Voluntary disclosure, disclosure bias and real effects*

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Abstract

Firms disclose information in order to reduce information asymmetry prior to issuing equity. We study a model in which a manager who is privately informed about the value of his firm's assets in place may issue equity to finance a profitable investment opportunity. In contrast to Myers and Majluf (1984) who do not consider any disclosure of information by the firm, we assume that the manager may voluntarily disclose his private information. If he chooses to do so, the manager is not confined to tell the truth but may bias his report at a cost.

The model shows that treating managers' disclosure and investment decisions both as endogenous and allowing managers to bias their voluntary reports yields qualitatively different predictions than when the disclosure and investment decisions are considered separately and disclosures are assumed to be truthful. The model predicts that managers may disclose good news and bad news but not intermediate news (contrary to traditional threshold equilibria of voluntary disclosure models) and that it is the managers with intermediate news who sometimes forego the profitable investment opportunity (in contrast to Myers and Majluf 1984). The model also predicts that (*i*) the underinvestment problem is more prevalent if the return on investment is low; and (*ii*) low-performing firms have (weakly) higher cost of capital than high-performing firms. As such, the paper highlights the importance of considering the interdependencies between firms' disclosure and investment decisions and provides new empirical predictions.

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1 Introduction

Firms' real decisions and disclosure decisions are closely linked. The theoretical literature that studies the relation between investment and disclosure decisions focuses on mandatory disclosure settings, i.e., settings in which managers are required to issue a signal/disclosure of their private information (e.g., Leland and Pyle 1977; Stein 1989). Empirical evidence, however, suggests that a substantial part of public information reaches markets through firms' voluntary disclosures (Beyer, Cohen, Lys and Walther 2010). In particular, prior to equity offerings, firms tend to increase both the quantity and the quality of their voluntary disclosures (e.g., Lang and Lundholm 1993, 2000; Marquardt and Weidman 1998).

When managers disclose information they can, and often do, bias their disclosures at some cost. Moreover, managers decide whether to disclose information and, if so, by how much to bias their disclosure, jointly with their firms' investment strategy. Therefore, it is important to consider management's incentives to issue voluntary reports and to bias such reports in a setting in which the firm's investment strategy is also endogenous. While costly reporting distortions and real effects have been widely studied in mandatory disclosure settings they have been largely ignored in the voluntary disclosure literature.[1] Instead, the voluntary disclosure literature mostly focuses on settings in which disclosures have no real effects and reports are assumed to be truthful/verifiable.[2] To the best of our knowledge, there exists no model of managers' decision whether to issue a report (voluntary disclosure decision) and, if so, what report to issue (biasing decision) when the firm's investment strategy is also endogenous (investment decision).

Our model studies the interdependencies between firms' voluntary disclosure decisions and their investment decisions and vice versa in a setting where managers can bias their reports at a cost. Most important, the model shows that the equilibrium characteristics of corporate investment and disclosure strategies are qualitatively different when studied jointly than when studied separately.

The paper develops a model similar to Myers and Majluf (1984) in which an entrepreneur, who is privately informed about the value of his firm's assets in place, may issue equity to finance a profitable investment opportunity. In contrast to Myers and Majluf (1984), in which firms lack the

[1] As far as we are aware, the only papers studying costly misreporting in a voluntary disclosure setting are Korn (2004) and Einhorn and Ziv (2010). Both of these papers do not consider real effects.

[2] There are a few papers that study product market competition in which voluntary disclosure has real effects (Vives 1984, Darrough 1993, Kanodia, Mukherji, Sapra and Venugopalan 2000, Hughes, Kao and Williams 2002, Fischer and Verrecchia 2004). These papers, however, do not consider investment decisions and restrict disclosures to be truthful. As such, the economic trade-offs considered are very different from the ones in our model.

ability to communicate any private information to potential investors, we assume that the manager may voluntarily disclose his private information. If he chooses to do so, he is not confined to tell the truth but may bias his report at a cost. In practice, managers enjoy reporting discretion due to the forward looking nature of many voluntary disclosures and the inherent flexibility in Generally Accepted Accounting Principles (GAAP). Empirical evidence suggests that managers indeed bias their reports (see for example, Burgstahler and Dichev 1997; Teoh, Welch and Wong 1998a,b; and Ajinkya, Bhojraj and Sengupta 2005). While most voluntary disclosure models assume that any disclosure has to be truthful (e.g., Verrecchia 1983, Dye 1985, Jung and Kwon 1988) some disclosure models take the opposite viewpoint and assume that misreporting is costless (cheap talk models in Newman and Sansing 1993, Gigler 1994, Stocken 2000 and Fischer and Stocken 2001). In this paper, we cover the "middle ground" that we believe is representative of the environment in which corporate disclosures take place.

The model illustrates that the manager sometimes withholds his private information in equilibrium even though he always obtains private information and there are no costs associated with making a disclosure per se. That is, a partial disclosure equilibrium evolves even though the manager can always issue a report without incurring any costs. The reason for partial disclosure to occur is that the manager does not incur any costs only if he reports truthfully. However, as common in costly signaling models, truth-telling can not be part of an equilibrium. Instead, the manager biases his report upwards whenever he makes a disclosure in order to increase investors' perception of the value of his firm's assets in place. This bias gives rise to endogenous disclosure costs. When the endogenous disclosure costs would be too high, the manager withholds his private information, resulting in a partial disclosure equilibrium.

The model further illustrates that due to the interaction of investment and disclosure decisions, the manager's disclosure strategy does not always take the common form of a "threshold" equilibrium in which the manager discloses information only when the value of his firm's assets is above the threshold. Instead, the manager sometimes discloses low and high values of assets in place but withholds intermediate values of assets in place. Similarly, and in contrast to Myers and Majluf (1984), the manager's investment strategy also no longer takes the form of a threshold below which the manager invests and above which he foregoes the profitable investment opportunity. Instead, the manager pursues the profitable investment opportunity when the value of the firm's assets in place is either sufficiently low or high but may forego the investment opportunity for intermediate values of assets in place. This is due to the interdependencies between the manager's disclosure and investment decision. Next, we elaborate on the model's predictions about the manager's disclosure and investment equilibrium strategies.[3]

First, the model predicts that if the investment opportunity is sufficiently profitable the manager always (i.e., for any value of the firm's assets in place) raises capital and pursues the investment opportunity. This is intuitive. If the profitability of the investment opportunity is sufficiently high, the expected return on investment outweighs the costs of being undervalued by investors or the costs from biasing the report. Since sufficiently high expected return on investment leads to the straight-forward case of efficient investment, we focus in the following discussion on the case of less profitable investment opportunities. For such investment opportunities, our model's predictions differ from Myers and Majluf (1984). In particular, our model predicts that a manager does not pursue the investment opportunity when the value of the firm's assets in place is in an intermediate range while the manager pursues the investment opportunity when the value of the firm's assets in place is either low or high.

Second, the model predicts that the manager issues a report when the value of the firm's assets in place is low or high but does not issue a report for intermediate values of asset in place. This prediction differs from the standard disclosure threshold equilibria in which good news are disclosed while bad news are withheld. The reason for the difference lies in the nature of the endogenous disclosure costs of our model, that differ from the commonly assumed constant (exogenous) disclosure costs. In the model, the endogenous bias and biasing costs turn out to be highest for intermediate values of assets in place. If the investment opportunity is not very profitable, the biasing costs of firms with intermediate values of assets in place would exceed the expected return on investment and, as a result, the manager opts to withhold his information and to forego the investment opportunity.

At first, it might seem surprising that in equilibrium the manager biases his report upwards by more when the value of assets in place is "intermediate" compared to when the value of assets in place is "high." Instead, one might expect the manager's bias to increase monotonically in his type as it is the case in standard signaling models in which the sender's payoff depends linearly on the receivers' perception of his type (e.g., Riley 1979, Miller and Rock 1985). In contrast, when equity is issued, the owner/manager's payoff is linear in the ownership fraction he must give up in exchange

[3] As common in the literature (e.g., Riley 1979, Miller and Rock 1985), we study equilibria in which – whenever a manager issues a report – it fully reveals the manager's private information to investors.

for the capital outside investors provide. The ownership fraction the manager is required to give up depends on investors' perception. The manager's benefit from marginally increasing investors' perception of the value of his firm's assets in place is higher when the value of those assets is lower. The reason is that the manager's benefit from making investors believe that the assets in place are marginally more valuable is greater when the "size of the pie" is smaller.[4] This, together with the standard result that the manager does not bias his report when the value of assets in place is lowest (zero), yields the prediction that the bias, and the biasing costs, are initially increasing and then decreasing in the firm's value of assets in place. Hence, the reporting bias and biasing costs are highest for intermediate values of asset in place. The fact that the bias function that emerges in this paper is different from the bias function in standard signaling models illustrates that modeling specific signaling settings and incorporating institutional details – such as financing needs and investment opportunities – can qualitatively alter predictions about properties of the sender's report and signaling costs.

Third, the model predicts that the underinvestment problem is more prevalent, in the sense that the manager foregoes the profitable investment opportunity more often, when the expected return on investment is lower. This is intuitive since it is less attractive for the manager to raise capital and invest and therefore the manager is less willing to incur biasing costs from issuing a report.

In the equilibrium described above, firms only raise capital after issuing a voluntary report. As a result, all firms that raise capital incur biasing costs (except for the firm whose assets in place are worth zero). It turns out that there exists an additional equilibrium, which is similar to the one described above, but in which firms with sufficiently low values of assets in place raise capital without issuing a voluntary report first. These firms "save" the biasing costs but may be pooled with firms whose assets in place are worth less and, hence, may have to give up a larger ownership fraction in order to raise the necessary capital. This equilibrium, which we refer to as "equilibrium with bad news undisclosed," gives rise to several additional predictions. First, the model predicts that there may exist two distinct non-disclosure intervals: in addition to firms with intermediate

[4] To see this, consider a manager whose firm's assets in place are worth x and who issues a report such that investors perceive the value of his firm's assets to be $x+\varepsilon$. If investors perceive the value of the firm's assets in place to be $x+\varepsilon$ they require fraction $\alpha = \frac{I}{x+\varepsilon+I+\mu_r}$ of the firm's ownership shares in exchange for providing capital I where μ_r denotes the expected return on investment. Since the assets are in fact worth x and not $x+\varepsilon$, the actual value of the shares investors obtain is $I\frac{x+I+\mu_r}{x+\varepsilon+I+\mu_r}$ rendering the manager's benefit from issuing a report that mimics a firm with assets worth $x+\varepsilon$ to be $I - I\frac{x+I+\mu_r}{x+\varepsilon+I+\mu_r} = I\frac{\varepsilon}{x+\varepsilon+I+\mu_r}$. Hence, the manager benefits less from mimicking a firm whose assets are marginally more valuable when the actual value of his firm's assets is higher.

asset values firms with sufficiently low asset values also do not issue a report. These two non-disclosure intervals, which are separated by an interval of values of assets in place for which firms disclose and invest, are distinct in terms of investment strategy. Firms with sufficiently low values of assets in place (left non-disclosure interval) raise capital and invest without issuing a report while firms with intermediate values of assets in place (right non-disclosure interval) do not raise capital because their costs from being pooled with lower types or from biasing their report would exceed the expected return on investment. Second, the model predicts that, if investors are risk-averse, firms that voluntarily disclose information prior to raising capital have lower cost of capital than firms that do not make such disclosure. The reason is that investors face greater uncertainty about the value of the firm's asset in place, and hence, investors require a higher expected return on their equity investment when the firm does not issue a report. Finally, the model predicts a negative association between firm performance and cost of capital. This association is driven by the fact that in equilibrium only firms with low values of assets in place raise capital without issuing a report while firms with higher values of assets in place always issue a report prior to raising capital.

In the equilibrium with bad news undisclosed, the firm is (weakly) more likely to raise capital and pursue the profitable investment opportunity than in the equilibrium discussed earlier. The fact that the underinvestment problem is less prevalent and the expected biasing costs are lower in the equilibrium with bad news undisclosed suggests that it is more efficient than the equilibrium with bad news disclosed. However, this conclusion is only valid if investors are risk-neutral. If investors are risk-averse they price the additional uncertainty they are exposed to when firms with relatively low values of assets in place raise capital without issuing a report. This may – depending on the relative risk-aversions of market participants – reduce social welfare to the extent that this equilibrium is less efficient than the equilibrium with bad news disclosed.

The remainder of the paper proceeds as follows. Section 2 provides a brief review of the literature. Section 3 outlines the setting of the model. Section 4 studies the equilibrium in which bad news are disclosed. Section 5 studies the equilibrium in which bad news remain undisclosed. Section 6 provides concluding remarks. All proofs are delegated to the Appendix.

2 Literature Review

Our paper studies a model that jointly considers a firm's voluntary disclosure strategy and the underinvestment problem in Myers and Majluf (1984). In Myers and Majluf (1984), a manager who acts on behalf of the existing shareholders decides whether to implement a new profitable investment opportunity. The firm does not have the internal funds, hence, it must raise equity capital in order to implement the investment. In particular, the manager decides whether to issue a fraction α of the firm's shares to outside investors in exchange for the required investment capital. The actual value of the shares offered to outside investors varies across firms because firms differ with respect to the value of their assets in place. Whether the manager chooses to issue equity depends on the value of the shares demanded by outside investors relative to the return the new investment opportunity is expected to generate. As a result, managers only issue shares if the value of those shares – or equivalently the value of his firm's assets in place – is relatively low. If the value of his firm's assets is higher the undervaluation of the shares he would have to issue in order to raise the required capital is too severe so that the manager prefers foregoing the investment opportunity instead. As a result, in Myers and Majluf (1984), the manager's equilibrium investment strategy is characterized by a threshold of values of assets in place up to which the manager pursues the investment opportunity and beyond which the manager foregoes it.

We extend the setting in Myers and Majluf (1984) to allow for communication between the manager and outside investors. In particular, we allow the manager to issue a (potentially biased) report prior to raising capital. Incorporating a disclosure decision into the Myers and Majluf-setting enables us to study real effects of voluntary disclosure. Prior literature on real effects of voluntary disclosure has focused almost exclusively on the decision of firms to disclose private information about market conditions to their competitors (e.g., Vives 1984, Darrough and Stoughton 1990, Wagenhofer 1990, Feltham and Xie 1992, Darrough 1993, Newman and Sansing 1993, Gigler 1994, Kanodia et al. 2000, Hughes et al. 2002, Fischer and Verrecchia 2004).[5] To the best of our knowledge, the only voluntary disclosure model that considers firms' investment decisions is Goex and Wagenhofer (2009) who derive the optimal disclosure rule firms commit to when they want to raise debt capital for investment purposes. Our model differs from Goex and Wagenhofer (2009) insofar as we do not assume that firms can credibly commit ex-ante to a certain disclosure policy.

