A Partial Equilibrium Model of Leverage Effects on Asset Prices:
Evidence from S&P 500 Index Put Option Prices

By

Robert Geske* and Yi Zhou+

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Abstract

The primary purpose of this paper is to show that leverage is important to asset pricing. By testing a partial equilibrium model that includes capital structure choices we demonstrate the importance of stochastic leverage. To our knowledge this is the first paper to attempt to directly isolate and analyze the effects of aggregate market leverage of the "market portfolio" on asset prices, here S&P 500 equity index options. To do this, first we introduce a methodology for measuring the daily *implied market value* of aggregate debt in the 500 firms comprising the S&P 500 index by using the known face value of debt and the market value of aggregate equity. We emphasize *implied market value* of aggregate debt (similar to implied volatility) from current market option prices and equity prices because this methodology is parsimonious, forward looking, and uses only contemporaneous prices. Second, we demonstrate significant effects of stochastic leverage and resultant stochastic equity volatility on equity index option prices. We isolate this pure leverage effect by making a standardized comparison to Black-Scholes, a partial equilibrium model without leverage, which is a special case of our model with leverage. This standardized comparison of *matched pairs* of option prices shows the improved results occur *because* of the inclusion of leverage, and are statistically and economically highly significant. Next, we demonstrate by similar standardized comparisons that our more parsimonious, stylized model results in significant statistical and economic improvements when compared to more complex models which omit leverage, but instead attempt to model the distribution of stock price changes by assuming a form of stochastic equity volatility and jumps. We explain the reasons for improvement can be attributed to including leverage which provides an implicit rather than explicit model of stochastic equity volatility. Thus fewer parameter estimates are required, aggregate measurement errors are reduced, and the inclusion of a term structure of volatility is possible, which is difficult to incorporate in explicit models of the equity distribution and difficult to test because of data limitations of contemporaneous option data.

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1. Introduction

The systemic effects of leverage have been a major focus of concern and study for financial markets because of the ongoing credit crisis which began in 2007/8. This period, called the "Great Recession", has been widely attributed to excess leverage. Thus, it is timely that this paper's main purpose is to investigate the systemic effects of corporate leverage on asset prices, specifically S&P 500 equity index option prices. Here the corporate market's leverage is that of the 500 firms comprising the S&P 500 index which is commonly used as the "market portfolio" by researchers in financial economics. Since options are ultra sensitive to volatility changes, and since changes in the strike prices of options which change the inherent option replicating leverage have been shown to cause changes in option volatility, options on options are ideal candidates for examining leverage effects on asset prices. S&P 500 equity index options, called SPX, are the world's most widely traded option[1], and are European, two important features for this study. The fact that SPX options are the world's most liquid option means they have minimal option price-stock price non-synchronicity. Furthermore, since SPX are European they are without the American early exercise feature which might mask differential leverage effects on options with different expirations.

Stylized option models have often been used by researchers to examine leverage effects on bond prices and credit spreads. However, to date this research has not been extended to equity index options. While stylized leverage models, like Merton's (1974) adaption of Black-Scholes, for example, cannot perfectly capture the complex nature of the balance sheet or bankruptcy conditions [2], they can and have provided useful insights about the effects of leverage on corporate bond prices and credit spreads in many academic publications and in practice by firms concerned with credit risk (c.f. Moody's-KMV). [3]

[1] See CBOE Market Stats 2004. For perspective, 50 million SPX contracts traded per year is about 200,000 contracts per trading day or 30,000 contracts per hour, which means these options trade almost continuously from open to close.

[2] It is well known that the "absolute priority" insolvency condition Merton's (1974) stylistic model imposes on corporate debt as an option boundary condition is a tremendous oversimplification of the actual bankruptcy, default or insolvency process.

[3] Simple stylized structural models incorporating leverage effects on bonds priced as options have often been used both by practitioners and academic researchers. First, note that Moody's purchased KMV's adaption of Merton's (1974) stylized

In Merton, for example, debt acts as the strike price for stock as a call option. It has been shown by many researchers that increasing in the strike price of a call option results in increases in the volatility of that call option price and return. This well known result can be understood by considering the option replicating portfolio where increasing the strike price is equivalent to borrowing more risk free bonds, D, and investing less in the risky equity, E, thus increasing leverage and the volatility of the option. [4]

When S&P 500 index option trading began in 1983, it was initially thought that the Black-Scholes model (1973) without leverage should do better at pricing options on a market portfolio of stocks than pricing options on the individual stocks in the market portfolio. Two reasons SPX index options should be priced better than individual options by Black-Scholes are because they are European options and because the limit distribution of the sum of returns on a large number of random variables is more likely to be the normal distribution. [5] However, research has shown Black-Scholes does not price these SPX options well. Much of this research, upon which our paper is based, is described herein.

