Keeping up with the Zhangs: household saving behavior as a function of relative wealth *

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November 17, 2011

Abstract

Economic research has consistently shown that households with greater incomes tend to have higher average saving rates. Using detailed Chinese household survey data, I demonstrate a new empirical finding: the real determinant of differences in saving rates among households is *relative* income. Specifically, I find that households that earn more than the average in their locality save a larger fraction of their income. That is, households with the same income are likely to consume more in high income locations. To explain this finding, I first argue that standard modeling features, such as household heterogeneity, income mean reversion and the correlation between average income and local prices, do not account for such relativistic household behavior. Instead, I propose a utility function that incorporates both relative consumption and utility from wealth. This function accounts for the higher rates of saving by the relatively rich and predicts the observed positive relation between economic growth and the aggregate saving rate.

*I am grateful to Alan Deardorff, John Laitner, Jagadeesh Sivadasan, Dmitriy Stolyarov and seminar participants at the University of Michigan and at the 2011 Midwest Macroeconomics Conference for their helpful comments and advice. All errors are my own.

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"In any community where goods are held in severalty it is necessary, in order to his own peace of mind, that an individual should possess as large a portion of goods as others with whom he is accustomed to class himself; and it is extremely gratifying to possess something more than others."

– *Thorstein Veblen, The Theory of the Leisure Class (1899)*

1 Introduction

It is well established in economic literature, going back to Kuznets (1953), that the rich save a larger share of their income. In this study, I use Chinese household survey data to show that not only do the rich save more, but wealth, or richness, is not absolute, and is perceived relative to local average income levels. That is, households in high income locations are likely to consume more of their disposable income, *ceteris paribus.* Using city identifiers, I estimate local average income, and construct a given household's relative income as the ratio of the household's income over the local average. I then demonstrate that the household's saving rate is more closely correlated to its relative income than to its absolute income.

To account for this relativistic household behavior, I construct a hybrid utility function that incorporates relative consumption, also referred to as "keeping up with the Joneses", and utility from wealth, also called the "capitalist spirit" after Weber (1905). Through analysis and simulations, I show that this hybrid utility generates higher rates of saving by the relatively rich. In addition, it is consistent with finding of high aggregate saving rates in rapidly growing economies.

The relative role of consumption, dating back to Veblen (1899) and the Duesenberry (1949) relative income hypothesis, has long been investigated in the field of happiness economics.[1] Harbaugh (1996) and Carroll et al. (1997) suggest that relative consumption may explain the positive correlation between economic growth and saving, although their analysis

[1] A seminal paper, presenting the 'Easterlin paradox', is Easterlin (1974). More recent papers discussing relative consumption are Luttmer (2005), Clark et al. (2008) and Oshio et al. (2010).

is using an infinite horizon setup, which greatly weakens the effect. With economic growth, the surrounding average level of consumption is constantly rising, and the agent's optimal consumption path is exponentially increasing. This trend places the bulk of consumption later in life, when income is low (due to retirement), thus necessitating more saving earlier. However, a relative consumption model by itself cannot explain why the relatively rich save more, as it generates identical saving rates across different levels of income. Similarly, utility from wealth by itself does not account for the fact that it is the *relative* wealth of a household that determines its average propensity to consume or save, as neither heterogeneity in time discounting nor income mean reversion suffice to explain the magnitude of savings. Consequently, I argue that combining utility from relative wealth with relative consumption presents a more comprehensive explanation of saving behavior.

To my knowledge, my model is the first to combine relative consumption and utility from wealth in creating a hybrid utility function that closely matches the stylized facts of increased saving rates with relatively higher income and the correlation between aggregate growth and saving to GDP ratios.

2 The Puzzles

2.1 The Relatively Rich Save Relatively More

That the rich (households in the upper income percentiles) save a larger share of their income is an accepted phenomenon in economic research. Kuznets (1953) shows it to be an extremely consistent in the US, and more recently it has been demonstrated by Dynan et al. (2004). Chamon and Prasad (2010) show a similar pattern for Chinese households. Out of multiple explanations suggested, Carroll (2000), Dynan et al. (2004) and Francis (2009) find the most likely answer to be a form of utility from wealth, much like the treatment of wealth as a luxury good. In this study I am agnostic about the specific reasons why households derive utility from wealth.

Instead, I argue that utility from wealth by itself fails to explain a crucial fact - that it is the relative, and not the absolute, wealth that determines the saving rate. As I show in Section 4, two households with the same income could have very different saving rates, depending on their relative wealth in their respective localities. I demonstrate this fact cross-sectionally using Chinese urban household and city-level data. Anecdotally, the relative nature of wealth and saving is also supported by the observation that while real income has seen a substantial increase in many countries over the second half of the 20th century, the saving rate in these countries has not increased. Saving rates seem to be much more influenced by the rate of development than by its level.

2.2 Economic Growth and Saving Behavior

The phenomenon of high saving rates accompanying rapid growth is well documented in the economic literature. Modigliani (1970) notes this relationship in the context of post World-War-II Europe. More recently it has been observed in the rapidly growing Asian economies, most notably China. As Deaton and Paxson (2000) state, a one percent increase in per-capita growth is accompanied by a two percent increase in the saving rate.[2]

Despite empirical evidence of the positive relationship between economic growth and saving rates, standard infinite horizon representative agent and life cycle models predict the opposite. That is, such models suggest that higher expected growth leads to greater borrowing, or consuming at the expense of future wealth. In the international context, these models suggest that a rapidly-developing country will run trade deficits, essentially borrowing from rich countries. However, the opposite is often observed in reality.[3]

In addition to its effect on aggregate and individual saving rates, rapid growth has two more closely related effects. First, Carroll and Summers (1991) find that households in rapidly growing countries experience a higher rate of increase in their consumption expendi-

[2]For empirical support for the relation between economic growth and the aggregate saving rate, see Carroll and Weil (1994) and Andersson (1999).

[3]An example in point is Chinese savings flowing into the US.

ture compared to their peers in lower growth countries. This phenomenon, observed in Lee et al. (2006), is inconsistent with the standard permanent income hypothesis (PIH) and life cycle models. The second phenomenon, observed in Carroll and Summers (1991), is that while different cohorts can have dramatically different levels of lifetime wealth, this does not cause a large difference in their consumption expenditure. For example, the lifetime (expected) wealth of a 60 year old is much lower than that of a 30 year old in a rapid growth environment, with this difference smaller in a slow growth location. Hence in a rapidly growing economy, we would expect a 60 year old to consume considerably less than a 30 year old. Interestingly, these differences do not seem to exist in data comparing US and Chinese expenditure data, as seen in Figure 1.

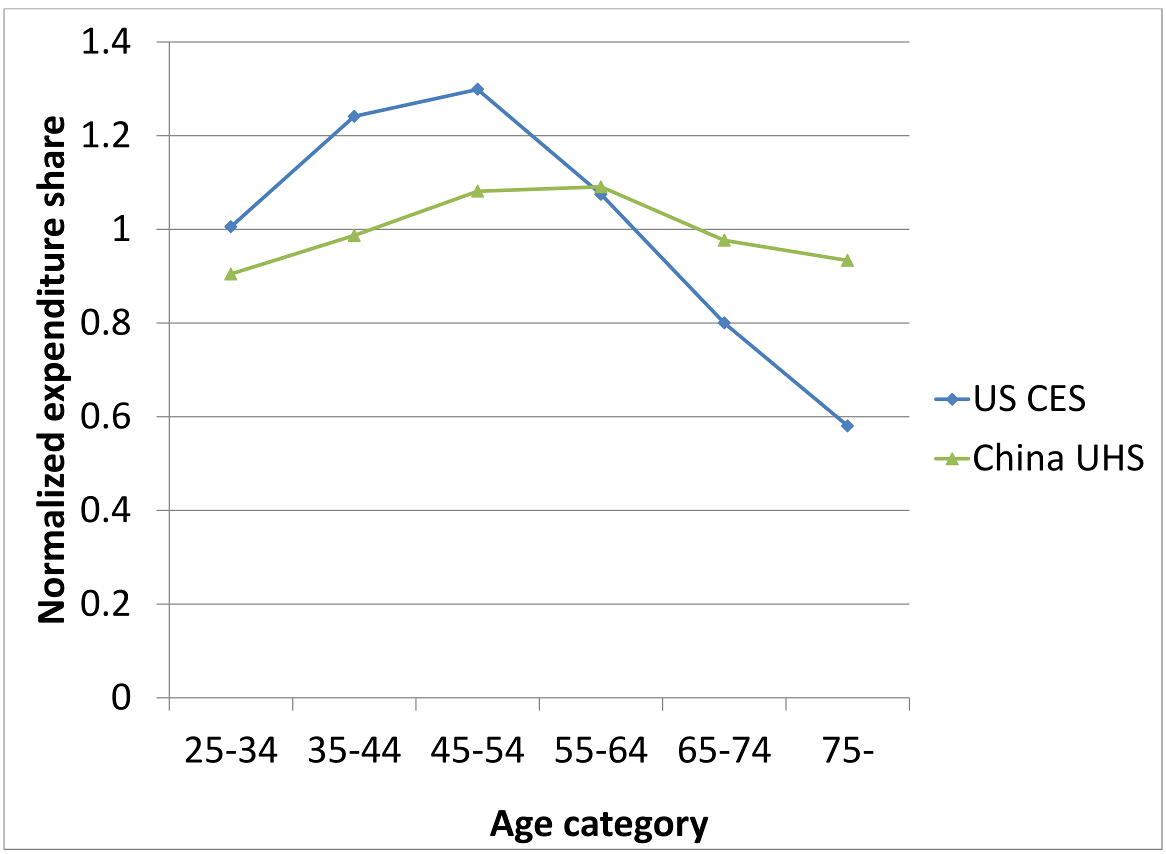

Figure 1: US and China Consumption-Age Cross-sections

3 The Data: China Urban Household Survey, Years 1993-1997

My data, acquired from the Universities Service Centre for China Studies in the Chinese University of Hong-Kong, contains detailed income and expenditure cross-sectional information on roughly 31,000 Chinese households, repeated from 1993 to 1997 (about 6,200 households per year), spread over 57 cities, across ten of China's provinces. The different cities provide significant heterogeneity in average income, both cross-sectionally and over time.

Using this income heterogeneity, I demonstrate the effect of local average income on a given household's expenditure level.

Key to this analysis is the inclusion of city identifiers, which enables me to estimate average income by locality. While this feature is unobtainable from many US household level databases due to privacy concerns, it is available in the Chinese data.[4] The availability of city identifiers and the significant average income heterogeneity between cities makes this data set an excellent testing ground for the effects of relative income. An added advantage to using Chinese data is the Chinese public record system (Dang'an tixi), which poses significant barriers to endogenous migration.

To begin the analysis, I provide a formal definition of saving. Specifically, I define the saving rate, s, of a given household, i, at a given time, t, as follows:

$$s_{i,t} = \frac{I_{i,t} - CE_{i,t}}{I_{i,t}}, \tag{1}$$

where I and CE are disposable income and consumption expenditure, respectively. Disposable income includes work income (including bonuses and benefits), capital income and incoming transfer payments (such as pensions, subsidies and family assistance). Consumption expenditure includes food, clothing, furniture, appliances, decorations, utensils, home

[4]For example, the US Consumer Expenditure Survey contains, at best, state-level identifiers, while US Public Use Microdata Sample (PUMS) includes geographical identifiers, but no expenditure data.

services (e.g., cleaning, cooking and child-sitting), medical expenses, communication, transportation, recreation, education and childcare, rent and other commercial goods and services. It does not include interest, loans paid, gifts, family assistance, house purchases or building expenses.