Moreover, with the exception of Newman and Sansing (1993) and Gigler (1994) who both consider cheap-talk settings, none of the above referenced papers allows managers to bias their reports. In order to study the empirical phenomenon of bias in corporate disclosures,[6] we relax the assumption of truthful reporting and study managers' propensity to bias voluntary reports prior to

[5] For a survey of the literature on accounting disclosure and real effects see Kanodia (2006).
[6] For a review of the earnings management literature see Dechow, Ge and Schrand (2009).

raising equity capital. Since empirical evidence suggests that managers bias their reports to a lesser extent if monitoring mechanisms are more effective, we assume that biasing reports is costly to the manager.[7] Costly misreporting has been widely studied in mandatory disclosure models.[8] However, in voluntary disclosure settings, costly misreporting has gotten little attention in the literature. To the best of our knowledge, the only papers studying costly misreporting in a voluntary disclosure setting are Korn (2004) and a recent working paper by Einhorn and Ziv (2010). Korn (2004) and Einhorn and Ziv (2010) assume that the manager maximizes the firm's share price net of his costs from biasing the report and do not consider any real effects. Both papers predict that, in equilibrium, the manager issues a voluntary report when his private information is sufficiently favorable and does not make a disclosure otherwise. In that sense, these models yield predictions similar to voluntary disclosure models in which managers are assumed to report truthfully and disclosure costs are fixed and exogenous (e.g., Jovanovic 1982; Verrecchia 1983). Our model differs from Korn (2004) and Einhorn and Ziv (2010) in that we consider the manager's voluntary disclosure decision jointly with the firm's investment decision and find that the manager not only discloses sufficiently favorable news but may also disclose unfavorable news.[9] Our model provides a potential alternative explanation for the empirical finding that firms voluntarily disclose bad news (e.g., Skinner 1994, 1997; Aboody and Kasznik 2000) and illustrates that considering real effects significantly alters the predictions about management's equilibrium disclosure strategy. Overall, we believe that the paper contributes to the literature by characterizing the interdependencies between firms' joint decisions: (1) whether to disclose, (2) given disclosure, whether and to what extent to bias the report and (3) whether to raise equity capital for investment purposes.

[7] For instance, Ajinkya, Bhojrai and Sengupta (2005) find that managers issue less optimistic earnings forecasts in firms with more outside directors and greater institutional ownership – suggesting that more effective monitoring limits managers propensity to bias their reports.

[8] Models of costly reporting distortions include models of earnings management by Stein (1989); Fischer and Verrecchia (2000); Sankar and Subramanyam (2001); Dye and Sridhar (2004); Guttman, Kadan and Kandel (2006) and Beyer (2009).

[9] There exist some other models that predict the disclosure of bad news for different reasons. In Wagenhofer (1990), Feltham and Xie (1992), Fischer and Verrecchia (2004) and Suijs (2005), the manager discloses negative news in order to deter entry or to alleviate other unfavorable actions by competitors or regulators. In Goex and Wagenhofer (2009), the manager commits to disclosing negative news in order to support inferences in the case of non-disclosure which are sufficiently favorable such that investors provide debt capital. Finally, in Einhorn (2007), the manager discloses negative news when his reporting objective is to minimize the firm's stock price.

3 Model setup

This section describes a parsimonious model of investment and voluntary disclosure. We start with a brief outline of the sequence of events in the model.

There are three points in time. An individual (called the "manager" in what follows) owns a firm with assets in place and with a new investment opportunity that requires external financing of $\$I$.[10] The net return of this investment opportunity is the realization of a random variable $\tilde{r}$ with expected value $\mu_r > 0$. At $t = 1$, the manager privately learns the value of his firm's assets in place, x.[11] Then, at $t = 2$, the manager simultaneously decides whether to voluntarily issue a report on his firm's asset value and whether to raise equity capital from outside investors to finance the new investment opportunity. Both the current assets in place and the new project (if carried out) will generate their final cash flows at $t = 3$.

We next provide more detail on the preceding outline of the model.

The value of the firm's assets in place is a realization of the random variable $\tilde{x}$ which is distributed over $[0, \infty)$ according to the probability density function $f(x) > 0$ for all x.[12] We restrict x to non-negative values based on the rationale that the assets in place have an abandonment option. At $t = 1$, the manager privately learns the realization of the value of his firm's assets in place. (In the following, we sometimes refer to the value of the firm's assets in place as the manager's "type.")

At $t = 2$, the manager simultaneously decides whether to issue a report to investors and whether to raise capital in the equity market. The investors observe the manager's report, if one has been issued, prior to the opening of the equity market. If the manager decides to issue a report on his firm's asset value, $x_R \in [0, \infty)$, he is not confined to tell the truth and may bias his report. We denote the manager's reporting bias by $b(x) = x_R(x) - x$. If the report differs from the true value of the current assets in place, the manager incurs a personal cost of manipulating the report. This cost is increasing in the difference between the report and the true value of the firm's

[10] Equivalently, we may assume that rather than owning the firm, the manager is hired by the firm's current owners and that there are no moral hazard problems between the manager and the current owners.

[11] We consider asymmetric information with respect to the value of the firm's assets in place but not with respect to the expected return of the new investment opportunity. We make this assumption since information asymmetry with respect to the value of the firm's assets in place is necessary and sufficient to obtain the underinvestment problem described in Myers and Majluf (1984). While the underinvestment problem in Myers and Majluf (1984) is robust to the additional information asymmetry with respect to the return on investment, it would add significant complexity in our model. In order to maintain tractability, we therefore assume that the manager and outside investors are symmetrically informed about the expected return on investment.

[12] x does not have to refer to the actual future cash flows that the firm's assets in place will generate but may rather denote the expected value of the firm's assets in place conditional on the manager's private information.

assets in place, x.[13] In particular, we assume that the manager incurs the cost $g(x_R - x)$ where the cost function $g(\cdot)$ is a well behaved U-shaped function, i.e., it is convex with $g(0) = 0$ and $g'(0) = 0$. The manager may raise equity capital whether or not he issues a report x_R. If the manager decides to raise equity capital, he offers a fraction α of the firm's ownership to investors in return for their investment of capital I with the firm. Investors may accept or reject the offer. We assume that investors are risk-neutral and that they accept the offer when they break even on average.[14] If investors accept the offer they contribute capital I and the manager pursues the investment opportunity. If investors do not contribute capital, the manager lacks the necessary funds to pursue the investment opportunity.

At $t = 3$, the firm's final cash flows are realized. If the manager did not pursue the investment opportunity, the firm's final cash flows equal x and the manager retains all of it. If the manager raised capital and pursued the investment opportunity, the firm's final cash flows are $x + I + r$, the manager retains fraction $(1 - \alpha)$ and investors are paid fraction α of the final cash flows.

In equilibrium, the manager simultaneously decides whether to issue a report, if so to what extent to bias the report, and whether to raise capital. If the manager decides to raise capital, he rationally anticipates investors' response and chooses the fraction α of the firm's ownership he offers to investors in exchange for their investment such that investors break even on average. The fraction α is determined by

$$I = \alpha \left(E\left[\widetilde{x} | \Omega\right] + I + \mu_r \right).$$

where Ω denotes the public information that is available to investors at $t = 2$. All parameters of the model, i.e., the required capital I, the expected return on investment μ_r, the cost function $g(\cdot)$ and the prior distribution of value of assets in place $f(\cdot)$ are common knowledge. Figure 1 summarizes the sequence of events of the model.

In the model, the manager jointly considers his disclosure and investment decisions. The reason is that the manager's disclosure decision depends on his investment decision and vice versa. On the one hand, voluntary disclosure can only be beneficial to the manager if he decides to raise capital. In the absence of equity issuance, outside investors' perception of the firm value is irrelevant and, hence, the manager cannot benefit from influencing investors' beliefs. On the other hand, the

[13] This is a standard assumption in the costly state falsification literature (e.g., Riley 1979; Lacker and Weinberg 1989; Stein 1989; Fischer and Verrecchia 2000; Guttman, Kadan, and Kandel 2006; Beyer 2009).

[14] We assume that investors are risk-neutral for simplicity only. If investors were risk-averse the model's predictions would remain qualitatively the same. In Corollaries 3 and 4, we discuss the implications of investors being risk-averse.

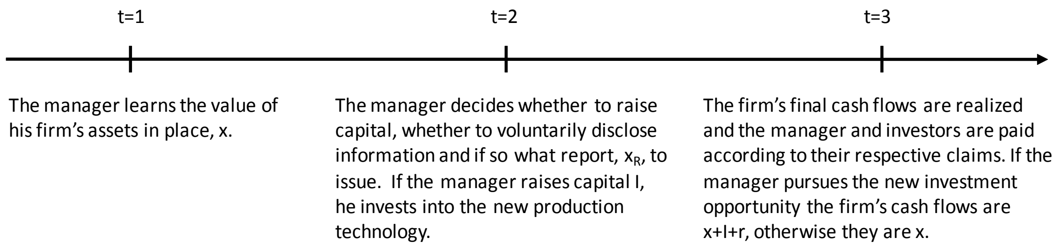

Figure 1: Timeline

profitability of the new investment opportunity to the manager depends on his voluntary disclosure decision, even though the expected profitability of the new investment opportunity is common knowledge. The reason is that the manager's report, or its absence, affects investors' perception of the value of the firm's assets in place and determines the fraction of shares the manager needs to give to investors in return for the capital they provide. Because of these interdependencies, it is essential to jointly consider firms' voluntary disclosure decisions and investment decisions. The following analysis studies these interdependencies and illustrates how the manager's decisions whether to issue a voluntary report – and if so to what extent to bias the report – affect and are affected by the manager's investment decision.

4 Equilibrium with bad news disclosed

In the model, a manager has to simultaneously decide whether to raise capital, whether to issue a report, and if so to what extent to bias the report. Before analyzing the manager's joint investment and disclosure decision, we focus on the manager's decision whether and to what extent to bias his report.

4.1 Disclosure bias

In this section, we study the extent to which the manager biases his report under the assumption that he always issues a report, raises capital and invests. As it is often the case in disclosure games with continuous support, multiple equilibria with pooling reports may evolve (e.g., Guttman et al. 2006). In line with the literature, we limit our analysis to a subset of equilibria in which the manager's report allows investors to perfectly infer the manager's private information.

The report issued by the manager affects his payoff in two ways. On the one hand, the report affects investors' beliefs about the value of the firm's assets in place, which determine the fraction α of equity investors require in exchange for providing capital I. On the other hand, the manager incurs disclosure costs whenever his report differs from the true value of assets in place. The manager's biasing costs are increasing in the magnitude of the bias. The trade-off between these two factors is reflected in the first order condition of the manager's optimization problem and determines the extent to which the manager biases his report. The following Lemma characterizes the manager's equilibrium bias strategy, $b(x)$.

Lemma 1 *The equilibrium bias in the manager's report is given by the solution to the differential equation*

$$b'(x) = \frac{I}{g'(b(x))(x + I + \mu_r)} - 1, \tag{1}$$

with the boundary condition $b(0) = 0$.

The equilibrium bias $b(x)$ *has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum and converges to zero as the value of the firm's assets in place goes to infinity.*

Figure 2 illustrates the equilibrium bias, $b(x)$, and the manager's equilibrium report, $x_R(x) = x + b(x)$. The figure is based on a quadratic cost function, $g(b) = \frac{1}{2}b^2$, and the parameter values $I = 1$ and $\mu_r = 0.25$.

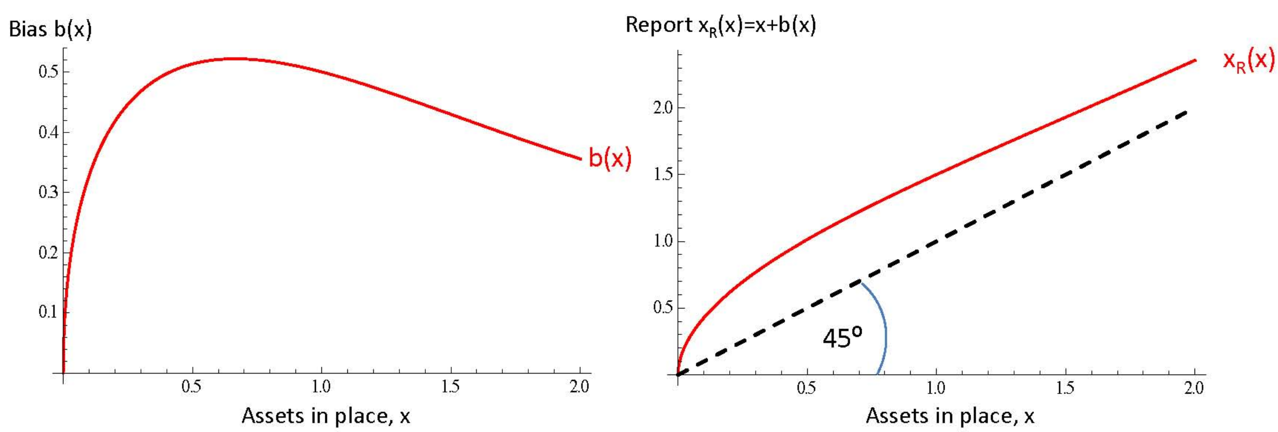

Figure 2: Bias function $b(x)$ and reporting function $x_R(x)$

In equilibrium, the manager biases his report upwards when making a disclosure. The reason is that investors associate higher reports with the firm's assets in place being more valuable. When investors perceive the firm's assets in place to be more valuable, investors require a smaller fraction of the firm's equity in exchange for investing capital I. In turn, the manager can keep a larger fraction of the firm's equity to himself. The extent to which the manager biases his report upwards depends on the benefits and costs associated with reporting a higher value for the assets in place. Three observations jointly explain the shape of the equilibrium bias function as shown in Figure 2.

First, a manager whose firm's assets in place are worth zero does not bias his report. This is intuitive since in equilibrium investors identify him as the lowest type and he is therefore not willing to bear any signaling costs.