It is often thought that biases in Black-Scholes arise because the underlying distribution of the state variable is not normal, and exhibits random, not constant or deterministic volatility. Although leverage can produce both effects from economic theory, many papers approach the non-normality of the distribution of stock returns by adding models of stochastic volatility. For example, Heston (1993) develops a closed-form stochastic volatility model with arbitrary correlation between volatility and asset returns, and demonstrates that this model has the ability to improve on the Black-Scholes biases when the correlation is assumed to be negative. [6] This model was difficult to implement because the volatility solution required complex numerical analysis, so Heston and Nandi (2000) develop a GARCH volatility to render the Heston model "closed form" and show this simplification produces the same results.

option approach to leverage. Moody's-KMV now licenses and uses the KMV's Merton approach to analyze leverage effects on both individual firms and portfolios of firms. Second, numerous academic articles have been published which derive and test simple stylized option models of corporate bond prices. See Merton (1974), Geske (1977), Longstaff-Schwartz (1984), Leland-Toft (1994), and Colin-Dufresne, Goldstein (1997), all summarized in Eom, Helwege, Huang (2004).

[4] See Cox and Rubinstein, 1985, for example. Change the strike price (debt D) changes the option price (equity E) without changing the underlying value (V = D+E)

[5] See Khintchine (1938) and Gnedko and Kolmogorov (1954), who show this convergence to normal is true even if the individual random variables are not IID.

[6] Wiggins (1978) and others have also developed arbitrary stochastic volatility models.

However, GARCH models rule out using possibly important volatility jumps. Pan (2002) finds that "…the stochastic volatility models of Heston (and Heston and Nandi) are not rich enough to capture the term structure of volatility implied by option prices"…, and argues a volatility term structure is important. [7] Dupire (1994) is cited with the first development of a lattice approach to best fit the cross-sectional structure of option prices wherein the volatility can depend on the asset price, strike price, and time. While this lattice approach of Dupire and others can include a term structure of deterministic volatility functions (DTV), these implied tree approaches have been found (Dumas) to work no better than *ad* hoc versions of Black-Scholes.

Bakshi, Cao, and Chen (BCC,1997) extend Black-Scholes (BS, 1973) to test three nested models of stochastic volatility (SV), stochastic volatility and jumps (SVJ), and stochastic volatility and stochastic interest rates (SVSI). They analyze index options and demonstrate some significant improvements to BS, but BCC has a large parameter advantage (9 to 1 for the best model SVJ) over BS because they do not allow BS to use a term structure of volatility since BCC cannot use one. We will show that a term structure of volatility is important, and when BS is implemented with one the BCC advantage is diminished. Bates (2000) develops more consistent tests of models with stochastic volatility and jumps which also show improvements over BS. However, Bates states, "these stochastic volatility models require extreme parameters that are implausible …, and while the stochastic volatility/jump diffusion model fits option prices better, its implicit distributions and jumps are inconsistent with the stock index price data."

Recently, Pan (2002) incorporates both volatility and stock price jumps, with either constant or state dependent jump intensity. In order to estimate these numerous additional parameters which exceed the number of available liquid contemporaneous index option prices, Pan must use a strategy which integrates the historical stock and option price data, and relies on implied-state, generalized method of moments estimation (IS-GMM) in order to measure these risk premia. While Pan finds more support for

[7] See Pan (JFE, 2002), page 31.

jump risk premia rather than volatility risk premia, she argues that better representation of the term structure of volatility is necessary, especially when pricing options of different maturities.[8]

As already mentioned much of the literature described above which followed BS is based on the idea that the distributional assumption of stock returns being normally distributed with a constant or deterministic conditional volatility is not realistic. For example, the equity distribution is observed to be asymmetric, with a fat left tail and thin right tail, which cannot occur with the normality assumption. There exists extensive empirical evidence of a *persistent* inverse relation between the level and returns on equity and the instantaneous conditional equity volatility, for both individual firms and for indexes (c.f. Christie, 1982, and Nelson, 1991). Black (1976) argued this inverse relation was caused by stochastic leverage induced by stock price changes.