4 Data Analysis and Empirical Results

As mentioned, it is well-documented that the wealthy save a larger share of their income.[5] China is no exception, as Figure 2 shows. This has been observed by Chamon and Prasad (2010), who show the difference in saving rates between income deciles in China to be increasing from 1995 to 2005. According to Chamon and Prasad (2010), in 2005, the average saving rate of the 1st income decile was around zero, while that of the 10th decile was about 30%.[6]

Next I examine the importance of relative wealth on saving choices. To do so, I divide the cities into quartiles based on the average household income per city. Table 1 gives the mean income and saving rates by income quartiles of the surveyed cities. To avoid household composition endogeneity issues (e.g., the choice of adult children to stay or establish their own household), household income is defined as the sum of the incomes of the household head and spouse.

Income Quartile	**Avg. Income (RMB)**	**Avg. Saving Rate**	**Obs.**
1	6090.1	.1361	7878
2	7627.8	.1397	7945
3	8912.6	.1124	8049
4	14334.3	.1538	7446
Total	9165.7	.1351	31318

Table 1: Average income and saving rates by city income quarters, years 1993-1997

[5]See Kuznets (1953), Carroll (2000), Dynan et al. (2004) and Francis (2009).

[6]In view of my results, a possible explanation for this increase in saving rate disparity is an increase in within-location income inequality over that period.

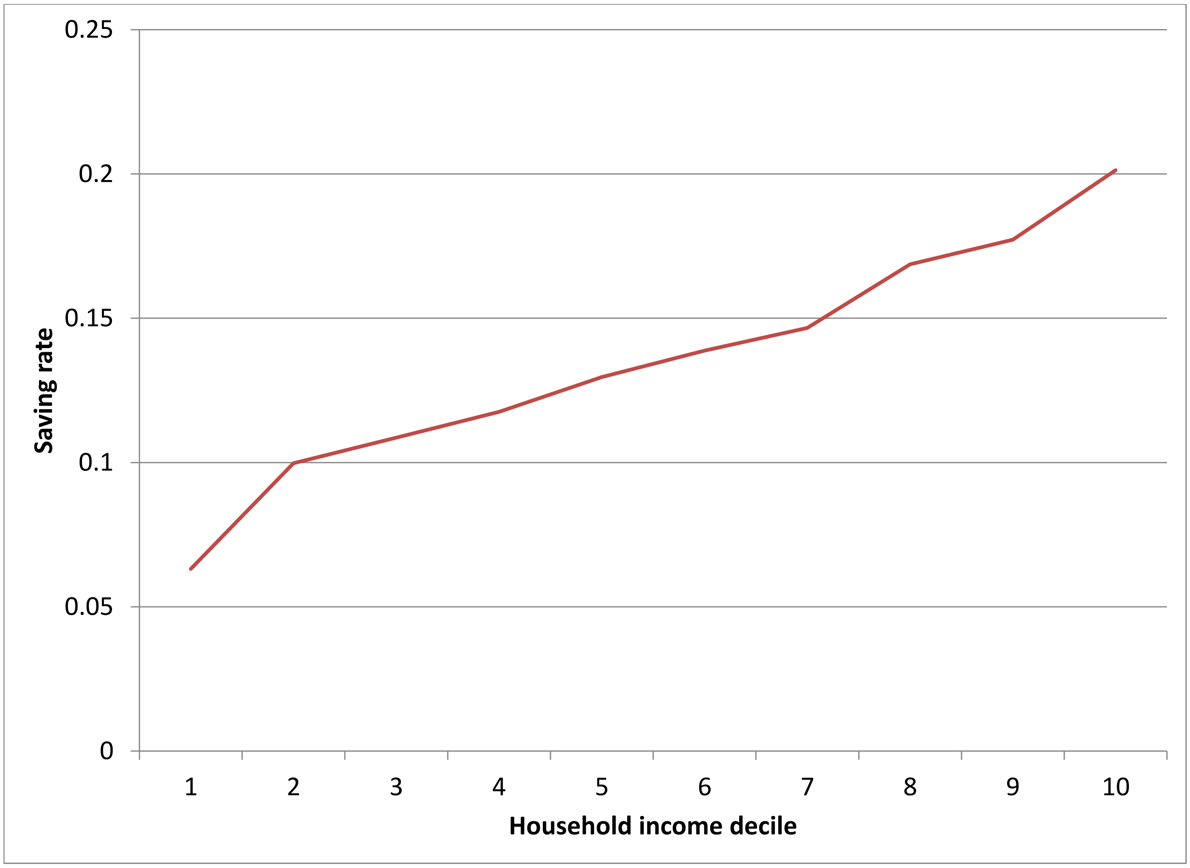

Figure 2: Saving rates by household income decile, general sample

Knowing the Chinese high national saving to GDP ratio, the average household saving rate in Table 1 may seem low. However, it is important to remember that since high saving rates are correlated with high income, the aggregate household saving rate is understated by averaging saving rates across households. It is also important to note that household surveys often undersample high income households.[7] Indeed, Chinese national accounts imply a higher aggregate household saving rate. In addition, the aggregate saving rate in China has further increased since the years the survey used in this work was taken.

[7]Chamon et al. (2010) demonstrates the inconsistency between Chinese household survey and national accounts data for saving rates over time.

Figure 3 presents the difference between the mean saving rate of households in each income bracket and the mean saving rate in each city income quartile. The results in Figure 3 indicate that a household within the same income bracket saves relatively more in a poor city (where it is rich by comparison), and relatively less in a rich city (where it is poor by comparison).

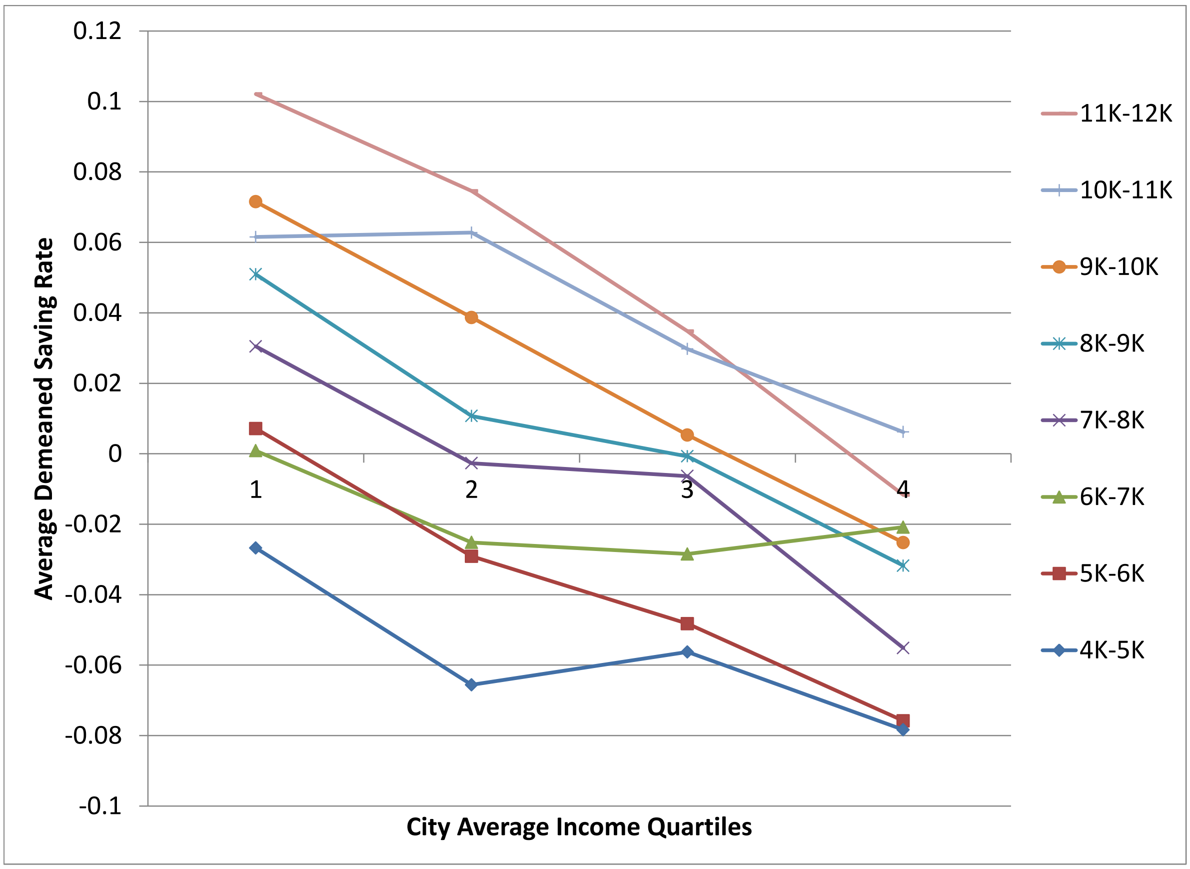

Figure 3: Saving rates given income by average income in location

Table 2 presents the results of regressing the household saving rate on different combinations of the log of household income, the log of average household income in the locality, the log of relative income (the difference between the two) and demographic covariates. All regressions are clustered by city.

As expected, column 1 in Table 2 shows that (absolute) household income has a positive effect on that household's saving rate. With a t-statistic over 6, this result is significant. Column 2 introduces the log of average income. With a t-statistic over 20, this result is also significant. Its inclusion causes the t-statistic of household income to increase from 6 to 10, and the R^2 to increase by about 50%, from 0.0425 to 0.0649. As expected, the sign of the coefficient on local average income is negative, meaning that higher local average income leads households to consume more and save less. Note that the size of the respective effects of income and average income is similar, so that the hypothesis that it is only the relative income that affects the household's saving rate cannot be rejected at a 5% statistical significance level. Column 3 includes household income and relative income. In this regression, absolute income is no longer statistically significant. Column 4 drops absolute income. The R^2 drops just barely, from 0.0649 to 0.0646. The coefficients of the rest of the variables are hardly affected. In column 5 I include city fixed effects, to absorb and city specific conditions, such as different price levels. The R^2 increases to 0.098, but the F-statistic drops from 78.8 to 73.3. City fixed effects are not included in the regressions presented in columns 1-3 because they would absorb the effect of average city income and make it impossible to discern between relative and absolute income.

To summarize, columns 1-5 indicate that, *ceteris paribus*, it is the relative income of a household compared to the local average that determines its saving rate, and that absolute income has no additional explanatory power.

This finding can be viewed graphically in Figure 4, which sorts households in the different city income quartiles into relative income deciles, with each household's income normalized by the average income in its location.

As can be seen from Figure 4, households making a certain ratio of the local average income in their respective city save a similar portion of their income, regardless of its absolute level.