Second, the manager's benefit from investors perceiving his firm's assets in place to be marginally more valuable than in fact they are depends on the actual value of those assets. When the value of the firm's assets in place is lower, the manager benefits more from mimicking a firm whose assets in place are marginally more valuable. The reason is that the effect of making investors believe that the firm's assets in place are marginally more valuable is greater when the "size of the pie" is smaller. To illustrate this further, we consider a manager whose firm's assets in place are worth x and who issues a report such that investors perceive the value of his firm's assets in place to be $\hat{x}$. If investors perceive the value of the firm's assets in place to be $\hat{x}$ they require $\alpha = \frac{I}{\hat{x}+I+\mu_r}$ shares in exchange for providing capital I. Hence, the actual value of the shares investors obtain is $I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r}$. Since investors provide capital I in exchange for those shares, investors overpay by $I - I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r} = I\frac{\hat{x}-x}{\hat{x}+I+\mu_r}$.[15] In turn, the manager's benefit from issuing a report that mimics a firm with assets worth $\hat{x}$ is also $I - I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r}$. The manager's benefit from inducing investors to believe that the firm's assets in place are worth marginally more than in fact they are is

$$\lim_{\hat{x}\to x} \frac{\partial}{\partial \hat{x}} \left(I - I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r} \right) = \frac{I}{x+I+\mu_r},$$

which decreases in the actual value of the firm's assets in place, x. Hence, the manager benefits less from mimicking a firm whose assets are marginally more valuable when the actual value of his firm's assets in place is higher.

Third, the costs that the manager incurs from marginally increasing investors' beliefs about the value of the firm's assets in place depends on (i) the overall magnitude of the bias (due to the convexity of the cost function $g(b)$) and (ii) the sensitivity of investors' inferences to changes in the manager's report (which determines the additional bias necessary in order to make investors believe that the value of his firm's assets in place is marginally higher). Letting $\hat{x}$ once again

[15] See also Footnote 4 in which ε denotes the difference beteween $\hat{x}$ and x.

denote investors' perception of the value of the firm's assets in place, $\frac{\partial x_R}{\partial \hat{x}}$ measures the additional bias necessary to marginally increase investors' beliefs about the value of the firm's assets in place and $\frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x)) \frac{\partial x_R}{\partial \hat{x}}$ measures the additional costs the manager incurs from marginally increasing investors' beliefs. In equilibrium, investors' inferences have to be consistent with the manager's reporting strategy, i.e., $\frac{\partial x_R}{\partial \hat{x}} = \frac{\partial x_R}{\partial x}$ or $\frac{\partial x_R}{\partial \hat{x}} = 1 + b'(x)$. So, in equilibrium the manager's cost of marginally increasing investors' beliefs is $\frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x))(1 + b'(x))$.[16]

These three observations jointly explain the shape of the equilibrium bias function: Initially, the bias is zero due to the fact that a manager with assets in place worth zero does not bias his report. Managers with assets slightly more valuable are willing to incur signaling costs and therefore bias their reports upwards. Since the bias is still relatively small, the marginal costs of biasing the report are also relatively small. In equilibrium, the manager's marginal costs of biasing his report must equal his marginal benefit from biasing his report. This implies that the marginal benefit of a manager with low asset values must also be relatively small. Since the marginal benefit of a manager with low asset values from increasing investors' beliefs is high, it must be that investors' beliefs are relatively insensitive to the manager's report. This is the case when investors attribute most of an increase in the report to an increase in the manager's bias and only a small part to an increase in the asset value. As a result, the bias function increases at a high rate when asset values are relatively low. As the bias continues to increase, the marginal costs of biasing the report increase as well. At the same time, the manager's marginal benefit from increasing investors' beliefs decreases. In order for the marginal costs to equal the marginal benefit from biasing the report, investors' beliefs must become more sensitive to the report. This implies that the rate of increase in the manager's bias must decrease. Since the marginal benefit from increasing investors' beliefs eventually approaches zero, the equilibrium bias decreases once assets in place reach a certain value.[17] In the limit, when the value of the firm's assets in place is very high the manager's benefit

[16] Note that these three observations are equivalent to Lemma 1. The first observation provides the boundary condition $b(0) = 0$. The second observation gives the marginal benefit from inducing investors to believe that the value of the firm's assets in place are worth marginally more than in fact they are, $\frac{I}{x+I+\mu_r}$, while the third observation provides the marginal costs, $\frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x))(1 + b'(x))$. Equating the marginal benefit and costs and rearranging terms yields the differential equation in Lemma 1.

[17] Assume that at $x = x^*$ the manager's bias is (weakly) decreasing. To see that for any $x > x^*$ the manager's bias is monotonically decreasing, suppose to the contrary that at some $x' > x^*$ the bias function starts to increase in x. The fact that the bias function starts to increase at x' has two implications. First, the manager's marginal cost from biasing the report is increasing in x at x'. Second, the bias function is increasing and convex in x at x', which implies that the sensitivity of investors' beliefs to the report decreases at x'. The decreased sensitivity of investors' beliefs combined with the fact that the benefit from marginally increasing investors' belief is decreasing in x implies that the manager's marginal benefit from increasing his bias is decreasing at x'. This leads to a contradiction, since in equilibrium the manager's marginal benefit from increasing the bias in his report must equal his marginal costs

from changing investors' beliefs goes to zero and therefore the manager biases his report upward by a vanishing amount, i.e., $\lim_{x\to\infty} b(x) = 0$.

When we characterize the manager's equilibrium disclosure and investment strategy in the following section, it will prove useful to establish how the bias function $b(x)$ characterized in Lemma 1 varies with the expected profitability of the investment opportunity μ_r.

Lemma 2 *The bias function $b(x)$ characterized in Lemma 1 is decreasing in the expected profitability of the investment opportunity, μ_r. That is, $\frac{\partial b(x)}{\partial \mu_r} < 0$ for all $x > 0$.*

Lemma 2 establishes that the effect of μ_r on the equilibrium bias $b(x)$ is such that higher expected return on investment cause the equilibrium bias to be lower for any given $x > 0$. The intuition is that as μ_r – which is common across all firms – increases, the difference in value of assets in place across firms becomes relatively less important. As a result, the manager is less willing to bear signaling costs and biases his report to a lesser extent in equilibrium.

The equilibrium bias described in Lemma 1 shows that – as standard in costly signaling settings – truth-telling is not an equilibrium and the manager ends up paying signaling costs even though he does not mislead investors in equilibrium. As a result, the manager always bears some costs when he makes a disclosure (except when his firm's asset in place are worth zero). The signaling costs the manager incurs differ from the signaling costs in standard signaling settings in which the sender (manager) maximizes his perceived type net of his signaling costs (e.g., Riley 1979, Miller and Rock 1985). In the standard signaling models, which consider only the disclosure decision of the manager, the marginal benefit of the manager from increasing investors' beliefs about his type is assumed to be constant. This property combined with a convex cost function yields increasing signaling costs that converge to a finite upper bound as the sender's type goes to infinity. Our model studies a different setting in which a manager that considers raising capital in order to finance an investment opportunity makes an investment decision in addition to the disclosure decision. The differences in the setting give rise to a qualitatively different disclosure behavior. This illustrates that modeling specific signaling settings and incorporating institutional details into the model can alter predictions about equilibrium properties of the sender's message and signaling costs in a qualitative sense.

from doing so for any x.

4.2 Joint investment and disclosure decision

The previous section characterized the properties of the manager's bias and resulting endogenous disclosure costs under the assumption that he issues a report prior to raising equity capital. However, the manager is not required to issue a report. Since the sole purpose of making a disclosure is to convince outside investors of the value of the firm's assets in place, the manager would prefer not to issue a report if the biasing costs outweigh the expected return the manager earns from the investment. That is, the manager is better off withholding his information and foregoing the investment opportunity if the bias is such that it were to impose costs of $g(b(x))$ that exceed the expected investment return, μ_r.

The previous section established that the bias and, as a result, the biasing costs are low for both small and large values of assets in place (see Figure 2). Hence, no disclosure may occur only for intermediate values of assets in place: either there exists an equilibrium in which the manager issues a report for all realizations of values of assets in place or there exists an equilibrium in which the manager issues a report for low and high values of assets in place but not for intermediate values of assets in place. Which form the equilibrium takes depends on the expected return on the investment opportunity, μ_r. For more profitable investment opportunities (i.e., for higher values of μ_r), the manager is willing to bear higher biasing costs in order to be able to realize the expected return on investment. The manager's willingness to bear higher biasing costs is reflected in a higher threshold of bias, $g^{-1}(\mu_r)$, up to which the manager is willing to bias his report in order to communicate the value of the assets in place to investors. This together with the fact that the bias function $b(x)$ is lower for higher values of μ_r (see Lemma 2) implies that the manager is less likely to withhold information when the expected return on investment is higher.

Proposition 1 formalizes the manager's equilibrium disclosure and investment strategy.

Proposition 1 *There exists an equilibrium which is characterized as follows.*

(*i*) *For $\mu_r \geq \mu_r^*$ the manager issues a report, raises capital and invests for all $x \geq 0$ ("Full disclosure and full investment" in Figure 3).*[18]

(*ii*) *For $\mu_r < \mu_r^*$ there exist two thresholds x_1^D and x_2^D which are uniquely defined by $b\left(x_1^D\right) = b\left(x_2^D\right) = g^{-1}(\mu_r)$ and $0 < x_1^D < x_2^D$. In equilibrium,*

[18] $\mu_r^* = g(b(x^*))$ where x^* is the value of assets in place for which $b'(x^*) = 0$ and $b(\cdot)$ is given in Lemma 1. That is, μ_r^* is the expected return of the investment opportunity for which the bias function in Lemma 1 is tangential to the horizontal line $g^{-1}(\mu_r^*)$.

the manager issues a report, raises capital and invests for $x \in [0, x_1^D] \cup [x_2^D, \infty)$*; and does not issue a report, does not raise capital and does not invest for* $x \in \left(x_1^D, x_2^D\right)$ *("Non-disclosure of intermediate news and partial investment" in Figure 4).*

When the manager issues a report he biases it according to Lemma 1*. Investors' off-equilibrium beliefs are as follows. If they observe an off-equilibrium report, investors believe that the manager biased his report according to Lemma* 1*. If they observe the manager raising capital without issuing a report, investors believe that the value of the firm's assets in place is zero.*

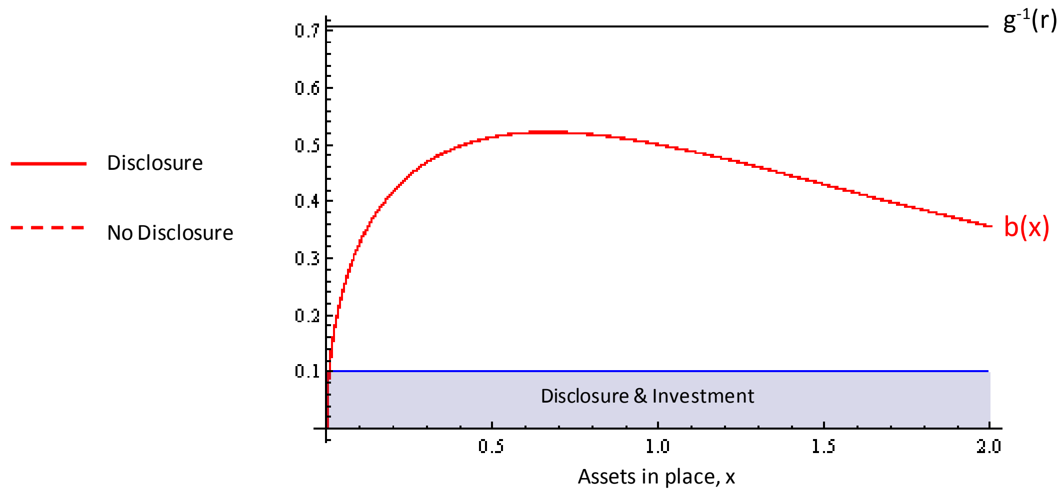

Figure 3: Disclosure and investment strategy for highly profitable investments ($\mu_r = 0.25$): Full disclosure and full investment

Part (i) of Proposition 1 establishes that if the expected return on investment is sufficiently high the manager always issues a report and pursues the investment opportunity. The intuition is as follows. For sufficiently high expected return on investment, the expected return always exceeds the cost of biasing the report, i.e., $\mu_r > g(b(x))$ for all values of assets in place, x. Hence, the manager prefers to issue a report and invest over forgoing the investment opportunity. However, the manager might still prefer to raise capital without first issuing a report. The off-equilibrium beliefs in Proposition 1 guarantee that even for low values of assets in place the manager is better

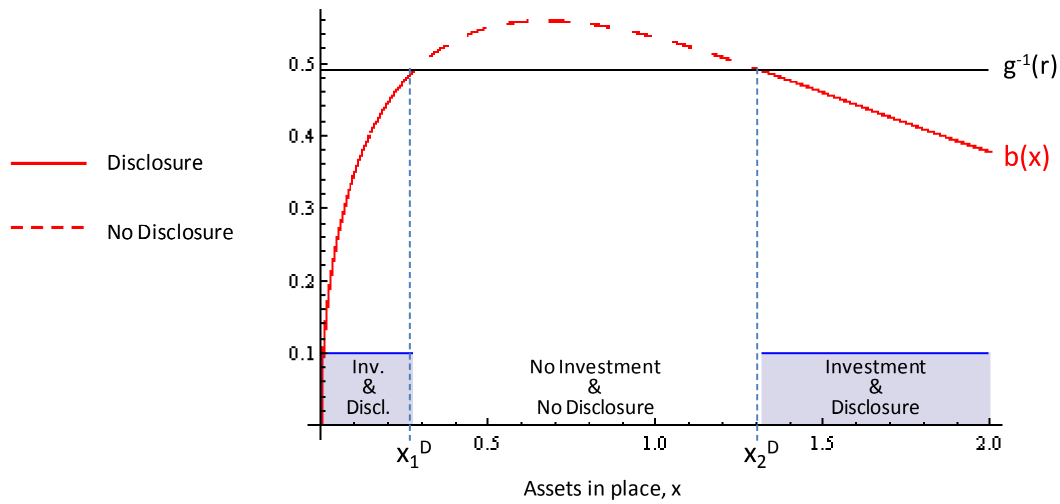

Figure 4: Disclosure and investment strategy for less profitable investments ($\mu_r = 0.12$): non-disclosure of intermediate news and partial investment

off pursuing the investment opportunity with, rather than without, issuing a report.[19] [20]

For the parameter values and cost function in Figure 2 ($\mu_r = 0.25$, $I = 1$, $g(b) = \frac{1}{2}b^2$), the manager optimally issues a report as long as the bias is less than $g^{-1}(\mu_r) = \sqrt{2\mu_r} = 0.71$. For these parameter values, it turns out that the maximum bias is 0.52 (for $x = 0.67$) and therefore full disclosure constitutes an equilibrium as characterized in part (i) of Proposition 1 and illustrated in Figure 3.