Three other papers that have empirically tested models of the effects of leverage on asset prices are by Toft and Prucyk (TP, JF, 1997), Ericsson and Reneby (ER, JB, 2002), and Geske and Zhou (GZ, 2008). TP (1997) adapt a version of Leland and Toft (LT, JF, 1996) to individual stock options, and using ordinary regression in cross-sectional tests they demonstrate significant correlations between their model's variables and the individual firm volatility level and slope for a 13 week period in 1994 for 138 firms in their final sample. However, TP do not investigate the extent of option pricing improvement attributable to leverage by comparison to either BS or more complex models which omit leverage. ER (2005) use a simple and consistent option approach and a maximum likelihood methodology developed by Duan (1999) to measure the value and volatility of the firm and the leverage effects on bond prices. GZ (2008) use the option on option approach of this paper to price options on the equity of individual firms rather than on the equity index, and again demonstrate the importance of leverage. This occurs because GZ demonstrate that cross-sectional variations in debt/equity ratios are much greater for individual firms.

[8] Pan (2002), page 32, states "To accommodate a richer term structure of volatility, one solution is to allow for multiple volatility factors with different rates of mean reversion."

Here we incorporate leverage in an index option pricing model which is parsimonious because it implicitly produces stochastic equity volatility and the observed inverse relation to equity returns without explicitly modeling stochastic volatility. BCC's explicit model of stochastic volatility requires four additional parameter estimates per volatility. Thus, a simple term structure allowing four volatilities would require, for example, at least sixteen parameters in addition to the eight other parameters necessary for BCC's best model, SVJ. Such a model would be much more theoretically complex, perhaps even require additional correlations between the stochastic volatility processes in the term structure, and introduce more measurement error. In addition, data limitations on liquid index options trading daily would prohibit the use of contemporaneous data to imply these volatility parameters.

To isolate the effects of leverage on asset prices we use standardized comparisons of otherwise similar models with and without leverage. First, we compare Geske's (G,1979) generalization of BS to include firm financing choices. We investigate how much different levels of leverage observed over time effect index option prices. Since Black-Scholes is a special case of Geske and both models are implemented in the same way, their differences must be attributed to leverage. Next, we compare both G and BS to BCC's more complex, nested models, which also generalize Black-Scholes by assuming many additional process parameters necessary to model a more complex equity process which includes a stochastic volatility process, a jump process, and a stochastic interest rate process. These joint comparisons reveal that leverage is very important to pricing equity index options.

In the following pages, Section 2 describes alternative models considered with emphasis on the three tested herein, Geske (G), Black-Scholes (BS), and Bakshi, Cao,and Chen (BCC). Section 3 describes the data and discusses how these three models are implemented and compared, Section 4 describes the results, and Section 5 concludes the paper.

2. Discussion of Alternative Models

Although the majority of public corporations in the S&P 500 use both debt and equity financing, most equity option pricing models do not consider the influence of a firm's choice of capital structure on option prices. However, in a recent paper, Eom, Helwege, and Huang (EHH, 2004) examine five structural models that do consider firm's financing choices for asset pricing when the assets are corporate bonds modeled as options. The five models they examine are those of Merton (1973), Geske (1977), Longstaff-Schwartz (1984), Leland-Toft (1996), and Colin-Dufresne-Goldstein (1997). For the sake of brevity when referring to these papers, we adopt their convention and refer to author names by their initials M, G, LS, LT, and CDG, and we also use the initials. [9] Of these five structurasl bond models only those of G (1977) and LT (1996) have been extended by G (1979) and TP (1997) to directly analyze the effects of the firm's choice of leverage on equity options, and only G for index options. [10]

If, as we show in this paper, the time series variations in the *implied market value* of aggregate leverage cause significant effects on the prices of equity index options, then leverage should also exhibit significant cross-sectional effects on the prices of options on the individual firms which comprise the index. This result should be expected even before the empirical demonstration because the cross section of individual firms which comprise the index will contain greater leverage extremes than the index which is an average of these firms. As previously mentioned there have been papers which examine differential cross-sectional effects of firm leverage on individual stock options. However, to the best of our knowledge there has been no published demonstration of empirical pricing improvements resulting from tests comparing models with and without leverage for pricing equity index options. Furthermore, we are unaware of any paper which presents a methodology that uses only contemporaneous, liquid

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[9] See Eom, Helwege, Huang (2004), for notation, page 500, and pricing, Table 3, page 512. As expected they show these models are much more accurate on price estimates than on estimates of yield or spread estimates.