	Absolute	Abs. and Local	Abs. and Relative	Relative	City FE
	(1)	(2)	(3)	(4)	(5)
log income	.089	.162	.013		
	(.014)***	(.008)***	(.013)		
log av. city income		-.150			
		(.014)***			
log rel. income			.150	.162	.163
			(.014)***	(.008)***	(.008)***
age of head	-.006	-.007	-.007	-.006	-.005
	(.001)***	(.001)***	(.001)***	(.001)***	(.0008)***
age squared	.0001	.0001	.0001	.0001	.0001
	(.00002)***	(.00002)***	(.00002)***	(.00002)***	(1.00e-05)***
high edu. head	-.002	-.014	-.014	-.013	-.012
	(.007)	(.007)**	(.007)**	(.007)*	(.006)**
high edu. spouse	-.013	-.023	-.023	-.022	-.023
	(.006)**	(.006)***	(.006)***	(.006)***	(.005)***
High school head	.006	-.0009	-.0009	-.0005	-.002
	(.004)	(.004)	(.004)	(.004)	(.004)
High school spouse	-.007	-.011	-.011	-.010	-.015
	(.005)	(.005)**	(.005)**	(.005)**	(.003)***
male	.008	.006	.006	.005	-.006
	(.008)	(.008)	(.008)	(.008)	(.004)
independent	.036	.035	.035	.035	.035
	(.015)**	(.016)**	(.016)**	(.016)**	(.014)**
adult children	.024	.026	.026	.026	.021
	(.004)***	(.003)***	(.003)***	(.003)***	(.003)***
minor children	-.027	-.027	-.027	-.027	-.030
	(.004)***	(.004)***	(.004)***	(.004)***	(.004)***
owner	.018	.018	.018	.018	-.0002
	(.008)**	(.008)**	(.008)**	(.008)**	(.005)
Const.	-.598	.114	.114	.225	.227
	(.128)***	(.120)	(.120)	(.018)***	(.013)***
Obs.	29402	29402	29402	29402	29402
R^2	.042	.065	.065	.065	.098
F statistic	29.165	73.535	73.535	78.834	73.324

Table 2: Saving rate, affected by absolute, local and relative income, in logs (clustered by city)

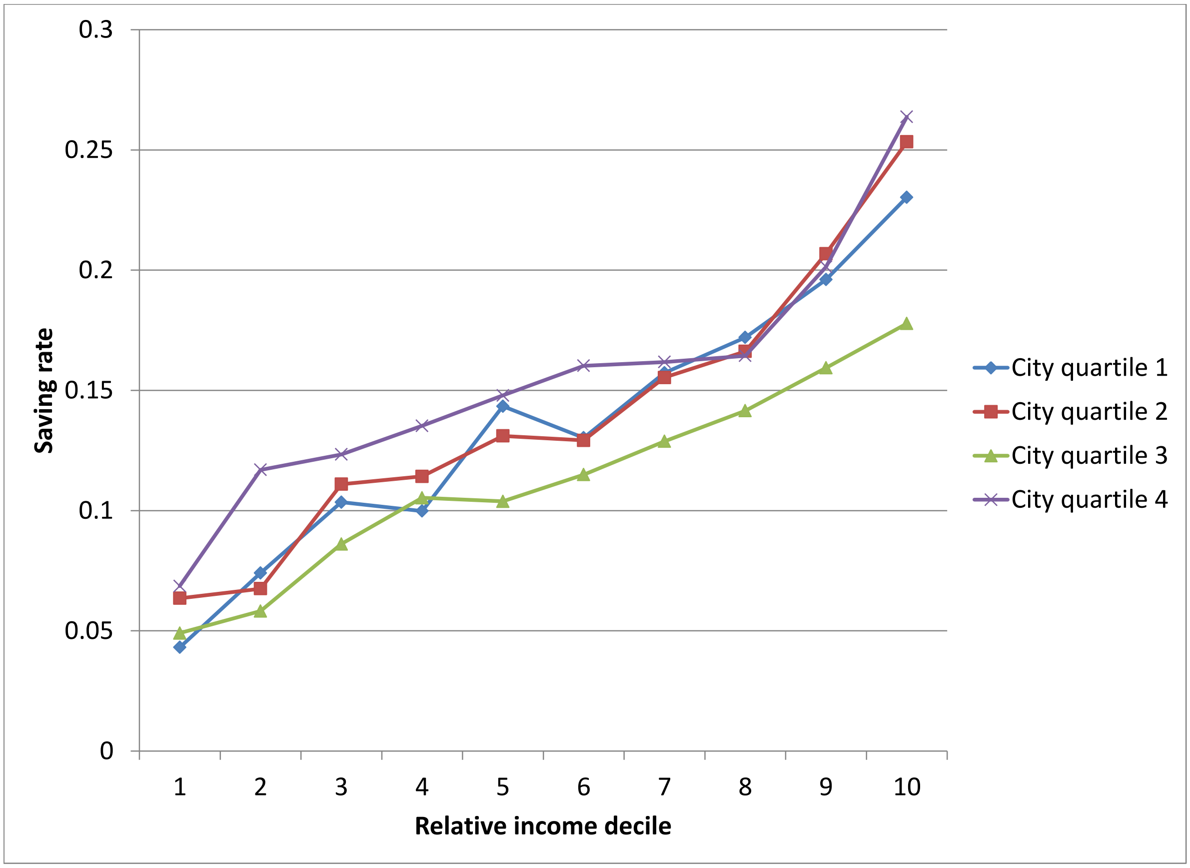

Figure 4: Saving rates by relative income deciles for city average income quartiles

5 The Hypothesis: Relative Consumption and Utility from Wealth

As noted in the introduction, the effect of comparative consumption on utility and happiness has long been a subject of research, with strong evidence that surrounding consumption levels affect individual utility. The present paper suggests that the elements of preference manifested in such behavior, namely relative consumption and utility from wealth, together explain differential saving rates. While each element already exists in the literature, this work is first to integrate the two into one model.

5.1 Reasons for Comparative Consumption Behavior

A common explanation for this competitive possessive behavior is the notion of status seeking, as portrayed by Veblen (1899). According to this explanation, individuals and households, when their survival is assured, seek to maximize utility from the social status that consumption and / or asset holdings bestow upon them.[8] In this context, underconsumption (compared to the local norm) may indicate low earning ability and social inadequacy. Conversely, overconsumption of luxury goods may be pointless in a 'poor' environment,[9] where it could be viewed as vain and reckless, and might make one a target for crime. In an example of how consumption choices may influence social status, Cole et al. (1992) and Wei and Zhang (2009) argue that wealth, or the amount of one's savings, determines status and is influential in important non-market matters such as the choice of spouse. While such behavior may be suboptimal from an individual 'rationality' perspective , Weber (1905) would argue that it is optimal at the society level.

In this paper I focus on local average consumption as the household's base of self-comparison. In another study, Charles et al. (2009) shows that ethnic groups are also a powerful reference for determining one's comparative status.

I would like to note that there may be reasons other than status behind the importance of relative consumption.

First, some studies suggest that strong local social networks can affect household saving decisions, in fear of friends and relatives asking for financial help.[10]

Second, a local market is most likely geared toward common consumption. In order to consume rare goods, one may have to pay high premiums. In addition, information about such goods is scarce – local knowledge cannot be relied upon to decide on specific goods. Thus, individuals may retain local consumption patterns regardless of income.

[8] Assuming wealth is somewhat observable, this explanation bears some resemblance to the modeling of conspicuous consumption in Charles et al. (2009).

[9] If no one else has a car, owning a basic one will give you just about the same 'rich' status as buying a Rolls-Royce.

[10] An example of a paper portraying such behavior is Goldberg (2010).

Third, if consumption choices are at least partly formed by imitation, it is reasonable to assume similar consumption profiles by locality.[11] In other words, due to the practice of local imitation, wealthier households may not know how to adjust the composition of their consumption in order to extract high utility from its increase, and may thus reach something resembling satiation.

Fourth, average local income can conceivably be highly correlated with the level of local infrastructure, which in turn may a consumption facilitator. For example, the lack of paved roads would inhibit the purchase of vehicles, the lack of Internet access would diminish the attractiveness of computers and the lack of restaurants would limit dining out.

The above elements can all be part of the explanation for the role of relative income in determining household saving behavior. I choose to be agnostic about the weights of the roles played by different elements, and instead focus on what they all have in common: the importance of relative income in determining saving behavior.

5.2 Alternative Explanations

This work offers preference-based explanations for the relationship between wealth and saving behavior. It is often argued that, as virtually any human phenomenon can be explained by creating the appropriate set of preferences, this approach is superficial. My answer to this point is twofold. First, social context intuitively is a plausible consideration for understanding preferences. Second, a preference-based approach provides a better explanation than alternative approaches. Below, I dispute three of the most intuitive alternatives.

- Household heterogeneity – reversing the causality, it could be argued that it is not that the rich save more, but rather that those who save more, become rich. In essence, this explanation suggests that there is something different about certain households

[11] It would seem that social networks propagate consumption patterns. For example, Christakis and Fowler (2007) find that obesity spreads over social networks, Christakis and Fowler (2008) find that smoking (and quitting smoking) is a network-based trait, and Rosenquist et al. (2010) show the same for alcohol consumption.

which causes them to both earn and save more. Thus, extra saving may enhance their earning ability (through financing private business, obtaining higher education etc.). An oft-mentioned form of such heterogeneity is the level of time discounting, or patience. That is, patient households save more early in their lifecycle. Additionally, patience and ability may be correlated, causing patient households to have a higher income on average. However, both Carroll (2000) and Dynan et al. (2004) find that patience heterogeneity cannot fully account for the increased saving rates of the rich. In addition, such household would show lower saving rates later in the lifecycle, which is not the case, as evident in Figure 5.

- Income Mean reversion – households may treat the part of their earning that is above the local average as transitory, and thus be more likely to save. Conversely, low income households will consume more, as they expect their income to increase to the mean.

  However, this explanation can be tested. To do so, I set Equation 2 as the base equation, presented in column 1 of Table 3, and also in column 2 in Table 2:

$$s_i = \beta_0 + \beta_1 y_i + \beta_2 \bar{y}_i + \beta_3 z_{1,i} + \epsilon_i, \tag{2}$$

  where s_i is the saving rate of household i, y_i its income, $\bar{y}_i$ is the average income in its location and $z_{1,i}$ represents demographic variables, as listed in Tables 2 and 3. As seen in Table 2, $\beta_1 + \beta_2 = 0$.

  I now use an auxiliary regression to break y_i into its predicted and unpredicted parts. Let

$$y_i = \gamma_0 + \gamma_1 z_{1,i} + \gamma_2 z_{2,i} + \epsilon_{y,i}, \tag{3}$$

  where z_2 represents the additional demographic variables, listed in Appendix A. Generating an R^2 of over 0.55, this regression provides income fitted values, which are the predicted income terms, $\hat{y}_i$, and residuals, which are the unpredicted terms, $\hat{e}_{y,i}$.

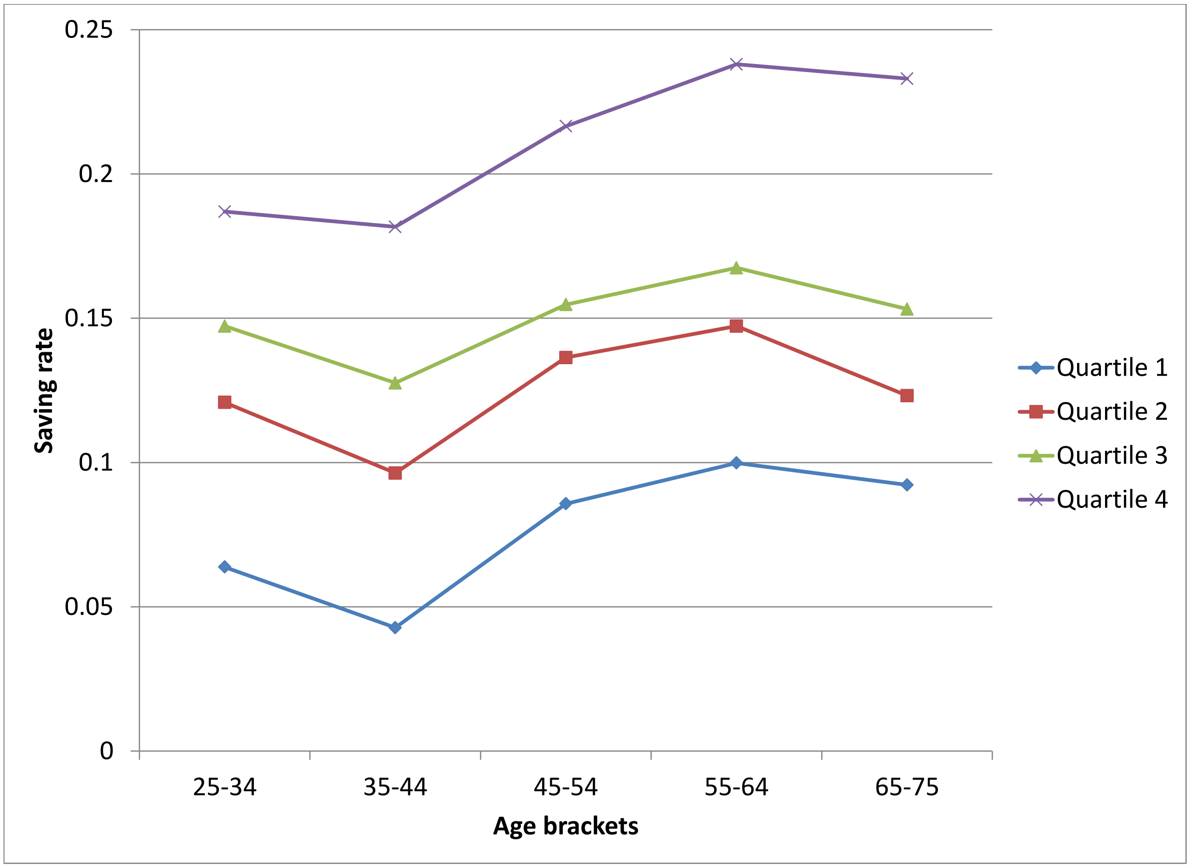

Figure 5: Saving rates over the lifecycle by household relative income quartiles (within age brackets)