When the investment is less profitable, the biasing costs exceed the expected return on investment for intermediate asset values. For instance, in Figure 4 where $\mu_r = 0.12$, the bias exceeds $g^{-1}(\mu_r) = \sqrt{2\mu_r} = 0.49$ for all intermediate asset values $x \in (0.28, 1.32)$. As a result, the manager does not make a disclosure if the value of his firm's assets in place falls into the intermediate range while the manager makes a disclosure when the asset value is either lower or higher. The prediction that the manager discloses unfavorable news but withholds intermediate news, even though disclosure is costly in equilibrium, differs from the prediction of models that focus exclusively on firms' voluntary disclosure decisions and predict that firms disclose information only if it is sufficiently

[19] To see that these off-equilibrium beliefs are necessary for the equilibrium to exist, suppose investors' off-equilibrium beliefs are such that they infer the firm's assets to be worth $x' > 0$ if the manager raises capital without making a disclosure. Then, a manager with assets worth less than x' can not only save on the disclosure costs by raising capital without making a disclosure but his firm's assets will also be perceived as more valuable than in fact they are. As a result, the manager would prefer to deviate and raise capital without making a disclosure.

[20] In section 5, we study an equilibrium in which raising capital without issuing a report is part of the manager's equilibrium strategy.

favorable (see Section 2 for review of the literature).

Part (ii) of Proposition 1 establishes that for the intermediate values of the firm's assets in place, for which the manager does not voluntarily disclose information, he also does not invest. At first, this behavior might appear suboptimal. The reason is that firms forego expected return μ_r when they do not pursue the investment opportunity. However, if a subset of firms with asset values between x_1^D and x_2^D were to raise capital and invest without disclosing information, then all firms with $x < x_1^D$ would prefer to mimic them by also raising capital without disclosing information. Hence, in equilibrium, all firms with asset values between x_1^D and x_2^D do not raise capital and investors' off-equilibrium beliefs are such that they infer that the firm is of the lowest type if it attempts to raise capital without disclosing information. The boundaries of the non-disclosure interval $\left(x_1^D, x_2^D\right)$ are uniquely defined by the cost of disclosure being such that they are exactly offset by the expected return on investment, i.e., $g\left(b\left(x_1^D\right)\right) = g\left(b\left(x_2^D\right)\right) = \mu_r$.[21]

Proposition 1 shows that inefficient investment behavior occurs when the investment opportunity is not too profitable. In contrast to Myers and Majluf (1984), it is the firms with intermediate asset values that forego the profitable investment opportunity and not the firms with high asset values. The reason is that for firms with high values of assets in place disclosure costs are relatively low such that they are outweighed by the expected return on investment. While the investment behavior characterized in Proposition 1 differs from the investment behavior in Myers and Majluf (1984), it is still the case that the underinvestment problem is less prevalent, in the sense that the manager foregoes the profitable investment opportunity less often, when the expected return on investment is higher. This is intuitive. As the expected return on investment increases, the manager is willing to incur higher costs (in the form of disclosure costs in this model or in the form of price discounts due to pooling in Myers and Majluf 1984) in order to raise capital and invest. The following Corollary formalizes this observation.

[21] In the equilibrium of Proposition 1, investors believe that the manger biases his report according to Lemma 1 if the manger issues an off-equilibrium report. There exist other equilibria that rely on different off-equilibrium beliefs. All these equilibria share the same qualitative characteristics as the equilibrium described in part (ii) of Proposition 1. In particular, the non-disclosure and non-investment interval $\left(x_1^D, x_2^D\right)$ will be characterized by the same lower bound x_1^D but a different upper bound, $x_2^{D\prime}$, which might be either higher or lower than x_2^D but which is always substantially greater than x_1^D, i.e., $x_2^{D\prime} >> x_1^D$. The minimum size of the non-disclosure and non-investment interval is given by $\left(x_1^D, x_2^{D\prime}\right)$ where $x_2^{D\prime}$ is the solution to

$$\left\{b'(x) = \frac{I}{g'(b(x))(x + I + \mu_r)} - 1,\ b'\left(x_2^{D\prime}\right) = 0,\ b\left(x_2^{D\prime}\right) = g^{-1}(\mu_r)\right\}.$$

Moreover, it can be shown that in any equilibrium in which the bias strategy is given by Lemma 1, there do not exist any equilibria in which there are more than two non-disclosure intervals (for a proof, see p. 40).

Corollary 1 *For $\mu_r < \mu_r^*$, x_1^D is increasing in μ_r and x_2^D is decreasing in μ_r. Hence, in the equilibrium characterized in Proposition 1, the underinvestment problem is less prevalent, in the sense that the probability of the manager raising capital and investing is higher, for higher expected returns on investment.*

5 Equilibrium with bad news undisclosed

In the previous section, we characterized an equilibrium in which the manager voluntarily discloses bad news. In that equilibrium, the manager raises capital and invests only when he voluntarily issues a report. In addition to the equilibrium discussed in the previous section, there exists an additional equilibrium. This additional equilibrium is similar to the one described in the previous section, except that firms with relatively low values of assets in place raise capital without issuing a voluntary report. These firms "save" the biasing costs but may be pooled with firms whose assets in place are even less valuable and, hence, may have to give up a larger ownership fraction in order to raise the necessary capital. This equilibrium, which we refer to as "equilibrium with bad news undisclosed," gives rise to several additional predictions.

The difference between the manager's disclosure strategy in the two equilibria implies that the firm is (weakly) more likely to raise capital and pursue the profitable investment opportunity in the equilibrium with bad news undisclosed than in the equilibrium in which bad news are disclosed (equilibrium in Proposition 1). Moreover, in the equilibrium with bad news undisclosed, the expected biasing costs are also lower. This suggests that the equilibrium with bad news undisclosed is more efficient than the equilibrium with bad news disclosed. However, this conclusion is not necessarily valid if investors are risk-averse. Risk-averse investors price the additional uncertainty they are exposed to when firms raise capital without issuing a report. This may – depending on the relative risk-aversions of market participants – reduce social welfare to the extent that this equilibrium is less efficient than the equilibrium with bad news disclosed. As a result, the two equilibria can not necessarily be ranked in terms of their efficiency.

We discuss this equilibrium and its predictions in Section 5.2. In Section 5.1, as a preliminary analysis, we study the manager's decision to raise capital and invest taking the manager's disclosure strategy as given.

5.1 Investment decision

In this section, we discuss the manager's investment decision while taking his disclosure strategy as given. In the equilibria we study, investors are able to infer the value of the firm's assets in place when the manager voluntarily discloses information. In such equilibria, a manager that voluntarily issues a report always raises capital and invests. However, for some values of assets in place the manager may choose not to issue a voluntary report. We denote the set of values of assets in place for which the manager does not issue a voluntary report by X_{nd}. In this section, we take the set of values of assets in place for which the manager does not issue a voluntary report as given and solve for the manager's investment strategy when he remains silent. This analysis is similar to the analysis in Myers and Majluf (1984).

When the manager does not disclose information, the information asymmetry between the manager and investors is not fully resolved. While the manager knows the precise value of the firm's assets in place, investors can only make inferences about the average value of the assets in place for which the manager does not issue a report. Based on their inferences, investors will require a share α_{nd} of the firm's equity in return for providing capital I when the firm raises capital without issuing a report. In equilibrium, the equity share α_{nd} will guarantee that the investors break even on average.

Although investors on average correctly value the equity capital they invest in, they overvalue some and undervalue other issues of equity capital. When investors undervalue the equity issued in exchange for their investment, the firm's manager must give up a larger fraction of equity ownership to new investors than seems necessary based on the manager's private information. As a result, when the actual value of the firm's assets in place is sufficiently higher than investors' inferences, the manager chooses not to raise capital even tough this means foregoing the profitable investment opportunity. Hence, whenever the value of the firm's asset in place exceeds a certain threshold, the manager will prefer passing up on the profitable investment opportunity over being pooled with firms that also invest without disclosing information but whose assets in place are worth less. We let x_I denote this threshold.[22] Of course, investors anticipate that a manager will only raise equity capital when the firm's assets in place are not particularly valuable and price new equity issues (as reflected in α_{nd}) accordingly. In equilibrium, the manager's investment and disclosure strategy

[22] x_I does not necessarily have to be in the set X_{nd}. If $x_I > x$ for all $x \in X_{nd}$ then all firms that remain silent raise capital and invest. If $x_I < x$ for all $x \in X_{nd}$ then no firm that remains silent raises capital and invests.

and the share α_{nd} investors demand in exchange for their investment I when the manager does not issue a voluntary report satisfy the following condition:

$$I = \alpha_{nd} E\left[\widetilde{x} + I + \mu_r | \widetilde{x} < x_I, x \in X_{nd}\right]. \tag{2}$$

In equilibrium, a manager whose assets in place are worth x_I has to be indifferent between investing without disclosing information and his next best option. When the manager invests without disclosing information, he retains an equity stake worth $(1 - \alpha_{nd})(x_I + I + \mu_r)$. The manager's next best option is either to voluntarily disclose information and invest, yielding an expected payoff of $x_I + \mu_r - g(b(x_I))$, or not to raise capital and forego the profitable investment opportunity, in which case the manager simply keeps x_I. In equilibrium x_I must satisfy

$$(1 - \alpha_{nd})(x_I + I + \mu_r) = \max\{x_I + \mu_r - g(b(x_I)), x_I\} \tag{3}$$

and only firms whose assets in place are worth less than x_I will raise capital and invest if they choose not to issue a report. Lemma (3) summarizes firms' investment behavior for a given disclosure strategy.

Lemma 3

(a) *In equilibrium, if a firm issues a report $x_R(x)$ it also raises capital and invests. The manager's expected payoff is $x + \mu_r - g(b(x))$.*

(b) *For any set of firms, X_{nd}, for which investors anticipate that the manager does not issue a report, there exists $\alpha_{nd} \in (0,1)$ and $x_I \in (0,\infty)$ that jointly satisfy equations (2) and (3). In equilibrium,*

- *for $x \geq x_I$, $x \in X_{nd}$ the manager does not raise capital and consequently does not invest. The manager's payoff is x.*
- *for $x < x_I$, $x \in X_{nd}$ the manager raises capital and invests. The manager's payoff is $(1 - \alpha_{nd})(x + I + \mu_r)$.*

Equation (2) illustrates that the fraction of equity investors demand in return for investing capital in the amount of I depends on their beliefs about the value of the firm's assets in place for which the manager does not issue a report but issues equity to raise capital. If the intersection of $\{x|x < x_I\}$ and X_{nd} is empty, we assume that investors believe that a firm that raises capital without voluntarily disclosing information has the lowest possible value of assets in place, i.e., $x = 0$. This scenario was analyzed in Section 4.2. When the intersection of $\{x|x < x_I\}$ and X_{nd} is non-empty, i.e., when the manager as part of his equilibrium strategy sometimes raises capital without making a disclosure, investors update their beliefs following Bayes' Rule. The following section analyzes the manager's equilibrium disclosure and investment strategy in this case.

5.2 Joint investment and disclosure decision

Since the manager makes both an investment and a disclosure decision, he can implement four qualitatively distinct strategies, depending on whether he discloses and whether he invests. As we've argued before, the manager never discloses without raising equity capital because disclosure is (weakly) costly and the sole purpose of making a disclosure is to gain access to cheaper capital by convincing investors of the value of the firm's assets in place. Hence, for any value of the assets in place the manager pursues one of the following three options: disclose/invest; not disclose/invest; not disclose/not invest. Before we characterize the manager's equilibrium strategy, as a preliminary step Lemma 4 shows that if it is optimal for the manager to raise capital and invest without making a disclosure when his assets are worth x' then it is also optimal for him to raise capital and invest without making a disclosure when his assets are worth less than x'.

Lemma 4 *In equilibrium, if the manager invests without issuing a report when the firm's assets in place are worth x' then the manager also invests without issuing a report when the firm's assets in place are worth $x < x'$.*

Recall that x_I denotes the highest value of assets in place for which the manager prefers investing without issuing a report in equilibrium. Lemma 4 establishes that if $x_I > 0$ the manager prefers raising capital without making a disclosure for all $x < x_I$. The intuition is as follows. In equilibrium, investors rationally infer the average value of assets in place for which the manager raises capital without issuing a report. While they price the firm's assets correctly on average, investors overvalue assets that are worth little and undervalue assets that are worth more. When his assets are undervalued, it is conceivable that, rather than raising capital without issuing a report, the manager may prefer to either (i) forego the profitable investment opportunity or (ii) to issue a (costly) report that allows investors to infer the actual value of his firm's assets. However, Lemma 4 establishes that this is never the case when the manager prefers raising capital without issuing a report for some higher values of assets in place. The footnote below provides the intuition for this result.[23]

What Lemma 4 does not specify is the highest asset value for which the manager prefers to raise capital without making a disclosure, i.e., x_I. Since, by definition, x_I is the highest value of assets in place for which the manager prefers to raise capital without making a disclosure, the firm's assets will be undervalued by investors when $x = x_I$. Depending on the extent of the undervaluation, the manager is willing to incur disclosure costs in order to communicate the true value of his firm's assets in place to investors. For any x, we let $b_I(x)$ denote the maximum bias a manager whose firm's assets in place are worth x is willing to pay for in order to separate himself from firms with lower asset values that also raise capital without disclosing information. The function $b_I(x)$ will prove useful in determining x_I. The following provides a formal definition of the function $b_I(x)$.

