,M} - a_{N,H}$ is increasing in t_H.

$$At\ t_H = t_M, a_{N,M} - a_{N,H} = -\frac{2(n-2)^2((n-2)t_M+2)^2}{(n^2t_M-4nt_M+3n+4t_M-4)\left(n^3t_M^2+n^2(5-4t_M)t_M+4n\left(t_M^2-3t_M+1\right)+8(t_M-1)\right)} < 0$$

Thus, $a_{N,M} < a_{N,H}$.

Proof of Corollary 2: It is a special case of Proposition 3 and at $t_M = 1$ and $t_H = 0$, $a_{N,H} = 1$. So no disclosure is never optimal. Moreover, $a_{H,N} = \frac{(3-n)(2-n)}{6-n(3+n)} < a_{N,H}$.

Proof of Proposition 4: We take the derivative of $a_{L,F}$ in t_H.

$$\frac{\partial a_{L,F}}{\partial t_H} = \frac{4(2-n)^2(1-n)n\left((t_M-t_H)\phi'(t_H)+\phi(t_H)\right)}{\left((n-2)^2\phi(t_H)+4(n-1)n(t_H-t_M)\right)^2} \tag{25}$$

We study the sign of the numerator of expression 25. We first show that $\phi(t_H) > 0$. We reduce the expression $\phi(t_H)$ to the same numerator.

$$\phi(t_H) = \frac{4(1-n)\left(-2(n-1)t_M(nt_H+n-2)+\overbrace{nt_H(n+t_H-2)}^{<0}+\overbrace{(1-n)(4-3n)}^{>0}t_M^2\right)}{(2-n)(nt_H+2t_M(1-n))(nt_H+2t_M(1-n)-(2-n))} \tag{26}$$

The numerator of expression 26 is a quadratic polynomial in t_M that admits two roots of opposite signs.

$$\begin{aligned}\text{At } t_M = 0, &\qquad 4(1-n)(nt_H(n+t_H-2)) < 0\\ \text{At } t_M = 1, &\qquad 4(1-n)(n\left(-nt_H+n+t_H^2-1\right)) < 0\end{aligned}$$

The numerator is negative. By inspection, the denominator is negative because $nt_H + 2t_M(1-n) - (2-n) < 0$. Thus, $\phi(t_H) > 0$.

We now turn to the derivative of $\phi(t_H)$ is t_H.

$$\phi'(t_H) = \frac{4(n-1)^2 n(t_H - t_M)(n(t_H(2t_M - 1) + (3 - 4t_M)t_M) + 4(t_M - 1)t_M)}{(n(t_H - 2t_M) + 2t_M)^2(nt_H - 2nt_M + n + 2t_M - 2)^2} \tag{27}$$

We collect the terms of $n(t_H(2t_M-1)+(3-4t_M)t_M)+4(t_M-1)t_M$ in t_M, which yields:

$$\underbrace{-nt_H}_{<0} + t_M(2nt_H + 3n - 4) + \underbrace{(4-4n)}_{>0} t_M^2$$

$$\text{At } t_M = 0,\ -nt_H < 0$$

$$\text{At } t_M = 1,\ -n(1-t_H) < 0$$

Thus, $\phi'(t_H)$ is negative if $t_H - t_M > 0$, otherwise positive. We conclude that expression 25 is positive because $(t_M - t_H)\phi'(t_H) > 0$ and $a_{L,F}$ is increasing in t_H.

i. The difference $a_{L,F} - a_{H,M}$ is increasing in t_H. At $t_H = 0$,

$$a_{L,F} - a_{H,M} = \frac{2(2-n)(2-3n)t_M \overbrace{(2-n-2(1-n)t_M)(4+n(3t_M-2)-4t_M)}^{>0}}{\left(8(1-n)^2 t_M^2 + n(2-n)t_M - 2(2-n)^2\right)\left(-n(n(t_M(4t_M-5)+2)+2(7-2t_M)t_M-8)+8t_M-8\right)}$$