Using the auxiliary regression results, Equation 4 estimates the effect of income on the saving rate using both the predicted and unpredicted parts, to eliminate the mean reversion effect, following Dynan et al. (2004). The results are given in column 2 of Table 3.

$$s_i = \delta_0 + \delta_1 \hat{y}_i + \delta_2 \hat{e}_{y,i} + \delta_3 \bar{y}_i + \delta_4 z_{1,i} + \epsilon_i. \quad (4)$$

	Base	Predicted and Unpredicted
	(1)	(2)
log income	.162	
	(.008)***	
predicted log income		.105
		(.032)***
unpredicted log income		.167
		(.008)***
log average city income	-.150	-.097
	(.014)***	(.026)***
age of head	-.007	-.005
	(.001)***	(.001)***
age squared	.0001	.00009
	(.00002)***	(.00002)***
high education head	-.014	-.002
	(.007)**	(.010)
high education spouse	-.023	-.011
	(.006)***	(.009)
high school head	-.0009	.006
	(.004)	(.005)
high school spouse	-.011	-.004
	(.005)**	(.006)
male	.006	.005
	(.008)	(.008)
independent	.035	.035
	(.016)**	(.016)**
adult children	.026	.027
	(.003)***	(.003)***
minor children	-.027	-.027
	(.004)***	(.004)***
owner	.018	.017
	(.008)**	(.008)**
const.	.114	.134
	(.120)	(.125)
obs.	29402	29402
R^2	.065	.066
F statistic	73.535	69.318

Table 3: Saving rate, affected by total income and predicted and unpredicted income, in logs (clustered by city)

From the results, it is evident that unpredicted income (closely related to the permanent income hypothesis notion of transitory income) has a higher positive effect on the saving rate compared to predicted (or permanent) income. Note that predicted income has a positive and significant effect on the saving rate, and that the sizes of the effects of predicted income and the local average are very close. That is, $\delta_1 + \delta_3 = 0$, so that relative income is still what determines the saving rate.

- Price levels – given the existence of non-tradables, it stands to reason that price levels in high income locations are relatively high. One could argue that, due to different price levels, a household with a given income in a high income location actually has less purchasing power than a household of comparable income in a low income location. Thus it is not being poorer, relative to the local mean, that makes the former household save less, but rather actually being poorer, in terms of purchasing power. As Table 2 shows, the effects of household income and local average income on the saving rate are similar in size. This is also evident in Figure 4.

  For the price level explanation to hold, prices would need to increase one-to-one with average local income. As long as prices do not increase one-to-one with average local income, the price level difference between localities does not weaken the explanatory power of relative income. To show this, let us first assume that P_i, the price level experienced by household i, is linearly correlated with local average income:

$$P_i = \phi \bar{y}_i, \qquad (5)$$

  where $\phi \geq 0$.

  Then, subtracting the price levels (in log terms) from both household income and average local income, cancels out, as $\beta_1 + \beta_2 = 0$:

$$s_i = \beta_0 + \beta_1(y_i - P_i) + \beta_2(\bar{y}_i - P_i) + \beta_3 z_{1,i} + \epsilon_i = \beta_0 + \beta_1 y_i + \beta_2 \bar{y}_i + (\beta_2 - \beta_1)P_i + \beta_3 z_{1,i} + \epsilon_i. \quad (6)$$

On the other hand, if $\phi \approx 1$, the price level can replace local average income entirely:

$$s_i = \beta_0 + \beta_1(y_i - P_i) + \beta_2 z_{1,i} + \epsilon_i = \beta_0 + \beta_1 y_i - \beta_1 P_i + \beta_3 z_{1,i} + \epsilon_i \approx \beta_0 + \beta_1 y_i - \beta_1 \bar{y}_i + \beta_3 z_{1,i} + \epsilon_i. \quad (7)$$

If ϕ nears unity, it would indicate that given prices, the average purchasing power is equal across all cities. Nevertheless, the data and some back of the envelope calculations indicate that $\phi < 1$.

To illustrate, I identify the price level P_i is a function of the prices and shares of tradable and non-tradable goods. I then assume p_t, the price of tradables (uniform throughout the economy), $\bar{P}$, the national price level and $\bar{y}$, the national average income, are all normalized to unity. Next I assume that $p_{nt,i}$, the local price of non-tradables (mostly services and real-estate), is one-to-one with average income:[12]

$$p_{nt,i} = \bar{y}_i, \quad (8)$$

where $\bar{y}_i$ is relative to the national average $\bar{y}$.

It follows that:

$$P_i = (1 - \tau_i) + \tau_i \bar{y}_i, \quad (9)$$

where τ_i is the share of non-tradables at locality i.

[12] This is a worst case scenario, as it is hard to justify prices going beyond one-to-one.

Evidently, the local price level P_i can only be argued to near a one-to-one relation with average local income ($\phi \approx 1$) if τ_i is close to unity. However, using expenditure shares from the Chinese urban household survey as presented in Appendix B, it is clear that the shares of food and clothing are consistently above 60% in all city groups. Therefore, τ, the share of services and other non-tradables, cannot approach unity. The consistency of expenditure shares also suggests that there is no major difference in relative prices between poorer and richer cities.

Consequently, it is unreasonable to assume that prices vary by the same scale as income, and thus a price-level explanation of the findings does not suffice. In addition, the price level explanation is insufficient to explain the fact that real income levels in developed countries have increased historically without a corresponding increase in saving rates.

6 The Model: Relative Consumption and Utility from Wealth in a Lifecycle Setting

Any model attempting to replicate the empirical observation of the effect of relative income on the saving rate should ideally meet both of the following conditions:

Condition 1 (Saving rate neutral in equal relative income) *If* $\{s_t\}_{t=1}^{T}$ *is optimal given* $\{y_t\}_{t=1}^{T}$ *and* $\{\bar{y}_t\}_{t=1}^{T}$*, then* $\forall x > 0$*,* $\{s_t\}_{t=1}^{T}$ *is optimal for* $\{x * y_t\}_{t=1}^{T}$ *and* $\{x * \bar{y}_t\}_{t=1}^{T}$*.*

Condition 2 (Saving rate increasing in relative income) $\forall x > 1$*, if* $\{s_t\}_{t=1}^{T}$ *is optimal given* $\{y_t\}_{t=1}^{T}$ *and* $\{\bar{y}_t\}_{t=1}^{T}$*, and* $\{s_t^*\}_{t=1}^{T}$ *is optimal given* $\{x * y_t\}_{t=1}^{T}$ *and* $\{\bar{y}_t\}_{t=1}^{T}$*, then* $s_t^* > s_t \forall t$*,*

where s_t and y_t are the saving rate and income of the household in period t respectively, and $\bar{y}_t$ is the average income in the locality of the household in period t.

To create a hybrid utility function which meets the above conditions, I use elements from two types of models present in the literature: the spirit of capitalism models and the relative consumption models. Separately, each of these model types is unable to meet conditions 1 and 2. However, I show that by integrating the two model types, both conditions can be met.

Standard spirit of capitalism models, such as the ones featured in Carroll (2000), Dynan et al. (2004) and Francis (2009), replicate the increased saving rate of the rich by introducing assets into an intra-period utility function. Let

$$u(c_t, a_t) = \frac{c_t^{1-\theta}}{1-\theta} + \lambda \frac{(a_t + \chi)^{1-\alpha}}{1-\alpha}, \tag{10}$$

where c and a denote consumption and asset holding, respectively; θ and α are the consumption and asset coefficients of relative risk aversion, $\theta > \alpha$; λ is the weight on utility from assets, $\lambda \geq 0$; and $\chi \geq 0$ is the Stone-Geary luxury factor for asset utility.

Since this functional form treats assets like a luxury good, the share of assets increases with lifetime wealth. Since χ is (weakly) positive, low income households will hold assets only for lifecycle saving purposes (preferring utility from consumption over utility from assets). By contrast, high income households will hold assets for utility (overcoming χ and preferring to hold assets over the forgone consumption), with richer households expending a larger share of their wealth on assets, due to the $\theta > \alpha$ condition.

Note that the spirit of capitalism model fulfills Condition 2, since the saving rate increases in income, but fails to meet Condition 1, as average local wealth is not included in the model. Equation 10 implies that as average lifetime wealth increases, so should the aggregate saving rate.

Can a utility function using relative consumption account for households with relatively higher income saving more? Consider the following function, which uses the Galí (1994) formulation:

$$u(c_{t,i}, \bar{c}_{t,i}) = \frac{{c_{t,i}}^{1-\theta} \bar{c}_{t,i}^{\theta}}{1-\theta}, \tag{11}$$

where $c_{t,i}$ and $\bar{c}_{t,i}$ are individual and average household consumption in locality i at time t, respectively.

As Harbaugh (1996) and Carroll et al. (1997) discuss, a relative consumption element may generate a causal relation between economic growth and the saving rate. However, for a given time profile of average consumption, the inclusion of an average local consumption term amounts to multiplying a standard Constant Relative Risk Aversion utility function by a set of constants. As such, it does not cause high income households to save differently. Thus, it meets Condition 1 (as would any standard CRRA utility function), but fails to meet Condition 2.

Overall, neither element, by itself, is able to account for the observed phenomenon of relative income influencing a household's saving rate. However, the following intra-period hybrid utility function, is able to replicate this fact:

$$u(c_{t,i}, \bar{c}_{t,i}, a_{t,i}, \bar{a}_{t,i}) = \frac{c_{t,i}^{1-\theta} \bar{c}_{t,i}^{\theta}}{1-\theta} + \lambda \frac{\left(a_{t,i} + \chi \bar{a}_{t,i}\right)^{1-\alpha} \bar{a}_{t,i}^{\alpha}}{1-\alpha}, \tag{12}$$

where $\bar{a}_{t,i}$ is the average asset holding in locality i at time t.

In this model, relative assets function as a luxury good, whose share increases with relative wealth.[13] Note that, as in equation 11, this utility function is homogeneous of degree 1. Growth, also as in equation 11, increases both $\bar{c}$ and $\bar{a}$, thus causing an increasing consumption path for the household.

I now use equation 12 within a lifecycle perfect foresight framework:

$$U = \sum_{t=1}^{T} \beta^{t-1} u(c_{t,i}, \bar{c}_{t,i}, a_{t,i}, \bar{a}_{t,i}). \tag{13}$$

[13] The relative terms of consumption and assets can be interpreted as status indicators, as in Bakshi and Chen (1996).

Assuming $1 + r = \frac{1}{\beta}$ for simplicity,[14] optimization yields the following Euler equations:

$$\left(\frac{c_{t,i}}{\bar{c}_{t,i}}\right)^{-\theta} = \left(\frac{c_{t+1,i}}{\bar{c}_{t+1,i}}\right)^{-\theta} + \lambda\left(\frac{a_{t,i}}{\bar{a}_{t,i}} + \chi\right)^{-\alpha} \quad \forall t < T, \tag{14}$$

$$\left(\frac{c_{T,i}}{\bar{c}_{T,i}}\right)^{-\theta} = \lambda\left(\frac{a_{T,i}}{\bar{a}_{T,i}} + \chi\right)^{-\alpha}. \tag{15}$$

Note that these Euler equations depend solely on consumption and asset ratios, and are neutral in levels.