Definition 1 *$b_I(x)$ denotes the bias that renders a manger whose assets in place are worth x indifferent between the following:* (1) *issuing a report with a bias $b_I(x)$ which reveals the value of his firm's assets in place and pursuing the investment opportunity; and* (2) *not issuing a report but investing when investors expect the firm's assets in place to be worth $E(\tilde{x}|\tilde{x} < x)$. Formally, $b_I(x)$ is defined by*

$$\left(1 - \frac{I}{x + I + \mu_r}\right)(x + I + \mu_r) - g(b_I(x)) = \left(1 - \frac{I}{E(\tilde{x}|\tilde{x} < x) + I + \mu_r}\right)(x + I + \mu_r)$$

In equilibrium, if the value of the assets in place is x_I the manager must be indifferent between raising capital without disclosing information and the next best option. The next best option is either to issue a report $x_R(x_I)$, which provides the manager with an expected payoff of $x_I + \mu_r - g(b(x_I))$, or not to disclose and forego the investment opportunity, which provides the manager

[23] It turns out that the manager never prefers option (i), i.e., he never prefers to forego the profitable investment opportunity for $x < x_I$. The reason is as follows. When the manager raises capital without issuing a report, the undervaluation of the firm when its assets are worth x is less than when its assets are worth x_I. Since the expected return on investment is sufficient to compensate the manager for the undervaluation when the assets are worth x_I it is more than sufficient if the assets are worth $x < x_I$. In addition, the manager never prefers option (ii), i.e., he never prefers to issue a report for any $x < x_I$ even though he would avoid the discount from being pooled with less valuable asset realizations. To see that, suppose to the contrary that the highest asset value (less than x_I) for which the manager preferred to issue a report were $x' < x_I$. Then, for asset values worth slightly more than x' the manager would prefer to issue the same report $x_R(x')$ over raising capital without making a disclosure – contradicting the assumption that x' is the highest asset value for which the manager prefers to issue a report. The intuition is as follows. Suppose the manager preferred to issue a report $x_R(x')$ when the assets are worth x' then his benefit from retaining a larger fraction of the firm must outweigh his disclosure costs. The costs of issuing a report $x_R(x')$ would be lower for asset values slightly higher than x' than they are for x' because the manager would have to bias his report upwards by less if his assets are worth slightly more. Moreover, the manager would benefit more from retaining a larger fraction of the ownership (as a result of reporting $x_R(x')$ rather than not issuing any report) when his firm's assets in place are worth more. Hence, for asset values slightly higher than x' the manager would strictly prefer to issue the report $x_R(x')$ over raising capital without issuing a report – contradicting the assumption that x' is the highest asset value for which the manager prefers to issue a report.

with a payoff of x_I. Hence, in equilibrium the maximum costs that the manager is willing to bear in order to separate himself from managers with lower realization of asset values is $g(b(x_I))$ or μ_r, whichever is lower. In other words, in equilibrium, $b_I(x_I)$ has to equal the lower of the equilibrium bias $b(x_I)$ and $g^{-1}(\mu_r)$. This guarantees that the manager is indifferent between raising capital without disclosing information and the next best option for $x = x_I$, and strictly prefers to raise capital without disclosing information for $x < x_I$.

So far, we characterized the equilibrium behavior when the value of assets in place are sufficiently low such that the manager raises capital without issuing a report. In order to fully characterize the equilibrium, we still need to characterize the manager's equilibrium strategy for higher values of assets in place. The analysis of the equilibrium strategy for such higher asset values follows closely the analysis of the equilibrium in Section 4.2. Using the definition of $b_I(x)$, Proposition 2 provides a full characterization of the equilibrium in which for sufficiently low values of assets in place ($x < x_I$) the manager raises capital without making a disclosure.

Proposition 2 *There exists an equilibrium which is characterized as follows. For $x \in [0, x_I)$, where the threshold $x_I > 0$ is uniquely defined by $x_I = \min\{x | b_I(x) = \min\{g^{-1}(\mu_r), b(x)\}\}$, the manager raises capital without issuing a report and invests. For $x \in [x_I, \infty)$, the manager's equilibrium strategy is given by one of the following:*

(i) For $\mu_r \geq \mu_r^*$ the manager issues a report, raises capital and invests[24]

("Non-disclosure of bad news and full investment" in Figure 5).

(ii) For $\mu_r < \mu_r^*$

a. if $x_2^D < x_I$ the manager issues a report, raises capital and invests

("Non-disclosure of bad news and full investment" in Figure 6);

b. if $x_1^D < x_I < x_2^D$ the manager does not issue a report, does not raise capital and does not invest for $x \in [x_I, x_2^D)$; and issues a report, raises capital and invests for $x \in [x_2^D, \infty)$

("Non-disclosure of bad news and partial investment" in Figure 7);

c. if $x_I < x_1^D$ the manager issues a report, raises capital and invests for $x \in \left[x_I, x_1^D\right]$; does not issue a report, does not raise capital and does not invest for $x \in \left(x_1^D, x_2^D\right)$; and issues a report, raises capital and invests for $x \in [x_2^D, \infty)$

[24] μ_r^* and the thresholds x_1^D and x_2^D below are the same as in Proposition 1.

(*"Two non-disclosure intervals and partial investment" in Figure 8*).

When a manager issues a report, he biases it according to Lemma 1. If investors observe an off-equilibrium report, they believe that the manager biased his report according to Lemma 1. Which of the above forms the equilibrium takes depends on the parameters of the model.[25]

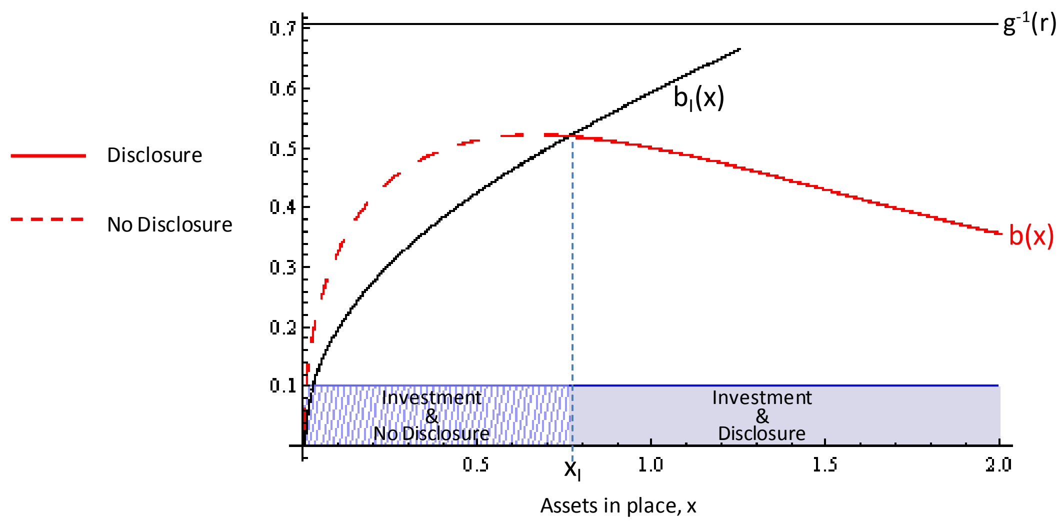

Figure 5: Non-disclosure of bad news and full investment ($\mu_r = 0.25, k = 3, \theta = 2$)

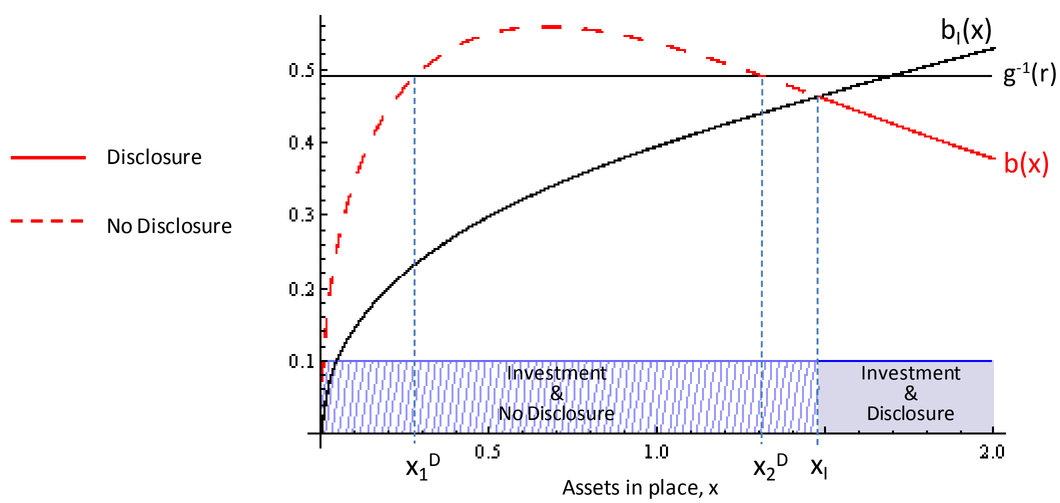

Figure 6: Non-disclosure of bad news and full investment ($\mu_r = 0.12, k = 7, \theta = 2$)

[25]Below and in Corollary 2, we characterize the effect of the model's parameters on the prevailing form of the equilibrium.

Proposition 2 part (i) shows that if the expected return on investment is sufficiently high, the manager raises capital and invests for any value of the assets in place. For asset values lower than x_I the manager does not issue a report, while for higher asset values he issues a report. Figure 5 illustrates such an equilibrium. This and the following figures use again the same cost function and parameter values as before ($g(b) = \frac{1}{2}b^2$, $I = 1$, $\mu_r = 0.12$ or 0.25) and assume that the value of assets in place is distributed according to a Gamma distribution with scale θ and shape k.[26]

If the expected return on investment is lower, the characteristics of the equilibrium are determined by how the thresholds x_I, x_1^D and x_2^D compare. If $x_I > x_2^D$ then $[0, x_I)$ is the unique non-disclosure interval because managers whose asset values exceed x_2^D prefer to disclose and invest. This case is characterized in part $(ii)\,a$ of Proposition 2 and is illustrated in Figure 6. If the intersection of $b(x)$ and $b_I(x)$ is such that $x_I \in \left(x_1^D, x_2^D\right)$ there is also a single non-disclosure interval but it spans $[0, x_2^D)$ rather than $[0, x_I)$. Firms with $x \in [0, x_I)$ invest while firms with $x \in \left(x_I, x_2^D\right)$ do not invest. This case is characterized in part $(ii)\,b$ of Proposition 2 and illustrated in Figure 7. Finally, if $x_I < x_1^D$, there are two distinct non-disclosure intervals. For $x < x_I$ the manager does not make a disclosure but raises capital and invests. For $x \in \left(x_1^D, x_2^D\right)$ the manager does not make a disclosure but also does not raise capital and does not invest. This case is characterized in part $(ii)\,c$ of Proposition 2 and is illustrated in Figure 8.

We have so far not specified which of the three forms illustrated in Figures 6-8 (or, equivalently in parts a-c in Proposition 2) prevails. Which form the equilibrium takes depends on how the threshold x_I compares to x_1^D and x_2^D. The location of x_I is determined by the properties of the prior distribution of $\tilde{x}$. When the prior distribution of $\tilde{x}$ assigns relatively low weight to small asset values, the manager is willing to pay less in order to separate himself from lower types. This is reflected in a downward shift of $b_I(x)$. As a result, x_I is farther to the right. In contrast, x_1^D and x_2^D are independent of the prior distribution of the value of assets in place, x, because both $g^{-1}(\mu_r)$ and $b(x)$ do not vary with the prior distribution of $\tilde{x}$.

Corollary 2 formalizes this intuition and establishes that x_I is higher for distribution h than for distribution f if h is a convex transformation of f such that $E_h[\tilde{x}|\tilde{x} < x'] > E_f[\tilde{x}|\tilde{x} < x']$ for all $x' > 0$. As a result, a firm with asset value x' is undervalued by less if investors' prior beliefs are that firms' asset values are distributed according to h rather than f. Then, a manager with asset value x' is willing to pay less for separating his firm from firms with lower asset values. This

[26] In the equilibrium of Proposition 1, the distribution of values of assets in place is irrelevant because firms raise capital only if they issue a report and, hence, there was no need to specify the distribution of $\tilde{x}$.

is reflected in the function $b_I(x)$ (which indicates the maximum bias for which a firm is willing to bear the costs in order to separate itself from firms with lower asset values) being lower, which maps into a larger value of x_I.

Corollary 2 *Let the probability density function $h(\cdot)$ be a convex transformation of $f(\cdot)$ in the sense that $h(x) = \frac{\pi(x)f(x)}{\int_0^\infty xf(x))dx}$ where $\pi(\cdot)$ is an increasing function of x. Then,*

$$x_I^h > x_I^f$$

where x_I^j denotes the highest value of assets in place for which the manager raises capital without issuing a report when the firm's asset values are distributed according to $j = f, g$.

Proposition 2 establishes that there always exists an equilibrium in which low types raise capital and invest without disclosing information. When the firm raises capital without disclosing information, investors can infer the average value of the firm's assets in place but investors are not able to infer the exact value of the firm's assets in place. In contrast, when the firm makes a disclosure before raising capital, investors are able to infer the exact value of the firm's assets in place and therefore obtain a more precise estimate of the return on their investment in the firm's shares. So far, we assumed that investors are risk-neutral. Risk-neutral investors do not price this additional uncertainty. If investors are risk-averse and the variation in the return on their equity investment contributes to their exposure to systematic risk, then investors will price equity offerings that are not accompanied by a voluntary disclosure at a discount. Due to the discount associated with non-disclosure, the firm is (weakly) less likely to withhold information when investors are risk-averse.[27]

Corollary 3 *If investors are risk-averse, the firm's cost of capital is on average lower when it makes a voluntary disclosure than when it does not. As investors' risk-aversion increases, the likelihood of the firm raising capital without disclosing information decreases.*

This prediction is consistent with several empirical studies analyzing the association between disclosures, financial reporting quality, and the cost of capital. Recent work documents that more extensive pre-IPO disclosures are associated with lower under-pricing using several proxies

[27] Equity offerings being priced at a discount in the absence of a voluntary report causes the $b_I(x)$ function to be higher. The reason is that, when investors are risk-averse, the manager is willing to bear even higher costs in order to separate himself from the pool of firms with lower assets in place and avoid the risk-premium.

for the level of disclosure a firm engages in prior to its IPO. For a sample of Canadian IPOs, Jog and McConomy (2003) use voluntary earnings forecasts that the Canadian securities commission allows, but does not require, issuers of IPOs to include in their prospectus as their measure of voluntary disclosure. Jog and McConomy (2003) find that firms that provide voluntary earnings forecasts experience less severe underpricing. Schrand and Verrecchia (2005) use the announcements contained in firms' press releases as a measure of disclosure level and find evidence consistent with lower underpricing of firms that voluntarily provide more information about their operating, investing and financing activities to the public in press releases prior to its IPO. Leone, Rock and Willenborg (2007) measure the detail an IPO issuer provides regarding their intended use of cash proceeds and first-day under-pricing and also find evidence of a negative association between the level of detail provided and first-day underpricing. Together, the evidence in these studies indicates that voluntary disclosure have a favorable and noticeable impact on the degree of underpricing and the post-issue return performance despite the comprehensive disclosures required by securities commissions prior to an IPO. This suggests that adverse selection is, at least in part, a determinant of cross-sectional variation in underpricing and that voluntary disclosure can reduce the adverse selection effect consistent with the prediction of our model.[28]

Our model also predicts an association between cost of capital and firm performance. Corollary 4 describes this association.

Corollary 4 *If investors are risk-averse, the firm's cost of capital is on average higher for low-performing firms than for high-performing firms. As investors' risk-aversion increases, the difference in cost of capital between low- and high-performing firms increases.*

The intuition for Corollary 4 is as follows. In the model, better performing firms, i.e., firms whose assets in place turned out to be higher, voluntarily issue a report while firms with low values of assets in place do not issue a report. This is consistent with empirical evidence provided by Miller (2002) who finds that high performing firms tend to issue voluntary reports more frequently than low performing firms. Since issuing a voluntary report reduces investors' risk-exposure, our model predicts that firm's cost of capital are lower when the firm issued a voluntary report (Corollary 3). Consequently, the model predicts that better performing firms have lower cost of capital.