We collect the terms in t_M for expression $-n(n(t_M(4t_M-5)+2)+2(7-2t_M)t_M-8)+8t_M-8$ and it yields:

$$\underbrace{\left(4n-4n^2\right)}_{>0} t_M^2 + \left(5n^2 - 14n + 8\right) t_M \underbrace{-2n^2 + 8n - 8}_{<0} \tag{28}$$

$$\text{At} \qquad t_M = 0,\ -8(1-n) - 2n^2 < 0$$

$$\text{At} \qquad t_M = 1,\ -2n - n^2 < 0$$

Similarly, we study the sign for expression $\underbrace{8(1-n)^2}_{>0} t_M^2 + n(2-n)t_M \underbrace{-2(2-n)^2}_{<0}$.

$$\text{At} \qquad t_M = 0,\ -2(-2+n)^2 < 0$$

$$\text{At} \qquad t_M = 1,\ n(-6+5n) < 0$$

Thus, at $t_H = 0$, $a_{L,F} - a_{H,M} > 0$.

$$\text{At } t_H = t_M,\ a_{L,F} - a_{H,M} = -\frac{4(1-n)t_M(2-2(1-n)t_M-n)}{\underbrace{8(1-n)^2}_{>0}t_M^2 + n(2-n)t_M \underbrace{-2(2-n)^2}_{<0}} \tag{29}$$

At $t_M = 0$, the denominator is equal to $-2(2-n)^2 < 0$ and at $t_M = 1$, it is equal to $n(-6+5n) < 0$. It follows that the denominator is negative. Thus, at $t_H = t_M$, $a_{L,F} - a_{H,M} > 0$.

We conclude that $a_{L,F} > a_{H,M}$.

ii. The difference $a_{L,F} - a_{H,N}$ is increasing in t_H because from 22, $a_{H,N}$ is decreasing in t_H and $a_{L,F}$ is increasing in t_H.

$$\text{At} \qquad t_H = t_M,\ a_{L,F} - a_{H,N} = \frac{-4(1-n)nt_M(2-n-2(1-n)t_M)}{\underbrace{8(1-n)^2n}_{>0}t_M^2 - (n(5n-12)+8)(n-2)t_M \underbrace{-4(n-2)^2}_{<0}} \tag{30}$$

At $t_M = 0$, the denominator of 30 is equal to $-4(2-n)^2$ and at $t_M = 1$, it is equal to $-n(2+n)(4-3n) < 0$. Thus, at $t_H = t_M$, $a_{L,F} - a_{H,N} > 0$. Thus, $a_{L,F} > a_{H,N}$ if $t_H > t_M$.

iii. The difference $a_{L,F} - a_{N,H}$ is increasing in t_H because $a_{N,H}$ is decreasing in t_H and $a_{L,F}$ is increasing in t_H.

$$\text{At} \qquad t_H = 0,\ a_{L,F} - a_{N,H} = \frac{\overbrace{-2nt_M(2-2(1-n)t_M-n)}^{<0}}{n(n(t_M(4t_M-5)+2)+2(7-2t_M)t_M-8)-8t_M+8} \tag{31}$$

Rearranging the denominator of expression 31 yields $\underbrace{-4n(1-n)}_{<0}t_M^2 + \left(-5n^2+14n-8\right)t_M + \underbrace{2(2-n)^2}_{>0}$. At $t_M = 0$, the denominator is equal to $2(2-n)^2 > 0$ and at $t_M = 1$, the denominator is equal to $n(2+n) > 0$. Thus, at $t_H = 0$, $a_{L,F} - a_{N,H} < 0$.

$$\text{At} \qquad t_H = t_M,\ a_{L,F} - a_{N,H} = \frac{\overbrace{-(2-n)nt_M(-(2-n)t_M+2)}^{<0}}{n\left(t_M\left((n-2)^2 t_M + 5n - 12\right) + 4\right) + 8(t_M - 1)} \qquad (32)$$

Collecting the terms in t_M for the denominator of expression 32 yields $\underbrace{-8+4n}_{<0}+(8-12n+5n^2)t_M+\underbrace{(-2+n)^2n}_{>0}t_M^2$. At $t_M = 0$, the denominator is equal to $-8+4n<0$ and at $t_M = 1$, it is equal to $n(-4+n+n^2)<0$. Thus, at $t_H = t_M$, $a_{L,F} - a_{N,H} > 0$. We conclude that, $a_{L,F} > a_{N,H}$ if $t_H > t_M$.