As standard, the within period budget constraint is:

$$y_t + (1 + r)a_{t-1} = c_t + a_t. \tag{16}$$

Finally, the household must have non-negative assets at the close of life:

$$a_T \geq 0. \tag{17}$$

Because households draw utility from their asset holdings, final period assets may be greater than zero. Since my model does not include utility from bequests, I assume all remaining assets are taken by the government, so that the income of existing households remains unaffected.

Other elements abstracted from the model include income uncertainty, timing-of-death uncertainty, private and public transfer payments (such as a pay-as-you-go pension), lifecycle weighted utility and household structure dynamics related to relative and absolute income levels (e.g., choice of number of children).

As the propositions below state, the hybrid utility function creates saving rate income level neutrality, so that only relative income affects the household saving behavior, and

[14] The assumption that r is constant, consistent with a small open economy, is needed in order to make the proofs in this paper tractable. I further assume that $1 + r = \frac{1}{\beta}$ for convenience, though this later assumption is not necessary for the proofs.

a household saving rate increasing with relative income. For the respective proofs, see Appendix C.

Proposition 1 (Saving rate neutral in equal relative wealth) *If* $\{s_t\}_{t=1}^T$ *is optimal given* $\{y_t\}_{t=1}^T$ *and* $\{\bar{y}_t\}_{t=1}^T$*, then* $\forall x > 0$ $\{s_t\}_{t=1}^T$ *is optimal for* $\{x * y_t\}_{t=1}^T$ *and* $\{x * \bar{y}_t\}_{t=1}^T$.

Proposition 2 (Asset share for all periods increasing in relative wealth) $\forall x > 1$*, if* $\{a_t\}_{t=1}^T$ *is optimal given* $\{y_t\}_{t=1}^T$ *and* $\{\bar{y}_t\}_{t=1}^T$ *and* $\{a_t^*\}_{t=1}^T$ *is optimal given* $\{x * y_t\}_{t=1}^T$ *and* $\{\bar{y}_t\}_{t=1}^T$*, then* $\frac{a_t^*}{x*y_t} > \frac{a_t}{y_t} \forall t$.

Note that Proposition 1 is identical to Condition 1, whereas Proposition 2 is necessary, but not sufficient, for Condition 2. Analytically, I am able to prove only that the ratio of assets to income increases with relative income. While this does not prove that the saving rate increases with relative income, my proof implies that the ratio of final period assets, which is the discounted value of the stream of lifetime savings, to lifetime wealth is increasing in relative wealth.[15]

7 Calibration and Simulation

Parameter	**Value**
T (length of household lifecycle)	55
θ (consumption utility coefficient)	3
α (assets utility coefficient)	1
λ (assets utility weight)	0.05
χ (assets utility Stone-Geary parameter)	0
β (time preference)	0.95
r (international real interest rate)	2%

Table 4: Calibration of Model Parameters

To test the ability of my model to replicate the empirical relation between relative wealth and the household's saving rate, I conduct a simulation. In this simulation, households live

[15] In addition, my numerical results show a strong and consistent increase of the saving rate in relative wealth, as shown in Figures 12 and 10.

for 55 periods (roughly ages 25 to 80). The timing of household death and future income flows are both known in advance. I assume a constant growth rate throughout the household's lifetime.[16]

I use the income-age profile in my data, as seen in Figure 6, for the simulation. Of special interest is late-age income, typically comprised of pensions and other transfer payments. The relation between working-age and retirement-age income is of paramount importance in determining saving behavior in a lifecycle model. The survey data indicate a fairly flat post-retirement income, which, in a high growth environment, would suggest that transfer payments increase with economic growth.[17]

Another important issue in designing my simulation is the potential survey selection bias: low income elderly people are more likely to join their children's households, leaving the sample. I choose to use the household income-age profile in the data for calibration. An alternative would be to use an individual income-age profile, thereby circumventing the endogeneity of household formation and dissolution. However, the option of joining a child's household may be of great value to a parent, equivalent to substantial transfer payments. An individual income profile would not capture this value.

While my profile choice is explained above, the specific profile does not drive my results. I experiment with several profiles (e.g., a model with no pension income, a flat cross-section income profile up to retirement at household age 40 and a profile based on US data), and obtain similar qualitative results.

To simulate income inequality, I draw income quantiles from a log-normal income distribution, with a mean and variance matching the those in the data. To reduce the effect of lifecycle income variation, I use only households with a head between 35 and 55 years old for calculating the income distribution's mean and variance. Income profiles are symmetric

[16] This is a simplification, as high-growth countries are likely to catch up with the development frontier, slowing down as they near it.

[17] While pensions are usually based on pre-retirement wage rates, and as such should not generally be affected by post-retirement growth, this does makes sense for private (within-family) transfers. If the income of working family members increases due to growth, they should be able to increase assistance for retired family members.

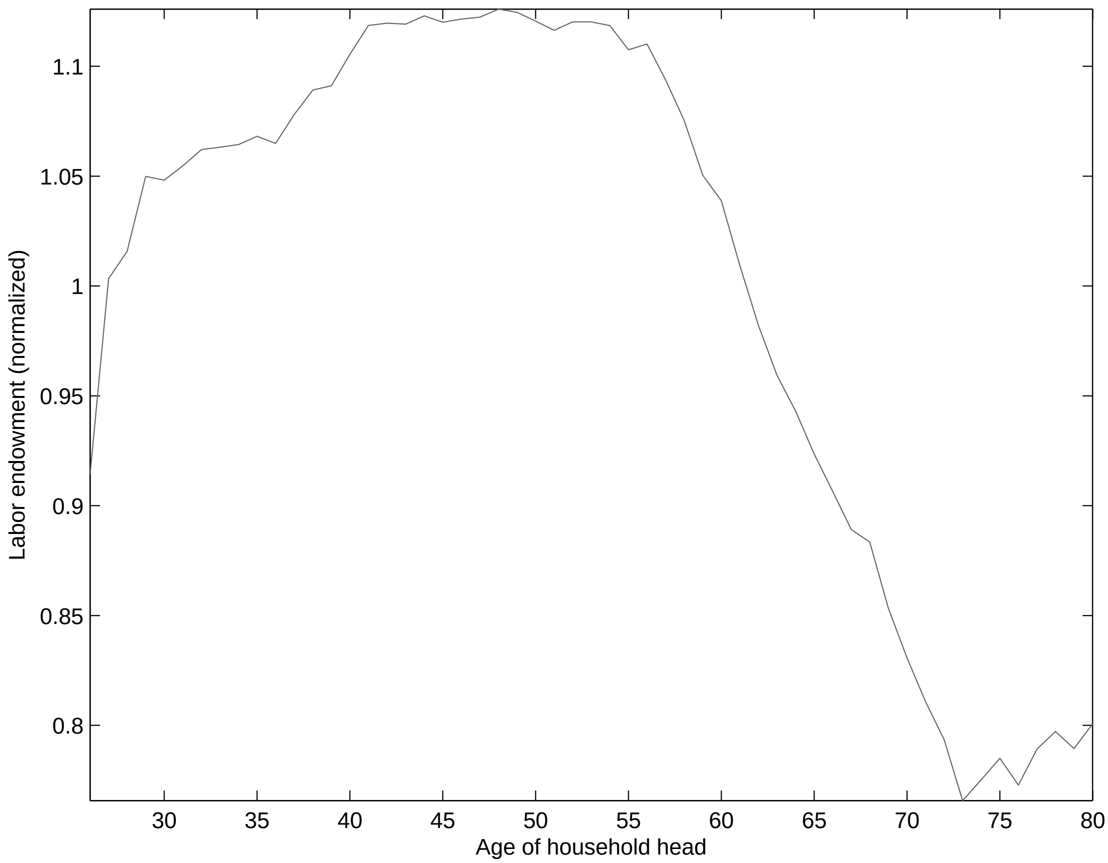

Figure 6: Labor endowment by age

between quantiles, as illustrated in Figure 7.

I simulate a range of growth rates. Growth affects household income directly, as seen in Figure 8. This is the context in which late-age income matters most, since rapid growth has a very strong effect on it.

For θ, the consumption risk-aversion parameter, I use a value of 3. I set α, the asset semi risk-aversion parameter, to one third of θ, a value of 1, in order to replicate the effect of greater asset holdings by the relatively rich as a share of lifetime wealth.

The remaining parameters, λ and χ somewhat offset each other. While λ encourages households to prefer asset holding to consumption and has a stronger effect on the rich, χ

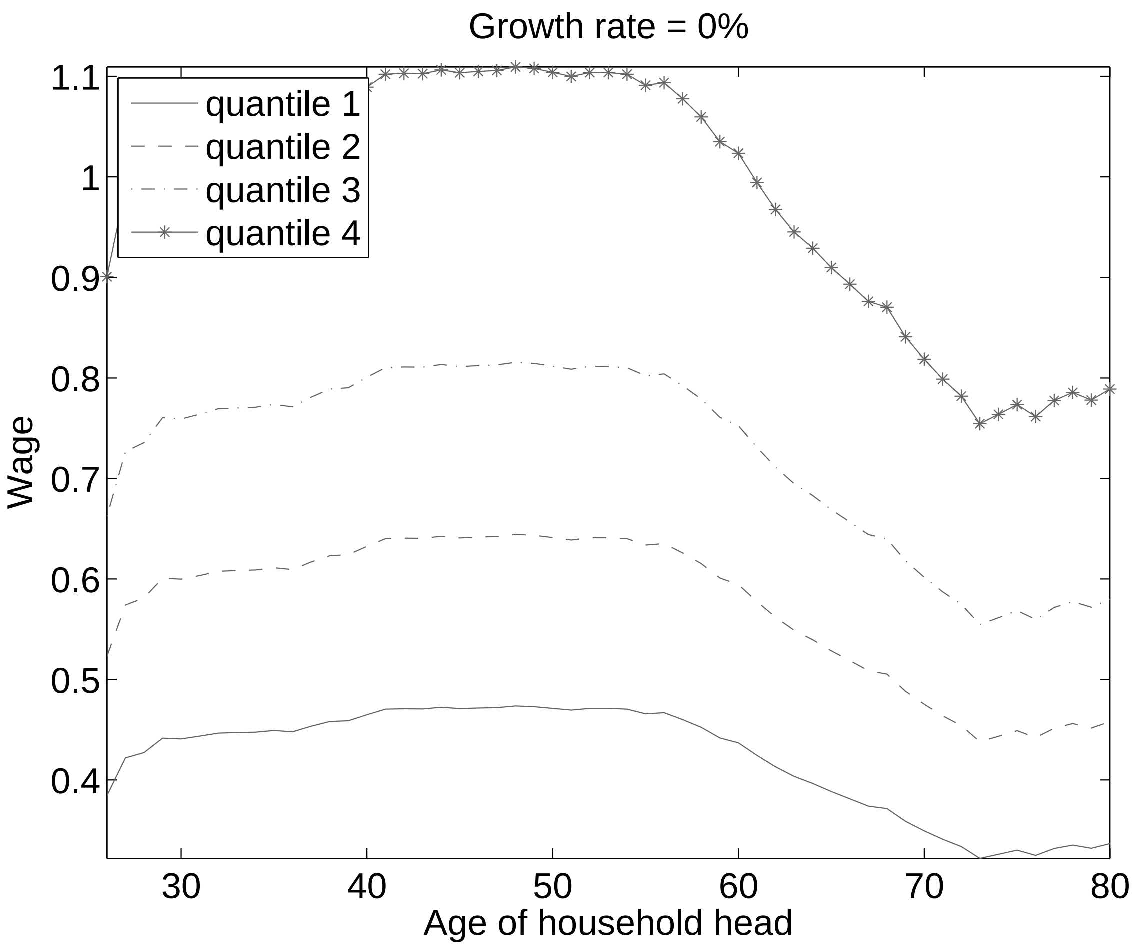

Figure 7: Income-age profile by quantile

discourages households from holding assets, and has a stronger effect on the poor. However, since these effects are disproportional, they cannot be reduced to a single variable. While a positive λ is imperative for creating utility from assets, adding a positive χ stops lower income households from preferring asset holding to consumption, creating a saving-rate kink as described in Carroll (2000).