[28] The association between firms' cost of capital and disclosure has been studied both empirically (e.g., Botosan 1997; and Botosan and Plumlee 2002) and theoretically (e.g., Lambert, Leuz and Verrecchia 2007; and Christensen, De La Rosa and Feltham 2010). The survey by Beyer et al. (2010) further discusses this literature.

6 Conclusion

The paper models managers' investment and voluntary disclosure strategies and their propensity to engage in costly reporting bias when managers disclose private information in order to raise equity capital for new investment opportunities. We contrast the predictions of the model with firms' optimal investment strategy when they lack the ability to communicate their private information (Myers and Majluf 1984) and with firms' optimal voluntary disclosure strategy when firms' investment behavior is taken exogenously.

Unlike most analytical models of voluntary disclosure, the model allows managers to bias their disclosures and to jointly optimize their investment and disclosure strategies. This seems to be the realistic case for two reasons. First, in practice, firms are often not able to issue perfectly credible or verifiable reports. Rather, managers can bias their reports because of the reporting discretion they enjoy due to the forward looking nature of many voluntary disclosures and the inherent flexibility in Generally Accepted Accounting Principles (GAAP).

Second, in practice, managers don't disclose information merely for its own sake. Instead, firms disclose information because it alters the information asymmetry between firms and investors and allows firms to raise equity capital at favorable rates. Since the price at which a firm can raise capital determines the feasibility and profitability of new investment opportunities, firms' disclosure decisions directly affect their investment decisions and the distribution of their future cash flows. That is, firms make disclosures because of their "real effects." In this paper, we focus on the real effects of firms' voluntary disclosure decisions.

The model shows that treating managers' disclosure and investment decisions both as endogenous and allowing the manager to bias his voluntary report yields qualitatively different predictions than when the disclosure and investment decisions are considered separately and truthful disclosure is assumed. In particular, the optimal investment and disclosure strategies are no longer characterized by a single threshold (in contrast to traditional voluntary disclosure models and Myers and Majluf, 1984). For example, the model predicts that managers sometimes disclose good news and bad news but do not disclose intermediate news. The model also predicts that (*i*) the underinvestment problem is more prevalent if the return on investment is low; and (*ii*) low-performing firms have (weakly) higher cost of capital than high-performing firms. As such, the paper illustrates the importance of considering the interdependencies between firms' disclosure and investment decisions and suggests that management's ability to bias its reports significantly affects firms' voluntary disclosure strategies. Future research that investigates the effect of bias in voluntary disclosures on firms' disclosure policies, investment strategies and other real decisions has the potential to contribute to our understanding of corporate disclosure policies.

Appendix

Proof of Lemma 1

In equilibrium, the manager's marginal benefit from biasing the report equals his marginal cost from biasing the report. The marginal benefit is

$$\frac{\partial}{\partial x_R}\left(1-\frac{I}{x_R-\widehat{b}\left(x_R\right)+I+\mu_r}\right)\left(x+I+\mu_r\right)=\frac{I\left(x+I+\mu_r\right)\left(1-\frac{\partial\widehat{b}\left(x_R\right)}{\partial x_R}\right)}{\left(x_R-\widehat{b}\left(x_R\right)+I+\mu_r\right)^2}.$$

On the equilibrium path, we can substitute $x_R-\widehat{b}\left(x_R\right)=x$. This, together with the marginal cost, yields the FOC (in terms of $\widehat{b}\left(x_R\right)$)

$$\frac{I}{x_R-\widehat{b}\left(x_R\right)+I+\mu_r}\left(1-\frac{\partial\widehat{b}\left(x_R\right)}{\partial x_R}\right)-g'\left(\widehat{b}\left(x_R\right)\right)=0 \tag{4}$$

We can expand $\frac{\partial\widehat{b}(x_R)}{\partial x_R}$ as $\frac{\partial\widehat{b}(x_R)}{\partial x}\frac{1}{\partial x_R/\partial x}=b'\left(x\right)\frac{1}{1+b'(x)}$ because, in equilibrium, $\frac{\partial\widehat{b}(x_R)}{\partial x}=b'\left(x\right)$. In addition, $\widehat{b}\left(x_R\left(x\right)\right)=b\left(x\right)$ and $x_R-\widehat{b}\left(x_R\right)=x$ on the equilibrium path. This yields the FOC (in terms of $b\left(x\right)$)

$$\frac{I}{x+I+\mu_r}\left(1-\frac{b'\left(x\right)}{1+b'\left(x\right)}\right)-g'\left(b\left(x\right)\right)=0 \tag{5}$$

Rearranging yields (1).

The equilibrium bias function is given by the solution to (1) with the boundary condition $b\left(0\right)=0$. We next want to show that there exists a solution to this initial value problem. We cannot invoke the Fundamental Theorem of Differential Equations in order to show that the solution to (1) exists and is unique since the RHS of (1) is not finite at $b\left(0\right)=0$. In order to show that the solution exists, we substitute $c\left(x\right)$ for $\frac{1}{2}\left(b\left(x\right)\right)^2$. This implies that $b\left(x\right)=\sqrt{2\left|c\left(x\right)\right|}$ and $b'\left(x\right)=\frac{c'(x)}{\sqrt{2|c(x)|}}$. Rewriting the differential equation in (1) in terms of $c\left(\cdot\right)$ yields

$$c'\left(x\right)=\frac{I}{x+I+\mu_r}\frac{\sqrt{2\left|c\left(x\right)\right|}}{g'\left(\sqrt{2\left|c\left(x\right)\right|}\right)}-\sqrt{2\left|c\left(x\right)\right|} \tag{6}$$

with the boundary condition $c(0)=0$. Let $F(x,c)=\frac{\sqrt{2|c|}}{g'\left(\sqrt{2|c|}\right)}\frac{I}{x+I+\mu_r}-\sqrt{2|c|}$ denote the RHS of (6). We want to show that $F(x,c)$ is continuous in x and c at $(0,0)$. While $F(x,c)$ is clearly continuous in x, it is only continuous in c if $\lim_{c\to 0}F(x,c)$ is finite.

$$\lim_{c\to 0}F(x,c)=\lim_{c\to 0}\frac{\frac{\partial}{\partial c}\sqrt{2|c|}}{\frac{\partial}{\partial c}g'\left(\sqrt{2|c|}\right)}\frac{I}{x+I+\mu_r}-0=\lim_{c\to 0}\frac{\frac{1}{\sqrt{2|c|}}}{g''\left(\sqrt{2|c|}\right)\frac{1}{\sqrt{2|c|}}}\frac{I}{x+I+\mu_r}=\frac{1}{g''(0)}\frac{I}{x+I+\mu_r}$$

which is finite because $g(\cdot)$ is strictly convex everywhere. Hence, there exists a continuous and differentiable solution to (6) with the boundary condition $c(0)=0$. Next, we show that $c(x)$ provides a solution to the manager's disclosure problem. That requires $c(x)\geq 0$ such that $b(x)$ is real. First note that $c(x)\geq 0$ for $[0,\varepsilon]$ and ε sufficiently small because $c'(0)=F(0,0)=\frac{1}{g''(0)}\frac{I}{I+\mu_r}>0$. Suppose there existed x' such that $c(x')<0$. Since $c(x)$ is continuous, there must exist $x''<x'$ such that $c(x'')=0$ and $c'(x'')<0$. However, $c(x'')=0$ implies that $c'(x'')>0$. Hence, $c(x)\geq 0$ for all $x\in[0,\infty)$. As a result, $b(x)=\sqrt{2c(x)}$ is real and provides a solution to the manager's disclosure problem. Moreover, the solution is unique. Suppose it were not unique and there existed two solutions, $b_1(x)$ and $b_2(x)$ which both satisfied the differential equation in (5) and $b_1(0)=b_2(0)=0$. Further, since $b_1(x)$ and $b_2(x)$ differ and are differentiable there must exist an interval (x',x'') for which $b_1(x)>b_2(x)$ and $b_1'(x)>b_2'(x)$. However, for a given x, b' is lower for higher values of b. As a result, $b_1(x)>b_2(x)$ and $b_1'(x)>b_2'(x)$ cannot hold for any x and the solution to the initial value problem is unique.

We next want to show that the equilibrium bias $b(x)$ has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum and converges to zero as the value of the firm's assets in place goes to infinity.

We have already shown that the equilibrium bias is continuous and strictly positive for $x>0$. From this and the boundary condition $b(0)=0$, it also follows that the bias is initially increasing. However, $b(x)$ cannot always be increasing. Suppose it were the case that $b'(x)>0$ for all x. Then, $1-\frac{b'(x)}{1+b'(x)}\in(0,1)$. Hence, the marginal benefit, $\frac{I}{x+I+\mu_r}\left(1-\frac{b'(x)}{1+b'(x)}\right)$, converges to 0 for $x\to\infty$. In equilibrium, the marginal cost must then also converge to zero. This cannot be the case if $b(x)$ is positive and always increasing for any x. As a result, for x sufficiently large $b(x)$ is decreasing $(-1<b'(x)<0)$ and hence $1-\frac{b'(x)}{1+b'(x)}>1$. Further, since $b(x)>0$ for all x, $b'(x)$ has to converge to 0 for $x\to\infty$. As a result, $\frac{I}{x+I+\mu_r}\left(1-\frac{b'(x)}{1+b'(x)}\right)$ converges to 0 for $x\to\infty$. Hence, the marginal cost must also converge to zero and therefore $b(x)\to 0$ for $x\to\infty$.

Finally, we want to show that $b(x)$ does not have a local minimum. We show that by proving that there does not exist any x for which $b(x)$ is weakly increasing and weakly convex. Suppose it were the case that $b'(x) \geq 0$ and $b''(x) \geq 0$ for a given x. As x increases, $b(x)$ weakly increases and hence $g'(b)$ weakly increases (due to $b(x) \geq 0$ and $b'(x) \geq 0$). Moreover, as x increases, $b'(x)$ weakly increases and hence $\frac{b'(x)}{1+b'(x)}$ weakly increases (due to $b''(x) \geq 0$). The latter implies that the marginal benefit, $\frac{I}{x+I+\mu_r}\left(1-\frac{b'(x)}{1+b'(x)}\right)$, strictly decreases in x. Since the marginal cost, $g'(b)$, weakly increases, this yields a contradiction. ■

Proof of Lemma 2

From the proof of Lemma 1 we know that $c'(0) = \frac{1}{g''(0)}\frac{I}{0+I+\mu_r}$ when the boundary condition is $c(0) = 0$. Hence, the slope of $c(x)$ is higher at $x = 0$ when μ_r is lower. Since the solution $c(x)$ is differentiable, there exists $\varepsilon > 0$ such that for all $x \in (0, \varepsilon)$ the slope of $c(x)$ is higher when μ_r is lower. Let $\mu_{r1} < \mu_{r2}$. From continuity it follows that $c(x; \mu_{r1}) > c(x; \mu_{r2})$ and equivalently $b(x; \mu_{r1}) > b(x; \mu_{r2})$ for some neighborhood $(0, \delta)$. Next, we want to show that $b(x; \mu_{r1}) > b(x; \mu_{r2})$ for all $x \in (0, \infty)$. Suppose there existed $x' > 0$ such that $b(x'; \mu_{r1}) = b(x'; \mu_{r2})$. From (1), we know that at this point $b'(x'; \mu_{r1}) > b'(x'; \mu_{r2})$. Hence, the slope of the lower bias curve (for $\mu_r = \mu_{r2}$) is lower than the slope of the higher bias curve (for $\mu_r = \mu_{r1}$) and hence it cannot be the case that the two curves intersect (or even touch). Hence, it follows that $b(x; \mu_{r1}) > b(x; \mu_{r2})$ for all $x \in (0, \infty)$. ■

Proof of Proposition 1

We start out by showing that $g(b(x)) - \mu_r$ is monotonically decreasing in μ_r. This implies that there exists $\mu_r^* > 0$ such that for $\mu_r \geq \mu_r^*$ the disclosure costs $g(b(x))$ are (weakly) less than the expected return on investment μ_r for all x and that for $\mu_r < \mu_r^*$ there are some values of x for which the disclosure costs $g(b(x))$ strictly exceed the expected return on investment μ_r.

Lemma 5 *For any given x, $g(b(x)) - \mu_r$ is monotonically decreasing in μ_r where $b(x)$ is given by (1) with $b(0) = 0$ as a boundary condition.*

Proof. We know from Lemma 2 that $b(x)$ is decreasing in μ_r. It follows that $g(b(x)) - \mu_r$ is decreasing in μ_r. ■

Next, we show that a necessary condition for an equilibrium with full disclosure to exist is that $\mu_r \leq \mu_r^*$ which implies $g(b(x)) \leq \mu_r$ for all x (Proposition 1, part *i*). Further, if $\mu_r > \mu_r^*$ then there exist some x for which $g(b(x)) > \mu_r$ which is a necessary condition for an equilibrium with intermediate news undisclosed to exist (Proposition 1, part *ii*). In addition, these conditions in conjunction with the off-equilibrium beliefs that managers that raise capital without issuing a report are of the lowest type ($x = 0$) are sufficient for the existence of the equilibrium in Proposition 1.

Lemma 6 *A necessary and sufficient condition for existence of a full disclosure equilibrium is that for any value of the assets in place, x, the following inequality holds*

$$\mu_r \geq g\left(b\left(x\right)\right). \tag{7}$$

Proof. In a full disclosure equilibrium, the manager always discloses, the firm equity is correctly priced and the firm invests in the positive NPV project. Hence, the firm's expected payoff is $x + \mu_r - g\left(b\left(x\right)\right)$. We need to show that none of the types has incentives to deviate. One potential deviation in a full disclosure equilibrium is for a type not to disclose and not to invest which yields a payoff of x. A necessary condition to preclude such deviation is that the disclosure cost of all types are lower than their expected return on the investment, μ_r. So, a necessary condition for the existence of a full disclosure equilibrium is that condition (7) holds for any x, or equivalently, $g^{-1}\left(\mu_r\right) > b\left(x\right)$.