[10] The "choice" of leverage may be thought as a joint decision made by firm management and by the market. As the market revalues a (or all) firm's cash flows daily, then firms may find they have more or less leverage than originally chosen. This happens to most firms in a severe market crisis.

stock and option prices to measure both the implied value of the market portfolio and the corporate debt component, and then uses this measure to examine the daily effects of the implied leverage changes on index option prices.

The previously mentioned version of the Leland (JF, 1994) optimal capital structure model which TP adapt to value individual equity options is not really appropriate for pricing index options. The reason this approach is inappropriate is the distinctions which Leland showed are important for individual firms are not important for an index whose characteristics are an average of individual firm attributes. For example, Leland's differences in individual firm total debt, firm volatility, firm tax rates, firm bankruptcy costs, and firm coupon and other payouts, are not important because they are averaged and disappear in an aggregate index. Furthermore, the distinction between short term, exogenous, protective covenant bankruptcy, and longer term debt, endogenous, stockholder decided bankruptcy is also less important for an index. For an index like the S&P 500, the notion of endogenous bankruptcy decided collectively by the stake-holders in the 500 firms in the S&P is probably not too pertinent. Also, the potential for both short term insolvency effecting some firms and long term insolvency effecting other firms is mitigated by the averaging process of an index. In addition, Geske and Delianedis (1995) show that incorporating options for both short term and long term insolvency generally increases the joint default probability and thus will make the leverage effect on option prices even stronger. This can be understood by an American option analogy where adding the opportunity to exercise twice generally increases the option value.

If we consider the S&P 500 index as "the market" and observe that this market portfolio exhibits systemic credit risk which may be characterized by events that impact market solvency, and these events include market liquidity, and market volatility, each accompanied and perhaps somewhat caused by changes in market leverage. [11] The credit risk associated with changes in the market value of aggregate

[11] Leverage here is defined as the debt-equity ratio (D/E). Leverage is the important variable because it is measured relative to the equity cushion and thus is a better measure of solvency than the absolute level of debt since what is "too much" debt obviously requires consideration of the equity support.

leverage is relevant to index option pricing because an efficient option market attempts to anticipate potential important systemic changes in market solvency, liquidity, and volatility which accompany changes in aggregate market leverage. The leverage effects can be severe regardless of whether the source arises collectively from internal firm decisions, from external revaluation of firm cash flows, or from external changes in the risky discount factor. The next few paragraphs describe our approach to provide evidence for this idea. [12]

Our stylistic solvency condition is leverage dependent and based on the relation between the promised payments, F, which is a book value, and the ability to pay, V=E+D, which is an implied market value. F is the face value of total aggregate promised payments outstanding, and V, the total implied market value of the ability to pay, equals the sum of the observed market value of the equity cushion, E, and the implied market value of the promised debt payment, D. Thus V derives it's "implied" characteristic from the debt, D, not from the observed equity, E. Insolvency occurs when V < F. This condition follows Merton (1974) and as mentioned is obviously heuristic since absolute priority does not occur in credit relations. Instead of absolute priority, there are negotiations between the stakeholders, which include all debt and equity holders, and in the case of a systemic market credit event, even the government is a stakeholder.[13]

Since the model assumptions underlying BS and BCC, and their resulting equations which omit leverage, are perhaps better known than G's (1979) option model which includes leverage, we begin the discussion of these three models with a review of G's model. G's model applied to listed equity index options characterizes the state variable as the total market value, V, of the 500 firms comprising the index (V = market debt D + market equity E), whereas BS and BCC choose the state variable to be the

[12] Systemic risk is certainly present in aggregate financial markets, as demonstrated in the depression in 1930's, the 1987 global equity market crash, the 1997 Asian currency crisis, the 1998 LTCM crisis, and most recently the 2007/8 "Great Recession" credit crisis. It is possible for a systemic market credit crisis to occur from too much leverage because the market anticipates the risk that the total market value of firms promised payments may be greater than their ability to pay.