Thus, to simplify the calibration, I set χ to zero. While this means that all households hold assets for utility, implicitly entailing a non-negativity asset constraint and a positive end-of-life asset holding, the poor will hold very little of their lifetime wealth in assets while the rich will hold a much larger share. This is similar to findings from the US Survey of

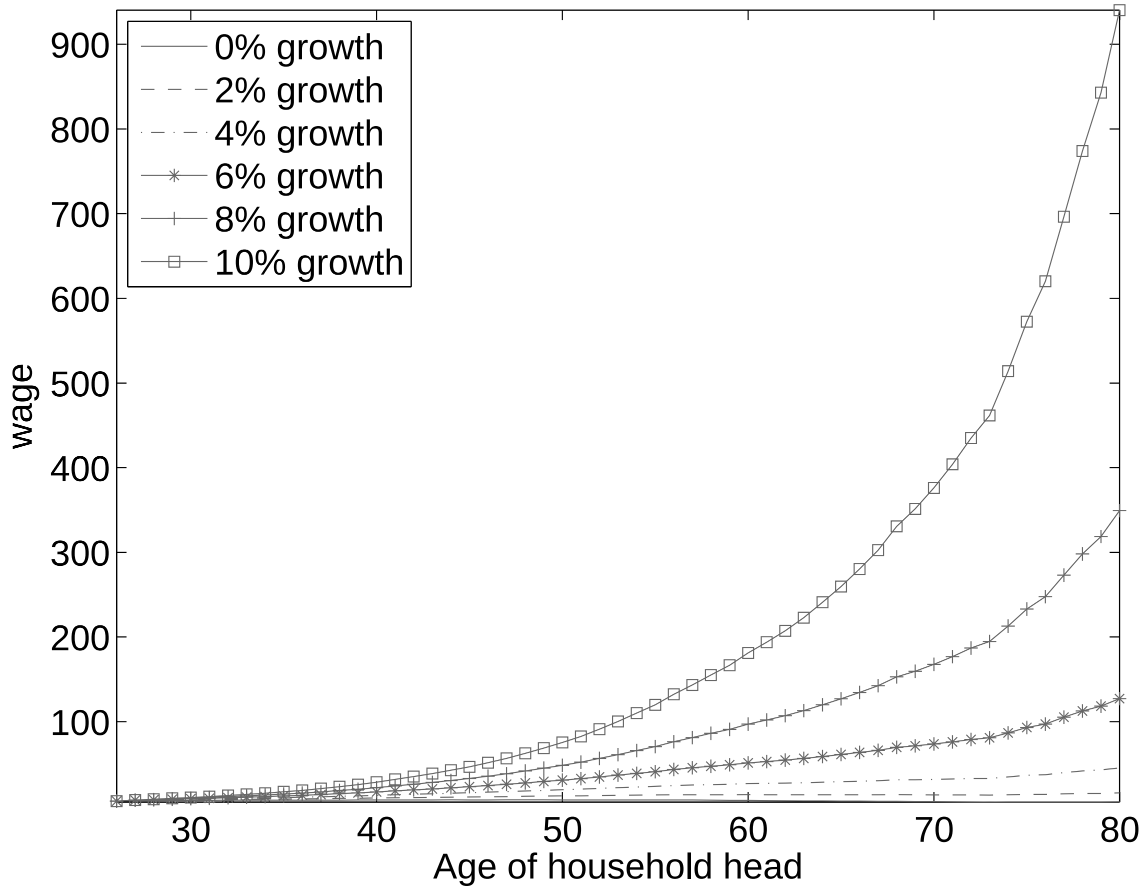

Figure 8: Average income-age profile by growth rate

Consumer Finances data, as shown in Carroll (2000).

I set λ to a value of 0.05, to match the saving rate by income quantile profile from Figure 4.

8 Comparing Simulation and Data

I find that the simulation is able to replicate the main stylized facts from the data. The 4% growth line is quite similar to that of the Chinese data presented in Figure 4.

As in the data shown in Figure 5, the simulated saving rate differentials between income quantiles are consistent throughout the lifecycle. Figure 10 displays the saving rates across

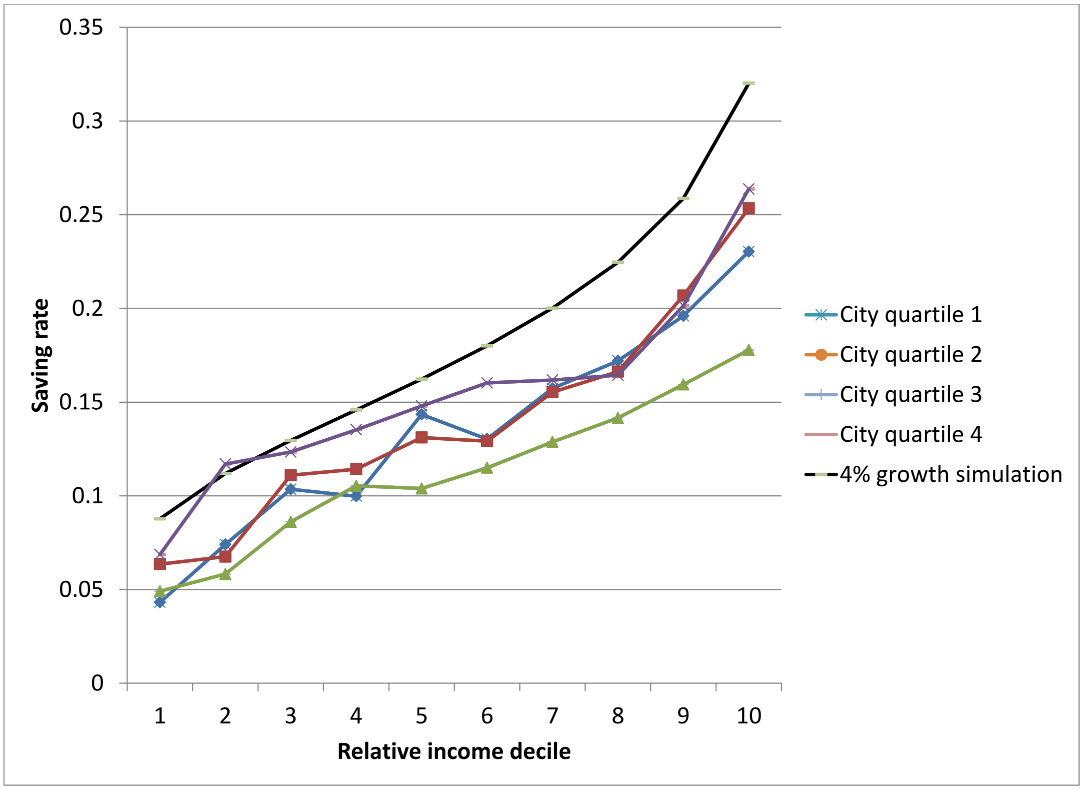

Figure 9: Saving rate by local income quantile

ages for different quartiles.

Simulation results are also consistent with the main stylized facts regarding the effects of growth on saving / consumption behavior. As Figure 11 shows, the consumption-age cross-section profile remains similar even across remarkably different growth rates, despite the inter-cohort lifetime wealth disparity created by growth.[18]

Finally, the simulation reinforces the point that the hybrid model is able to generate a positive relation between output growth and the aggregate saving rate. As Figure 9 shows, the saving rate increases for both income quantile and economic growth rate. Note that, for the zero growth line, the lower quantile households save almost nothing.

[18]This conforms to the data, as discussed in Section 2.2.

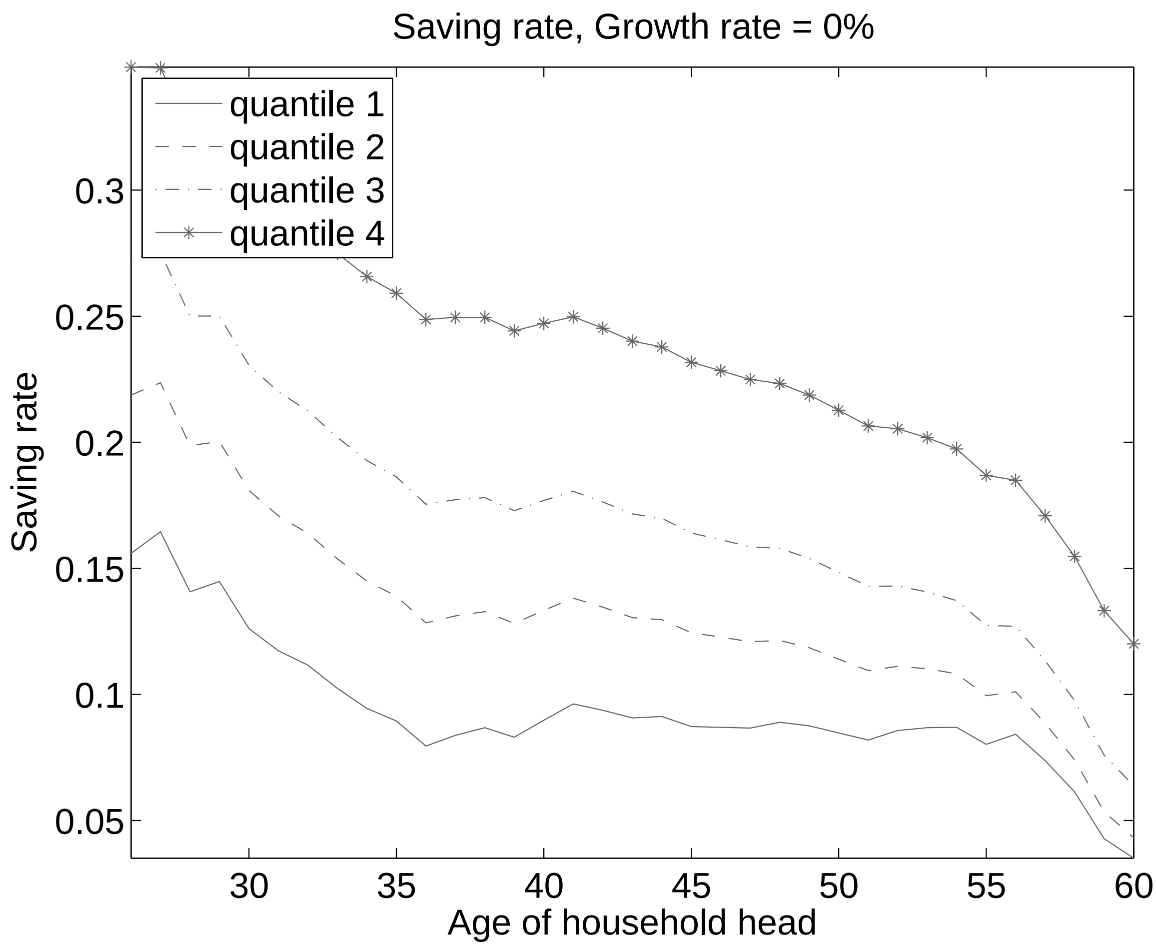

Figure 10: Saving rates over the lifecycle by quantiles

Figure 13 presents the simulated aggregate saving rate for a range of economic growth rates, compared with a corresponding international cross-section. The countries used for the international comparison are listed in Appendix D. Growth rates are averaged over ten years, as they are much more volatile than the saving rate.

The simulated values of the aggregate saving rate by growth present a trend similar to the data, but are generally lower. This can be explained, at least partly, by the role of the other determinants of the aggregate saving rate, i.e., retained earnings by firms, and government savings.