Condition (7) is not only necessary, but also sufficient for existence of a full disclosure equilibrium. To show the sufficiency of this condition one needs to preclude any form of deviation. There are two other types of potential deviations: (i) disclosure of a report other than the type's equilibrium report and investment and (ii) non-disclosure and investment. By construction of the bias function $b\left(x\right)$ in Lemma 1, any deviation to a report of another type is precluded. To preclude deviation to non-disclosure, we need to assign off-equilibrium beliefs that guarantee that no type wants to deviate to a non-disclosure and investment strategy. The only off-equilibrium beliefs that preclude such deviation are the ones under which investors believe that the manager is of the lowest type if he raises capital without issuing a report. Note that these off-equilibrium beliefs are natural since in equilibrium the only type who is always indifferent between investing with and investing without issuing a report is the lowest type. ■

Lemma 7 *A necessary and sufficient condition for existence of an equilibrium with intermediate news undisclosed is that there exists a value of assets in place, x, such that*

$$\mu_r < g\left(b\left(x\right)\right). \tag{8}$$

Proof. Condition (8) implies that there are exactly two values of assets in place, x_1^D and x_2^D, for which $b\left(x_1^D\right) = b\left(x_2^D\right) = g^{-1}\left(\mu_r\right)$. We want to show that the following equilibrium exists if condition (8) holds: For all $x \in [0, x_1^D)$ and $x \in [x_2^D, \infty)$, the manager invests and discloses according to Lemma 1 and for all $x \in [x_1^D, x_2^D)$ the manager does not invest and does not disclose. If investors observe an off-equilibrium report they believe that the manager reported according to Lemma 1. These off-equilibrium beliefs together with the manager's reporting strategy guarantee that any type that issues a report in equilibrium does not deviate and issue a different report (either on or off the equilibrium path). As before, we assume that if a firm invests without issuing a report investors believe that the firm is of the lowest type, i.e., the value of the firm's assets in place is zero. Together, this shows that condition (8) is sufficient for an equilibrium with intermediate news undisclosed to exist. To show that condition (8) is also necessary, note that if condition (8) does not hold then any type that is supposed not to disclose and not to invest would deviate and issue a report according to Lemma 1 and invest.[29] ■

Proof of Corollary 1

Recall that x_1^D and x_2^D are given by $g\left(b\left(x_1^D\right)\right) = g\left(b\left(x_2^D\right)\right) = \mu_r$. Differentiating this condition with respect to x_i^D and μ_r for $i = 1, 2$ yields

$$g'\left(b\left(x_i^D\right)\right) b'\left(x_i^D\right) dx_i^D + \left(g'\left(b\left(x_i^D\right)\right) \frac{\partial b\left(x_i^D\right)}{\partial \mu_r} - 1\right) d\mu_r = 0.$$

Rearranging yields

$$\frac{dx_i^D}{d\mu_r} = -\frac{g'\left(b\left(x_i^D\right)\right) \frac{\partial b\left(x_i^D\right)}{\partial \mu_r} - 1}{g'\left(b\left(x_i^D\right)\right) b'\left(x_i^D\right)}.$$

$g'\left(b\left(x_i^D\right)\right)$ is positive and Lemma 2 implies that $\frac{\partial b\left(x_i^D\right)}{\partial \mu_r}$ is negative. Hence, the numerator is always negative and the denominator takes the sign of $b'\left(x_i^D\right)$. For x_1^D $\left(x_2^D\right)$ the bias is increasing and hence $\frac{dx_1^D}{d\mu_r} > 0$ $\left(\frac{dx_2^D}{d\mu_r} < 0\right)$. ■

Proof of Lemma 3

Suppose, for a given X_{nd}, $E\left[\widetilde{x} + I + \mu_r | x \in X_{nd}\right] = \bar{x}_{nd}$. Further, if $\{x | x < x_I\} \cap X_{nd} = \emptyset$ then $E\left[\widetilde{x} | \widetilde{x} < x_I, x \in X_{nd}\right] \equiv 0$. Hence, $\alpha_{nd} = I / E\left[\widetilde{x} + I + \mu_r | \widetilde{x} < x_I, x \in X_{nd}\right]$ decreases weakly in x_I,

[29] This arguments relies on off-equilibrium beliefs that equal the full disclosure beliefs, however, it is easy to show that if condition (8) does not hold an equilibrium with intermediate news undisclosed cannot exist for any off-equilibrium beliefs.

$\alpha_{nd}(x_I = 0) = \frac{I}{I+\mu_r}$ and $\lim_{x_I \to \infty} \alpha_{nd}(x_I) = \frac{I}{\bar{x}_{nd}+I+\mu_r}$. Hence, $\alpha \in \left(\frac{I}{I+\mu_r}, \frac{I}{\bar{x}_{nd}+I+\mu_r}\right) \subset (0,1)$. Substituting equation (2) into equation (3) and rearranging yields

$$\frac{I}{E[\widetilde{x}|\widetilde{x} < x_I, x \in X_{nd}] + I + \mu_r} x_I + \max\{0, \mu_r - g(b(x_I))\} = \left(1 - \frac{I}{E[\widetilde{x}|\widetilde{x} < x_I, x \in X_{nd}] + I + \mu_r}\right)(I + \mu_r).$$

Both the LHS and the RHS are continuous in x_I. Moreover, for $x_I = 0$ the LHS equals 0 and the RHS equals μ_r while for $x_I \to \infty$ the LHS approaches ∞ and the RHS equals $\frac{\bar{x}_{nd}+\mu_r}{\bar{x}_{nd}+I+\mu_r}(I + \mu_r)$ which is finite. Hence, there exists $x_I \in (0, \infty)$ such that equation (3) holds when α_{nd} is given by $\alpha_{nd} = I/E[\widetilde{x} + I + \mu_r|\widetilde{x} < x_I, x \in X_{nd}]$. ■

Proof of Lemma 4

As an intermediate step, we prove the following lemma.

Lemma 8 *If there exists a non-disclosure interval, $\left(x_1^D, x_2^D\right)$ such that there exists x' which is ε to the left of x_1^D that issues a report and invests then any $x \in \left(x_1^D, x_2^D\right)$ does not invest.*

Proof. Let $y \in \{0, 1\}$ be the investment decision where $y = 1$ indicates that the firm pursues the investment opportunity and $y = 0$ otherwise. Suppose type x_1^D were indifferent between issuing a report $x_R\left(x_1^D\right)$ and investing and not issuing a report and investing, i.e.,

$$\begin{aligned} &\left(1 - \frac{I}{E\left(\tilde{x}|x_R\left(x_1^D\right)\right) + I + \mu_r}\right)\left(x_1^D + I + \mu_r\right) - g\left(x_R\left(x_1^D\right) - x_1^D\right) \\ &= \left(1 - \frac{I}{E\left(\tilde{x}|nd, y = 1\right) + I + \mu_r}\right)\left(x_1^D + I + \mu_r\right) \end{aligned} \tag{9}$$

For x_1^D to be indifferent, investors' beliefs have to be such that $E\left(\tilde{x}|x_R\left(x_1^D\right)\right) > E\left(\tilde{x}|nd, y = 1\right)$ because the disclosure costs are strictly positive (this follows from the bias being strictly positive; note that zero bias is not feasible for finite $x > 0$). We also know that type x' prefers to invest and disclose over non-disclosure and investment, i.e.,

$$\begin{aligned} &\left(1 - \frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right) + I + \mu_r}\right)\left(x' + I + \mu_r\right) - g\left(x_R\left(x'\right) - x'\right) \\ &> \left(1 - \frac{I}{E\left(\tilde{x}|nd, y = 1\right) + I + \mu_r}\right)\left(x' + I + \mu_r\right) \end{aligned}$$

Since x' is sufficiently close to x_1^D it is the case that $E\left(\tilde{x}|x_R\left(x_1^D\right)\right) > E\left(\tilde{x}|x_R\left(x'\right)\right) > E\left(\tilde{x}|nd, y = 1\right)$. The difference between the payoff of type x_1^D from mimicking x' and his payoff from investing without disclosing is given by

$$\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right)-g\left(x_R\left(x'\right)-x_1^D\right) \qquad (10)$$
$$-\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right).$$

In order to arrive at a contradiction, we want to show that the expression in (10) is positive (which implies that type x_1^D wants to deviate and mimic x')

$$\begin{aligned}
&\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right)-g\left(x_R\left(x'\right)-x_1^D\right)\\
&-\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right)\\
=&\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\left(x'+\varepsilon+I+\mu_r\right)-g\left(x_R\left(x'\right)-x^{'}-\varepsilon\right)\\
&-\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x'+\varepsilon+I+\mu_r\right)
\end{aligned}$$

which yields

$$A+\left(\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\varepsilon+g\left(x_R\left(x'\right)-x'\right)-g\left(x_R\left(x'\right)-x^{'}-\varepsilon\right) \qquad (11)$$

where A denotes the LHS expression in (**??**). Note that

$$\left(\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\varepsilon>0$$

because $E\left(\tilde{x}|x_R\left(x'\right)\right)>E\left(\tilde{x}|nd,y=1\right)$ and

$$g\left(x_R\left(x'\right)-x'\right)-g\left(x_R\left(x'\right)-x^{'}-\varepsilon\right)>0,$$

which follows from the bias being positive and

$$\begin{aligned}
x_R\left(x'\right)-x'&>x_R\left(x'\right)-x^{'}-\varepsilon\\
x_R\left(x'\right)&>x_R\left(x'\right)-\varepsilon.
\end{aligned}$$

This implies that the expression in (11) is positive. As a result, type x_1^D prefers mimicking the report of x' which contradicts the assumed equilibrium behavior. ■

To complete the proof of Lemma 4, we have to show that if there exists a type x that prefers to invest without issuing a report then all types to the left of x also prefer to invest without issuing a report. Based on Lemma 8, type x belongs to the left-most non-disclosure interval. Hence, all types to the left of x do not issue a report. We need to show that if x prefers investing without issuing a report to non-investing without issuing a report then all types to the left of x also do. This follows from the fact that lower types give up the same fraction in exchange for the same expected return on investment μ_r but that (the fraction of) their firm is worth less. ■

Proof of Proposition 2

We start by showing that for sufficiently small x, $b(x)$ is strictly greater than $b_I(x)$. From this it follows that $b(x)$and $b_I(x)$ intersect at least once.

Lemma 9 *There exists $\varepsilon > 0$ such that $b(x) > b_I(x)$ for $x \in (0, \varepsilon)$.*

Proof. From Definition 1, it follows that

$$b_I(x) = g^{-1}\left(I\frac{x - E(\tilde{x}|\tilde{x} < x)}{E(\tilde{x}|\tilde{x} < x) + I + \mu_r}\right)$$

and

$$b_I'(x) = \frac{1}{g'(b_I(x))}\frac{I}{E(\tilde{x}|\tilde{x} < x) + I + \mu_r}\left(1 - \frac{x + I + \mu_r}{E(\tilde{x}|\tilde{x} < x) + I + \mu_r}(x - E(\tilde{x}|\tilde{x} < x))\frac{f(x)}{F(x)}\right),$$

where $F(x)$ is the cumulative distribution function of the value of the firm's assets in place. We first compute $\lim_{x \to 0} \frac{b'(x)}{b_I'(x)} > 1$ where $b'(x)$ is given by equation (1).

$$\begin{aligned}
\lim_{x \to 0} \frac{b'(x)}{b_I'(x)} &= \lim_{x \to 0} \frac{\frac{I}{g'(b(x))(x+I+\mu_r)} - 1}{\frac{1}{g'(b_I(x))}\frac{I}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}\left(1 - \frac{x+I+\mu_r}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}(x - E(\tilde{x}|\tilde{x} < x))\frac{f(x)}{F(x)}\right)} \\
&= \lim_{x \to 0} \frac{\frac{I}{(x+I+\mu_r)} - g'(b(x))}{\frac{I}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}\left(1 - \frac{x+I+\mu_r}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}(x - E(\tilde{x}|\tilde{x} < x))\frac{f(x)}{F(x)}\right)} \lim_{x \to 0} \frac{g'(b_I(x))}{g'(b(x))} \\
&= \frac{\frac{I}{I+\mu_r} - 0}{\frac{I}{I+\mu_r}\left(1 - \lim_{x \to 0}(x - E(\tilde{x}|\tilde{x} < x))\frac{f(x)}{F(x)}\right)} \lim_{x \to 0} \frac{g'(b_I(x))}{g'(b(x))} \\
&= 2 \lim_{x \to 0} \frac{g'(b_I(x))}{g'(b(x))}
\end{aligned}$$

where the last equality follows from

$$\begin{aligned}
\lim_{x \to 0} x\frac{f(x)}{F(x)} &= \lim_{x \to 0} \frac{f(x) + xf'(x)}{f(x)} = 1 + \lim_{x \to 0} \frac{xf'(x)}{f(x)} = 1 \text{ for } f(0) > 0 \text{ and } \left|f'(0)\right| < \infty \\
\lim_{x \to 0} E(\tilde{x}|\tilde{x} < x)\frac{f(x)}{F(x)} &= \lim_{x \to 0} \frac{f(x)}{F^2(x)}\int_0^x zf(z)\,dz = \lim_{x \to 0} \frac{f'(x)\int_0^x zf(z)\,dz + f(x)\,xf(x)}{2F(x)\,f(x)} \\
&= \lim_{x \to 0} \frac{f'(x)}{2f(x)}E(\tilde{x}|\tilde{x} < x) + \lim_{x \to 0} \frac{xf(x)}{2F(x)} = \frac{1}{2}.
\end{aligned}$$

We have just shown that

$$\lim_{x\to 0}\frac{b'(x)}{b_I'(x)}\frac{g'(b(x))}{g'(b_I(x))}=2.$$

First, consider the case when $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)}\leq 1$. Then, $\lim_{x\to 0}\frac{g'(b(x))}{g'(b_I(x))}\geq 2$. Which implies that there exists an interval $(0,\varepsilon)$ for which $b(x)>b_I(x)$ (which can only hold when $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)}=1$; if $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)}<1$ we arrive at a contradiction). Next, consider the case when $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)}>1$. Then, there must be an interval for which $b(\cdot)$ is steeper than $b_I(\cdot)$ because both functions are differentiable for $x>0$, and as a result there exists an interval $(0,\varepsilon)$ for which $b(x)>b_I(x)$. Hence, there always exists an interval $(0,\varepsilon)$ for which $b(x)>b_I(x)$. ■

Next, we show that there always exists an equilibrium in which low types do not disclose but invest. The remaining characteristics of the equilibrium in Proposition 2 follow immediately from Proposition 1 and Lemma 4.