[13] See Lau & Santos, IMF 2010, for a similar application of compound option theory to total country debt and country insolvency risk. Responsible governments are corporate stakeholders, and the loss of expected corporate taxes may provide another reason for government "bailouts".

aggregate equity market value, E. Thus, G includes debt D which BS and BCC omit. As both BS (1973) and Merton (1974) have concluded, adding financing choices to the option model which allows asset pricing to be approached at the more fundamental firm level should make equity option pricing more consistent with the theory of the firm. It also will allow the examination of credit risk effects in option markets which are the markets thought to convey the most information (Black, 1976). G's model is consistent with the no arbitrage ideas of Modigliani and Miller (V=D+E). The boundary condition for exercise of the index option is an identity which depends on the critical market value, V*, defined by known option exercise prices at each expiration date. If we assume markets are perfect with respect to information, there are no transaction costs, there are no riskless arbitrage opportunities, an index options are functions of aggregate total market value, V, and time, t, and describe V in equation (1) to be following the stochastic process,

$$dV/V = \mu_v dt + \sigma_v dZ_v \tag{1}$$

where all index options expire at some T_i before the debt is to be repaid at T_D, then we can derive G's equations for pricing S&P 500 index put, P(V,t), and call, C(V,t), options:[14]

$$P = Fe^{-r_{Ftd}(T_D - t)} N_2(-h_1, h_2; -\rho) - VN_2\left(-(h_1 + \sigma_{vTi}), h_2 + \sigma_{vTi}; -\rho\right) + Ke^{-r_{Fti}} N_1(-h_1) \tag{2}$$

$$C = VN_2\,(h_1 + \sigma_{vTi}, h_2 + \sigma_{vTi};\ \rho) - Fe^{-r_{Ftd}(T_D - t)}\, N_2\,(h_1, h_2;\ \rho) - \ K\ e^{-r_{Fti}(T_i - t)} N_1(h_1) \tag{3}$$

where $$h_1 = \frac{\ln(V/V^*) + (r_{FTi} - 1/2\sigma_{vTi}{}^2)(T_i - t)}{\sigma_{vTi}\sqrt{T_i - t}}$$

[14] See Geske (1979) for more detail. Our exact implementation technique is described in more detail in Section 3. Since V = E+D and E is known, solving for D is similar to solving for V when we use observed E and solve for E+D = V. N_2 and N_1 are bivariate and univariate cumulants of the normal distribution whose limits are h_1 and h_2 and in the bivariate case ρ is the correlation between the option payoff event and the debt payoff event.

$$h_2 = \frac{\ln(V/F) + (r_{FTD} - 1/2\sigma_{vTD}{}^2)(T_D - t)}{\sigma_{vTD}\sqrt{T_D - t}}$$

and

$$\rho = \sqrt{(T_i - t)/(T_D - t)}$$

In terms of the number of unknowns, solving for numerically for V is similar to solving for D since V=E+D and E is known.[15] However, since the volatility of V is an appropriate combination of the volatilities of E and D, and because equity, E is much more volatile than debt, D, substituting the observed value for E will reduce the volatility and thus the expected range of numerical search for V = E+D. This will improve the option pricing by reducing measurement error from implied parameters. Also, since market prices of both equity and options on the equity are used in the solution and are dependent on V, this will also insure the consistency between the observed stock and option market prices because the larger and more volatile component of V is known.

G's two primary unknowns are V and σ_{vTi} (or E+D and σ_{vTi}). The third unknown, V*, is defined as the critical aggregate market value required for exercise, and is solved for simultaneously with V=E+D and σ_{vTi}. V^* is primarily based on the known option strike price, K_J, and thus known equity value E_J*, for all j strike prices, where E_J* must equal K_J, or $E_J^* = K_J$. Thus, V* is the critical total market value which determines whether each put or call option with strike price $K_j = E_j^*$ and time to option expiration T_i will be exercised. At each option expiration T_i for each exercise price K_j , if V < V* then E<K, and P = K-E (C = 0), and if V > V*, E>K, and P = 0 (C = E-K). The parameter F is the duration based face value of all 500 firm's total debt outstanding, and T_D is the duration date for the solvency check for this aggregate debt.[16] Since V* = E* + D*, and since E* is equal to known and constant K_J for all T_i, the only changes in V* at different T_i will arise from changes in D*. Thus, in

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[15] The substitution of E_t+D_t for V_t and use of the known, observed E_t is important because it is consistent to use the same E_t (and thus V_t) for all options at any specific time t.
[16] Note that Compustat balance sheet data is used to find the duration and amount of each firm's outstanding promised debt payments, and then this is aggregated to find the duration and amount of the total debt of the 500 firms in the S&P.

most cases the variation in V* will be small and dependent on changes in the discounted value. Lastly, for G's model there are two correlated options, one at time T_i for the index put option exercise, and one at time T_D for the market's solvency. Their correlation is measured by ρ where the index option expiration T_i is less than or equal to the market's debt duration T_D.