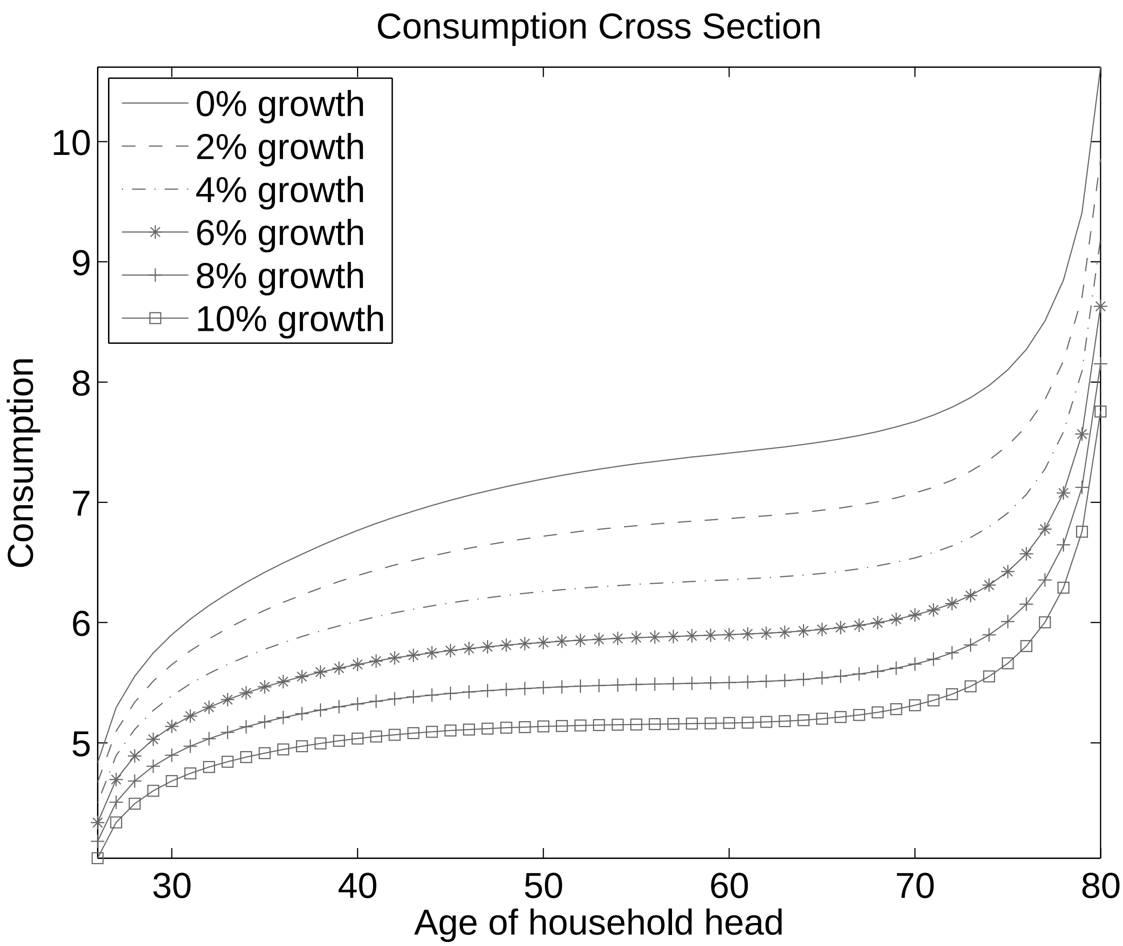

Figure 11: Consumption cross-section

9 Conclusion

Using Chinese household data, I show that relative income plays a crucial role in determining a given household's saving rate. While it is well established in the literature that the rich save a larger share of their income, the fact that it is *relative* wealth that determines the saving rate is a new empirical finding.[19]

The explanation I suggest for this phenomenon is based on the importance of relative consumption and relative asset holdings in households' preferences. Specifically, I create a hybrid utility function model, weighting both the household's consumption and utility

[19]This has been suggested by Duesenberry (1949), but without substantial empirical support.

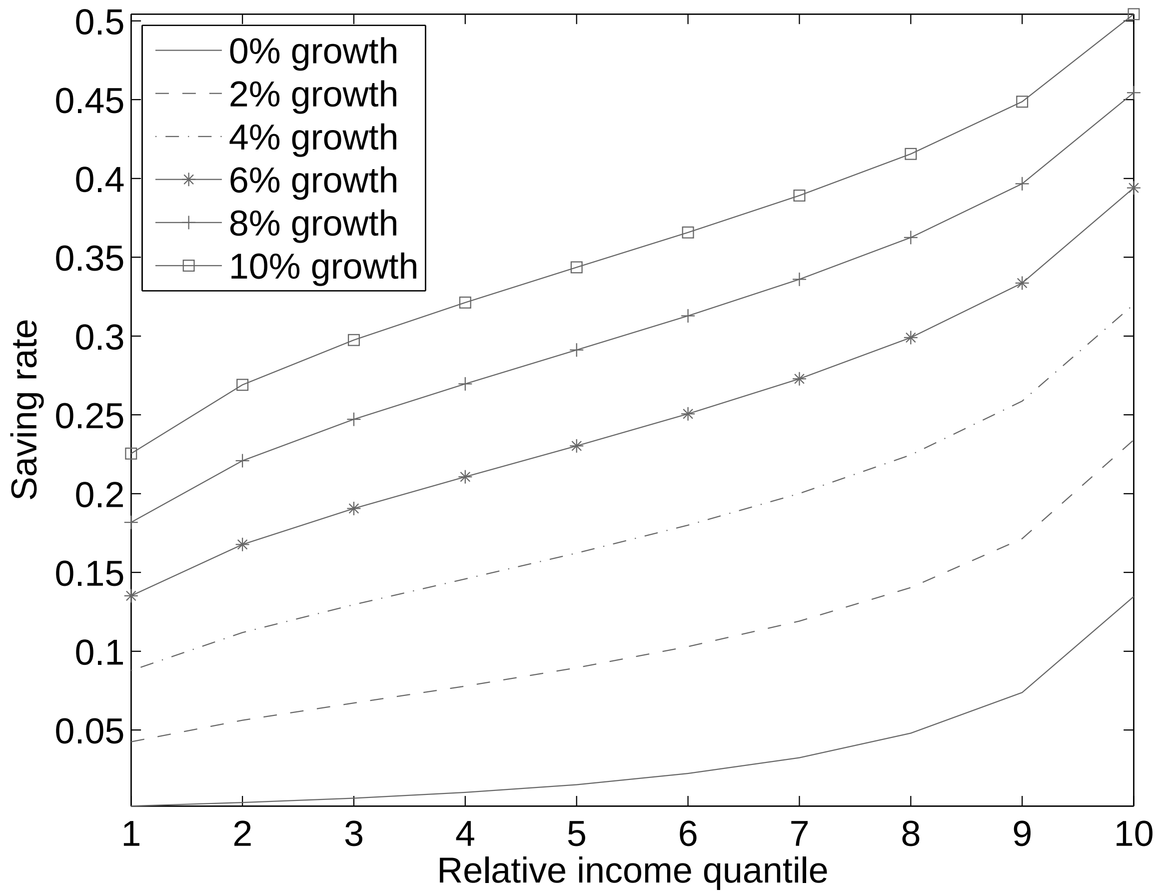

Figure 12: Saving rate by local income quantile and aggregate growth rate

from assets with average local consumption and asset holding levels. This utility function is shown, both analytically and via simulation, to recreate behavior evident in the data.

Overall, my findings suggest that social status may play a large role in determining households' economic behavior, with local rankings in consumption and asset holding functioning as indicators of social success. Such preferences are instrumental in explaining differences in saving behavior not only between rich and poor locations, but also between high and low growth environments. With economic growth, a household may direct more wealth to future use, keeping up with the increase in average income, and commensurately, average consumption and asset holdings. Thus, these preferences, constructed to account for the ef-

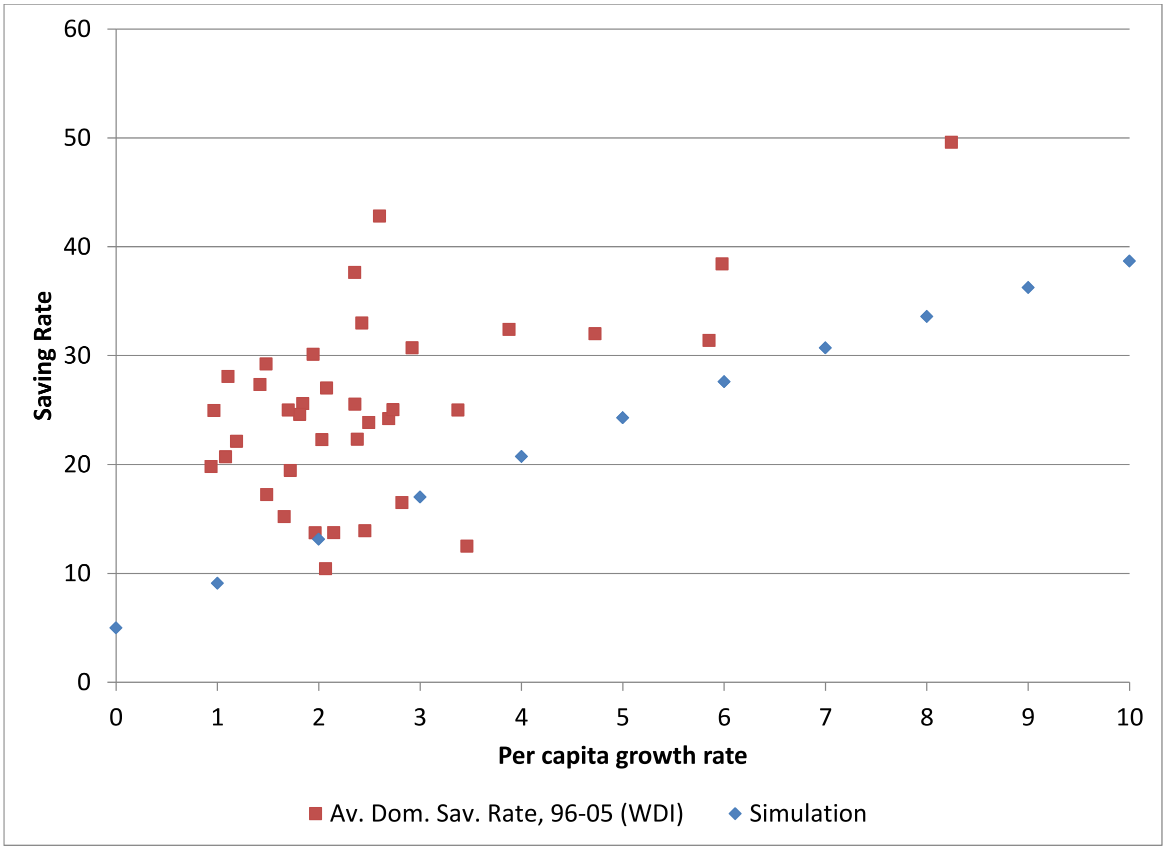

Figure 13: Aggregate saving rate by growth, simulation and WDI data

fect of relative income on household saving rates, are also able to match the empirical effects of rapid growth on saving behavior. I am able to generate age-consumption cross-sections symmetric across growth levels, and simulate a positive relation between aggregate growth and saving rates that closely resembles the data.

Looking forward, the hybrid model has interesting implications for the dynamic behavior of households of different relative wealth levels in response to aggregate and idiosyncratic income shocks. In addition, it provides a framework of analysis for the economic behavior of households in different economic growth, income distribution and social identity environments.