Lemma 10 *There always exists an equilibrium in which low types do not disclose but invest.*

Proof. First, we consider the case in which the parameters are such that $b(x)\leq g^{-1}(\mu_r)$ (i.e., $\mu_r\geq\mu_r^*$). Let x' be the highest value of the firm's assets in place for which the manager raises capital without issuing a report. Define $u_1(x')$ as the difference between the payoff of type x' if he raises capital without issuing a report and the payoff if he raises capital and issues a report $x_R(x')=x'+b(x')$, i.e.,

$$u_1(x')=\frac{E[\tilde{x}|x<x']+\mu_r}{E[\tilde{x}|x<x']+I+\mu_r}\left(x'+\mu_r+I\right)-\left(x'+\mu_r-g\left(b\left(x'\right)\right)\right).$$

We want to show that there exists $x'>0$ such that $u_1(x')=0$. From the definition of $u_1(x')$ it follows that $\lim_{x'\to 0}u_1(x')=0$ and $\lim_{x'\to\infty}u_1(x')=-\infty$. Moreover,

$$\begin{aligned}
\left.\frac{\partial u_1(x')}{\partial x'}\right|_{x'=0} &= \left.\frac{\left(E[\tilde{x}|x<x']+I+\mu_r\right)\frac{\partial E[\tilde{x}|x<x']}{\partial x'}-\left(E[\tilde{x}|x<x']+\mu_r\right)\frac{\partial E[\tilde{x}|x<x']}{\partial x'}}{\left(E[\tilde{x}|x<x']+I+\mu_r\right)^2}\left(x'+\mu_r+I\right)\right|_{x'=0}\\
&\quad+\left.\left(\frac{E[\tilde{x}|x<x']+\mu_r}{E[\tilde{x}|x<x']+I+\mu_r}-\left(1-g'(b(x))\,b'\left(x'\right)\right)\right)\right|_{x'=0}\\
&=\left.\left(\frac{I\left(x'+\mu_r+I\right)}{\left(E[\tilde{x}|x<x']+I+\mu_r\right)^2}\frac{\partial E[\tilde{x}|x<x']}{\partial x'}\right)\right|_{x'=0}\\
&\quad+\left.\left(\frac{E[\tilde{x}|x<x']+\mu_r}{E[\tilde{x}|x<x']+I+\mu_r}-\left(1-\frac{I}{x'+I+\mu_r}\frac{b'(x')}{1+b'(x')}\right)\right)\right|\\
&=\frac{I}{I+\mu_r}\frac{\partial E[\tilde{x}|x<x']}{\partial x'}+\frac{\mu_r}{I+\mu_r}-\left(1-\frac{I}{I+\mu_r}\right)\\
&=\frac{I}{I+\mu_r}\frac{\partial E[\tilde{x}|x<x']}{\partial x'}>0.
\end{aligned}$$

Hence, continuity of $u(x')$ implies that there exists at least one $x' > 0$ such that $u_1(x') = 0$. Let

$$x_I = \min\left\{x' \left| u_1\left(x'\right) = 0, x' > 0\right.\right\}$$

We have shown that $\lim_{x' \to 0} u_1(x') = 0$, $\lim_{x' \to \infty} u_1(x') = -\infty$, and that $u_1(x')$ is increasing in x'. By construction of x_I, for all $x' \in [0, x_I)$ we have $u_I(x') \geq 0$. Since $\frac{E[\tilde{x}|x<x']+\mu_r}{E[\tilde{x}|x<x']+I+\mu_r}$ is increasing in x' it follows that

$$\frac{E\left[\tilde{x}|x < x_I\right] + \mu_r}{E\left[\tilde{x}|x < x_I\right] + I + \mu_r}\left(x' + \mu_r + I\right) - \left(x' + \mu_r - g\left(b\left(x'\right)\right)\right) \geq 0.$$

Hence, for all $x' \in [0, x_I)$, the manager prefers investment without disclosure to investment with disclosure. We further need to show that no type $x' > x_I$ wants to deviate and invest without disclosure. We know that $x' > x_I$ does not mimic x_I by issuing the report $x^R(x_I)$ and investing. That is

$$x' + \mu_r - g\left(b\left(x'\right)\right) \geq \frac{x_I + \mu_r}{x_I + I + \mu_r}\left(x' + \mu_r + I\right) - g\left(b\left(x_I\right) - \left(x' - x_I\right)\right).$$

We want to show that type $x' > x_I$ prefers issuing the report $x^R(x_I)$ to investment without disclosure, i.e., we want to show that

$$\frac{x_I + \mu_r}{x_I + I + \mu_r}\left(x' + \mu_r + I\right) - g\left(b\left(x_I\right) - \left(x' - x_I\right)\right) > \frac{E\left[\tilde{x}|x < x_I\right] + \mu_r}{E\left[\tilde{x}|x < x_I\right] + I + \mu_r}\left(x' + \mu_r + I\right). \quad (12)$$

First, note that we can restrict the analysis to x' that do not need to bias their report downwards in order to mimic x_I. The reason is that x' that would have to bias its report downward in order to mimic x_I can mimic a higher type without incurring any biasing costs. Hence, we restrict attention to $x' \in (x_I, x_I + b(x_I))$. Taking the derivative of the LHS of (12) net of the RHS of (12) with respect to x' yields

$$\frac{x_I + \mu_r}{x_I + I + \mu_r} + g'\left(b\left(x_I\right) - \left(x' - x_I\right)\right) - \frac{E\left[\tilde{x}|x < x_I\right] + \mu_r}{E\left[\tilde{x}|x < x_I\right] + I + \mu_r},$$

which is positive for $x' \in (x_I, x_I + b(x_I))$. We know that (12) holds with equality for x_I. Hence, the inequality in (12) holds for all $x' \in (x_I, x_I + b(x_I))$. This proves that no type $x' > x_I$ wants to deviate and invest without disclosure. The manager's equilibrium strategy is as follows: $x \in [0, x_I)$ invest but do not disclose and $x \in [x_I, \infty)$ invest and disclose.

Next, we consider the case in which $b(x) > g^{-1}(\mu_r)$ for some x. We define

$$u_1\left(x'\right) = \frac{E\left[\tilde{x}|x < x'\right] + \mu_r}{E\left[\tilde{x}|x < x'\right] + I + \mu_r}\left(x' + \mu_r + I\right) - \left(x' + \max\left\{0, \mu_r - g\left(b\left(x'\right)\right)\right\}\right).$$

By the same argument as above, there exists at least one $x' > 0$ such that $u_1(x') = 0$. If $\widehat{x} = \min\{x' \,|u_1(x') = 0, x' > 0\}$ is such that $b(\widehat{x}) \leq g^{-1}(\mu_r)$, then $x_I = \min\{x' \,|u_1(x') = 0, x' > 0\}$. In equilibrium, $x \in [0, x_I)$ invest but do not disclose and $x \in [x_I, \infty)$ invest and disclose. If $\widehat{x} = \min\{x' \,|u_1(x') = 0, x' > 0\}$ is such that $b(\widehat{x}) > g^{-1}(\mu_r)$, then $x_I = \min\left\{x' \,\middle|b(x') = g^{-1}(\mu_r)\right\}$. Let $x_D = \max\left\{x' \,\middle|b(x') = g^{-1}(\mu_r)\right\}$. In equilibrium, $x \in [0, x_I)$ invest but do not disclose; $x \in [x_I, x_D)$ do not invest and do not disclose and $x \in [x_D, \infty)$ invest and disclose. ■

Proof of Corollary 2

The cumulative distribution function of $h(x) = \frac{\pi(x)f(x)}{\int_0^\infty \pi(y)f(y)dy}$ is given by

$$H(x) = \int_0^x h(z)\,dz = \int_0^x \frac{\pi(z)\,f(z)}{\int_0^\infty \pi(y)\,f(y)\,dy}dz = \frac{F(x)\,E_f[\pi(\tilde{x})\,|\tilde{x} < x]}{\int_0^\infty \pi(y)\,f(y)\,dy}$$

We want to show that $E_f[\tilde{x}|\tilde{x} < x] < E_h[\tilde{x}|\tilde{x} < x]$. We can rewrite $E_h[\tilde{x}|\tilde{x} < x]$ as

$$E_h[\tilde{x}|\tilde{x} < x] = \frac{1}{H(x)} \int_0^x \frac{z\pi(z)\,f(z)}{\int_0^\infty \pi(y)\,f(y)\,dy}dz = \frac{1}{F(x)\,E_f[\pi(\tilde{x})\,|\tilde{x} < x]} \int_0^x z\pi(z)\,f(z)\,dz$$

Hence, $E_f[\tilde{x}|\tilde{x} < x] < E_h[\tilde{x}|\tilde{x} < x]$ is equivalent to

$$\begin{aligned}
E_f[\tilde{x}|\tilde{x} < x] &< \frac{1}{F(x)\,E_f[\pi(\tilde{x})\,|\tilde{x} < x]} \int_0^x z\pi(z)\,f(z)\,dz \\
E_f[\tilde{x}|\tilde{x} < x]\,E_f[\pi(\tilde{x})\,|\tilde{x} < x] &< E_f[\tilde{x}\pi(\tilde{x})\,|\tilde{x} < x] \\
0 &< Cov_f[\tilde{x}, \pi(\tilde{x})\,|\tilde{x} < x]
\end{aligned}$$

which always holds for increasing functions $\pi(x)$ because (we omit the condition $\tilde{x} < x$ and the subscript f for readability)

$$\begin{aligned}
Cov[\tilde{x}, \pi(\tilde{x})] &= E[(\tilde{x} - \mu_x)(\pi(\tilde{x}) - E[\pi(\tilde{x})])] \\
&= E[(\tilde{x} - \mu_x)(\pi(\tilde{x}) - \pi(\mu_x))] + E[\tilde{x} - \mu_x](\pi(\mu_x) - E[\pi(\tilde{x})]) \\
&= E[(\tilde{x} - \mu_x)(\pi(\tilde{x}) - \pi(\mu_x))].
\end{aligned}$$

The last expression is non-negative for every x because $\pi(x)$ is increasing. ■

Proof of Claim in Footnote 21

Claim 1 *In any equilibrium in which the reporting strategy is given by Lemma 1, there are at most two non-disclosure intervals.*

We start by proving the following lemma.

Lemma 11 *If the highest type of a non-disclosure interval, x', does not invest then there does not exist an additional non-disclosure interval to the right of x'.*

Proof. x' must be indifferent between disclosing and investing and not disclosing and not investing. This requires $b(x') = g^{-1}(\mu_r)$. Suppose there exists an additional non-disclosure interval, $\left(x_1^D, x_2^D\right)$, where $x' < x_1^D < x_2^D$. Then, the bias must be decreasing over the interval $\left[x', x_1^D\right]$ because the bias can not exceed $g^{-1}(\mu_r)$ and the bias never increases in x once it has decreased. This implies that the bias at x_1^D is strictly lower than $g^{-1}(\mu_r)$. Since x' prefers not to invest if he does not disclose, x_1^D must strictly prefer not to invest if he does not disclose. x_1^D is indifferent between disclosing and investing and not disclosing and not investing only if $b\left(x_1^D\right) = g^{-1}(\mu_r)$. This yields a contradiction because we argued that $b\left(x_1^D\right) < b(x') = g^{-1}(\mu_r)$. ■

From Lemma 8 it follows that the highest type of the second non-disclosure interval, x_2^D, never invests. Following an argument similar to the proof of Lemma 11, it follows that there are at most two non-disclosure intervals. ■

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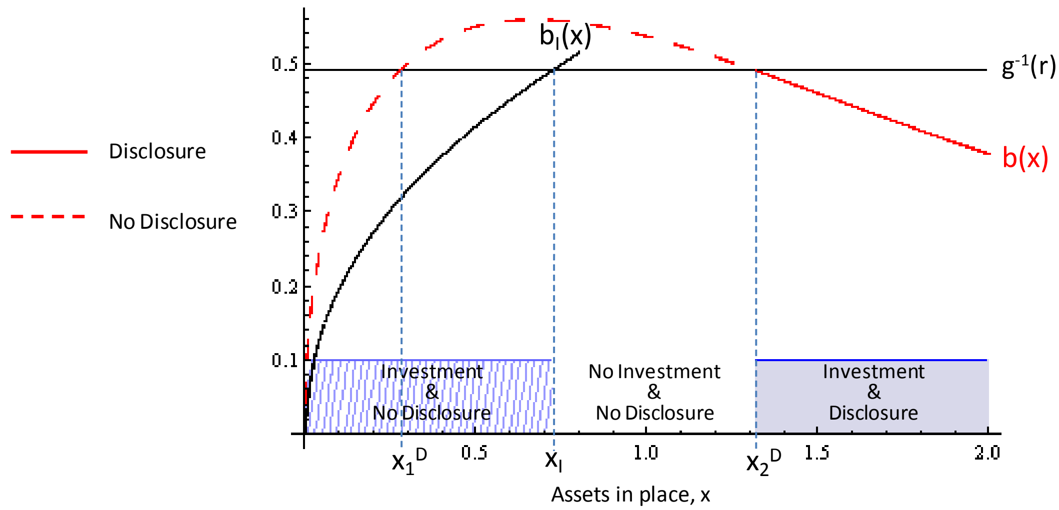

Figure 7: Non-disclosure of bad news and partial investment ($\mu_r = 0.12, k = 3, \theta = 2$)

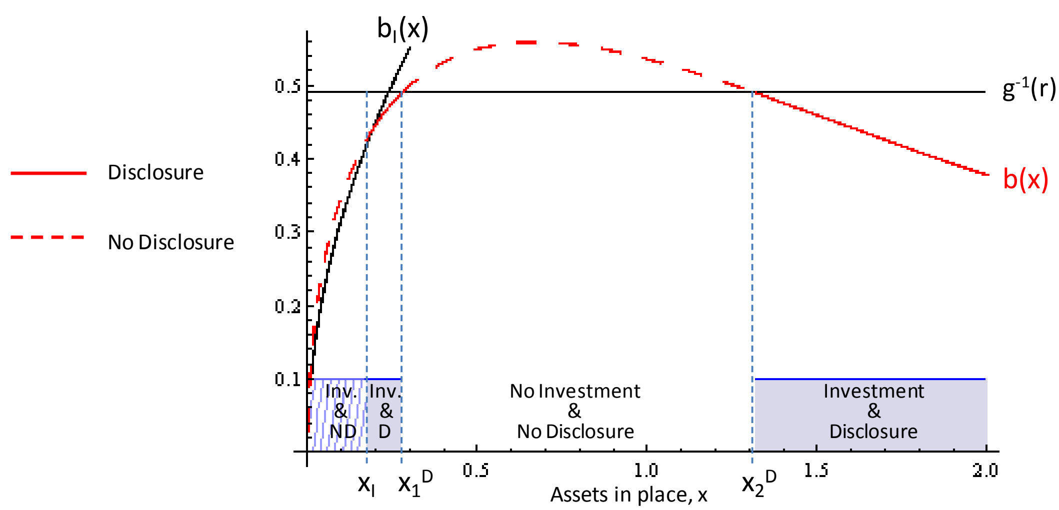

Figure 8: Two non-disclosure intervals and partial investment ($\mu_r = 0.12, k = 0.75, \theta = 2$)