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A Auxiliary Regression

log income	
age of head	.020 (.0009)***
age squared	-.0003 (.00002)***
high education head	.129 (.007)***
high education spouse	.193 (.007)***
high school head	.074 (.005)***
high school spouse	.109 (.005)***
male	-.061 (.005)***
independent	.003 (.018)
work sector dummies	
work status dummies	
profession dummies	
city dummies	
year dummies	
const.	7.818 (.364)***
R^2	.558
F(112, 29289)	329.77
Obs.	29402

Table 5: Auxiliary regression on household income

B Consumption Shares

Table 6: **Food**

qtile	mean	p50	N
1	.5593268	.5630299	7878
2	.5407054	.5411122	7945
3	.536771	.5361337	8049
4	.5169927	.5125776	7446
Total	.5387406	.5389511	31318

Table 7: **Clothing**

qtile	mean	p50	N
1	.1513846	.1388798	7878
2	.1516372	.1372549	7945
3	.1363584	.1236035	8049
4	.1055242	.0894131	7446
Total	.1366833	.1218562	31318

Table 8: **Home Equipment, Goods and Services**

qtile	mean	p50	N
1	.0542762	.0282269	7878
2	.061942	.0337838	7945
3	.064296	.0340283	8049
4	.078116	.0455057	7446
Total	.0644641	.0348963	31318

Table 9: **Medical Expenditure**

qtile	mean	p50	N
1	.0334353	.0163043	7878
2	.02977	.0148002	7945
3	.0278337	.0130197	8049
4	.0300433	.0153396	7446
Total	.0302593	.0147758	31318

Table 10: **Transportation**

qtile	mean	p50	N
1	.0147247	.0050909	7878
2	.017311	.0088028	7945
3	.018942	.0101331	8049
4	.0241895	.0118173	7446
Total	.018715	.0087756	31318

Table 11: **Communication**

qtile	mean	p50	N
1	.0110864	0	7878
2	.0151562	0	7945
3	.0233213	.0013078	8049
4	.0242935	.0083831	7446
Total	.0184034	.0004126	31318

Table 12: **Entertainment Durables**

qtile	mean	p50	N
1	.0121275	0	7878
2	.0139232	0	7945
3	.0144456	0	8049
4	.0233325	0	7446
Total	.0158429	0	31318

Table 13: **Education**

qtile	mean	p50	N
1	.0477914	.0262692	7878
2	.0497142	.0292281	7945
3	.0461365	.0262985	8049
4	.0435007	.0258181	7446
Total	.0468337	.0267708	31318

Table 14: **Culture and Recreation**

qtile	mean	p50	N
1	.0149257	.0094613	7878
2	.0176527	.0119488	7945
3	.0210411	.0146362	8049
4	.0290694	.0225761	7446
Total	.020552	.0140692	31318

Table 15: **Rent and Home Repairs**

qtile	mean	p50	N
1	.0192804	.0053653	7878
2	.0206443	.0112423	7945
3	.0226388	.0129579	8049
4	.0315875	.013605	7446
Total	.0234156	.0108458	31318

Table 16: **Utilities**

qtile	mean	p50	N
1	.0489261	.0428028	7878
2	.0461125	.0426292	7945
3	.0487464	.0437873	8049
4	.0466288	.0403951	7446
Total	.0476199	.0425022	31318

Table 17: **Miscellaneous**

qtile	mean	p50	N
1	.032715	.0204039	7878
2	.0354313	.0226931	7945
3	.0394693	.0265168	8049
4	.0467219	.0321755	7446
Total	.0384702	.025039	31318

C Proofs

Proposition 1 (Saving rate neutral in equal relative wealth) *If $\{s_t\}_{t=1}^T$ is optimal given $\{y_t\}_{t=1}^T$ and $\{\bar{y}_t\}_{t=1}^T$, then $\forall x > 0$ $\{s_t\}_{t=1}^T$ is optimal for $\{x * y_t\}_{t=1}^T$ and $\{x * \bar{y}_t\}_{t=1}^T$.*

Assumption 1 *Assume $\bar{c}_t = \gamma_1 \bar{y}_t$, $\bar{a}_t = \gamma_2 \bar{y}_t$, so that if $\{\bar{y}_{2,t}\}_{t=1}^T = \{x * \bar{y}_{1,t}\}_{t=1}^T$, then $\{\bar{c}_{2,t}, \bar{a}_{2,t}\}_{t=1}^T = \{x * \bar{c}_{1,t}, x * \bar{a}_{1,t}\}_{t=1}^T$.*

Proof 1 *By Assumption 1, if $\{c_t, a_t\}_{t=1}^T$ is optimal given $\{\bar{y}_t, y_t\}_{t=1}^T$, then $\{x * c_t, x * a_t\}_{t=1}^T$ is optimal given $\{x * \bar{y}_t, x * y_t\}_{t=1}^T$ (where optimality is defined by equations 14-17), so that the saving rate profile is identical.*

Since the saving rate profile is identical, Assumption 1 holds true. ■

Lemma 1 (End-of-life asset share increasing in relative wealth) *Given $\{y_t\}_{t=1}^T$, define $Y = \sum_{t=1}^T (1+r)^{T-t} y_t$. If $y_{2,t} = x * y_{1,t} \forall t$, where $x > 1$ (household 2 has higher income) and $\bar{y}_2, t = \bar{y}_1, t \, \forall t$ (both households are in a similar average income locality), then $\frac{a_{1,T}}{Y_1} < \frac{a_{2,T}}{Y_2}$ (household 2 will save a larger portion of its lifetime income).*

Proof 2 *By equation 15, we get:*

$$\hat{a}_T = \frac{\theta}{\alpha} * \frac{\frac{a_T}{\bar{a}_T} + \chi}{\frac{a_T}{\bar{a}_T}} * \hat{c}_T, \tag{18}$$

where hat notations denote a rate of change. Since $\frac{\theta}{\alpha}$, $\frac{\frac{a_T}{\bar{a}_T} + \chi}{\frac{a_T}{\bar{a}_T}} > 0$, $\hat{a}_T > \hat{c}_T$. However, we need to show that $\hat{a}_T > \hat{Y}$. Defining $C = \sum_{t=1}^T (1+r)^{T-t} c_t$, we get $a_T = Y - C$, so we then need to show $\hat{a}_T > \hat{C} = \sum_{t=1}^T (1+r)^{T-t} c_t \hat{c}_t / \sum_{t=1}^T (1+r)^{T-t} c_t$.

Combining equations 14 and 15, we get:

$$\left(\frac{c_t}{\hat{c}_t}\right)^{-\theta} = \lambda \sum_{j=1}^{T-t+1} \left(\frac{a_{T+1-j}}{\bar{a}_{T+1-j}} + \chi\right)^{-\alpha}. \tag{19}$$

Therefore the change rate of consumption at time t is a weighted average of the change rates of asset holdings at periods t to T, multiplied by numbers smaller than unity:

$$\hat{c}_t = \frac{\alpha}{\theta} \frac{\sum_{j=1}^{T-t+1} \left(\frac{a_{T+1-j}}{\bar{a}_{T+1-j}} + \chi \right)^{-\alpha} \frac{\frac{a_{T+1-j}}{\bar{a}_{T+1-j}}}{\frac{a_{T+1-j}}{\bar{a}_{T+1-j}} + \chi} \hat{a}_{T+1-j}}{\sum_{j=1}^{T-t+1} \left(\frac{a_{T+1-j}}{\bar{a}_{T+1-j}} + \chi \right)^{-\alpha}}. \tag{20}$$

By equation 16 (the budget constraint):

$$y_t \hat{y}_t + (1+r) a_{t-1} \hat{a}_{t-1} = c_t \hat{c}_t + a_t \hat{a}_t. \tag{21}$$

By definition (in lemma 1), $\hat{y}_t = x - 1 = \hat{y} \, \forall t$.

(proof by contradiction)

Assume $\hat{a}_T < \hat{y}$. From equation 18, $\hat{a}_T > \hat{c}_T$. Define accordingly $1 > \delta_{1,T}, \delta_{2,T}$ so that $\hat{a}_T = \delta_{1,T} \hat{y}$ and $\hat{c}_T = \delta_{2,T} \hat{y}$. Then:

$$a_{T-1} \hat{a}_{T-1} = \frac{1}{1+r} (\delta_{1,T} a_T + \delta_{2,T} c_T - y) \hat{y}. \tag{22}$$

Since $a_{T-1} > \frac{1}{1+r} (\delta_{1,T} a_T + \delta_{2,T} c_T - y)$, as evident from equation 16, then $\hat{a}_{T-1} < \hat{y}$. Therefore, as $\hat{a}_T$, $\hat{a}_{T-1} < \hat{y}$, then, by equation 20, $\hat{c}_{T-1} < \hat{y}$.

Define accordingly $1 > \delta_{1,T-1}, \delta_{2,T-1}$ so that $\hat{a}_{T-1} = \delta_{1,T-1} \hat{y}$ and $\hat{c}_{T-1} = \delta_{2,T-1} \hat{y}$. Show that $\hat{a}_{T-2}$, $\hat{c}_{T-2} < \hat{y}$ and repeat until $t = 1$.

Therefore, if $\hat{a}_T < \hat{y}$, then $\hat{c}_t < \hat{y} \, \forall t = 1, ..., T$, so that $\hat{C} < \hat{y}$. But since $Y = C + a_T$, it cannot be that $\hat{a}_T$ and $\hat{C}$ are both less than $\hat{y}$.

Thus, we have a contradiction, and it must be that $\hat{a}_T > \hat{y}$. ∎

Proposition 2 (Asset share for all periods increasing in relative wealth) *$\forall x > 1$, if $\{a_t\}_{t=1}^T$ is optimal given $\{y_t\}_{t=1}^T$ and $\{\bar{y}_t\}_{t=1}^T$ and $\{a_t^*\}_{t=1}^T$ is optimal given $\{x * y_t\}_{t=1}^T$ and $\{\bar{y}_t\}_{t=1}^T$, then $\frac{a_t^*}{x * y_t} > \frac{a_t}{y_t} \forall t$.*

Proof 3 *Assume $\hat{a}_t < \hat{y}$. Then $\hat{C}_t > \hat{y}$, and there exists $j \leq t$ such that $\hat{C}_j$, $\hat{c}_j > \hat{y}$. By* *equation 14, we have the following:*

$$\hat{c}_{t+1} = \hat{c}_t + \frac{\lambda\left(\frac{a_t}{\bar{a}_t} + \chi\right)^{-\alpha}}{\left(\frac{c_{t+1}}{\bar{c}_{t+1}}\right)^{-\theta}} \left(\hat{c}_t - \frac{\alpha}{\theta}\frac{\frac{a_t}{\bar{a}_t}}{\frac{a_t}{\bar{a}_t} + \chi}\hat{a}_t\right). \tag{23}$$

Since $\hat{c}_j > \hat{y} > \hat{a}_j$*, then by equation 23* $\hat{c}_{j+1} > \hat{c}_j$ *and* $\hat{C}_{j+1} > \hat{y}$*. By induction,* $\hat{C}_s > \hat{y} > \hat{a}_s \, \forall s \geq j$*. But in lemma 1 we showed that* $\hat{a}_T > \hat{y}$*. Therefore, we have a contradiction and* $\hat{a}_t > \hat{y} > \hat{C}_t \, \forall t$*.* ∎

D Country List

Country	96-05 per capita growth av.	Av. Dom. Sav. Rate, 05
Argentina	1.42	27.34
Australia	2.49	23.85
Austria	1.84	25.58
Belgium	1.81	24.60
Brazil	0.94	19.81
Canada	2.36	25.54
Chile	2.92	30.71
China	8.24	49.60
Denmark	1.70	25.00
Finland	3.37	24.99
France	1.72	19.46
Germany	1.19	22.13
Greece	3.46	12.49
Hong Kong, China	2.43	33.00
India	4.73	31.99
Indonesia	1.48	29.23
Ireland	5.98	38.43
Italy	1.08	20.68
Japan	0.97	24.95
Korea, Rep.	3.88	32.42
Malaysia	2.60	42.82
Mexico	2.38	22.32
Netherlands	2.08	27.02
New Zealand	2.03	22.26
Norway	2.35	37.64
Pakistan	1.66	15.21
Philippines	2.07	10.42
Portugal	1.96	13.71
South Africa	1.49	17.23
Spain	2.69	24.20
Sweden	2.73	25.01
Switzerland	1.10	28.09
Thailand	1.94	30.13
Turkey	2.82	16.49
United Kingdom	2.45	13.89
United States	2.15	13.73
Vietnam	5.85	31.39

Figure 14: International Growth and Saving Rates, Source: World Development Indicators, World Bank