The Black-Scholes option pricing model does not consider the firm's capital structure. Instead BS assume a stochastic process for the equity, dE/E, which includes the assumptions of constant or deterministic equity volatility, and given interest rate assumptions, no arbitrage conditions, and the defined strike price boundary condition for put options, results in the following well known equation for valuing put options[17]:

$$P = -E(1 - N_1(d_1 + \sigma_{ETi}\sqrt{T_1 - t})) + Ke^{-r_{F1}(T_1 - t)}(1 - N_1(d_1)) \tag{4}$$

Where d_1 and d_2 are similar in form to h_1 and h_2, but here E is substituted for V, K is substituted for V*, and σ_{ETi} is substituted for σ_{VTi} .

We also use the Merton (M, 1974) version of the BS model for stock as a call option on the firm assets V, which has the same state variable as equation (1), and is expressed by the following equation:

$$S = V\,N_1(h_2 + \sigma_{vTD}\sqrt{T_D - t}) - Fe^{-r_{FTD}(T_D - t)}N_1(h_2) \tag{5}$$

where h_2 is defined above.

The notation for these three models, G, BS, and Merton can be summarized as follows:

P/C = current market value of an SPX index put/call option,

E = equity index level net of dividends d,

D = current market value of aggregate debt in the 500 firms in the S&P

[17] In this paper we only present results for index put options as the title indicates, but the results also hold for index call options.

V = current total (debt D+ equity E) market value of 500 firms in the S&P 500,

V* = defined as the critical total market value where $E_{Ti} \geq K$ implies $V_{Ti} \geq V_{Ti}^*$

F = face value of market debt (debt outstanding for S&P 500 firms),

K_j = strike price of the j^{th} option,

r_{Ft} = the risk-free rate of interest to date t,

σ_{vTi} = the instantaneous volatility of the total market return for options expiring at T_i

σ_{vTD} = the instantaneous volatility of the total market return for debt maturing at T_D

σ_{ETi} = the instantaneous volatility of the equity index return for options expiring at T_i,

t = current time,

T_i = expiration date of the i^{th} option,

T_D = duration of the aggregate market debt,

$N_1(.)$ = univariate cumulative normal distribution function,

$N_2(\ldots)$ = bivariate cumulative normal distribution function,

ρ = correlation between the two exercise opportunities at T_i and T_D.

d = dividends

It is important to include BS in the model comparisons of G and BCC. Since BS is a special case of G, which reduces to BS when there is no leverage (F=0), we will show that any differences between BS and G must be directly attributed to leverage. Similarly, since BS is also a special case of BCC, and BCC's models reduce to BS if the volatility and interest rates are constant and there are no jumps, then any differences between BS and BCC models must be attributed to these additional parameters. This standardized method of comparison allows us to examine the relative improvements of G to BS and compare them to those of BCC to BS. The above discussion completes our brief descriptions of G and BS. Next we briefly describe more details about the BCC model.

In order to compare G to more complex models which omit leverage but instead attempt to characterize the complexity equity distribution with other explicit stochastic processes, we implement the nested versions of the three BCC (1997) models, stochastic volatility, stochastic interest rates, and stochastic volatility with jumps (SV, SVSI, SVJ). We use identical implementation techniques as described in their paper, and we replicate BCC's Table 3, to confirm our matching their methodology. BCC implement the following nested equation (6) where their notation letters and subscripts *V*, R, J, and λ refer to volatility diffusion component V, spot interest rates R, jumps size J, and jump intensity λ. By sequentially making constant or setting to zero the relevant terms for i) R, λ, and J, or ii) λ and J, or iii) R, we are left with simpler equations for either i) stochastic volatility (SV), ii) stochastic volatility and stochastic interest rates (SVSI), or iii) stochastic volatility and jumps (SVJ), respectively.

$$\frac{1}{2}\, VS^2\, \frac{\partial^2 C}{\partial S^2} + \left[R - \lambda\mu_J\right] S\, \frac{\partial C}{\partial S} + \rho\sigma_v\, VS\, \frac{\partial^2 C}{\partial S \partial v} + \frac{1}{2}\, \sigma_v^2\, \frac{\partial^2 C}{\partial v^2} + \left[\theta_v - \kappa_v V\right] \frac{\partial C}{\partial v} \tag{6}$$

$$+ \frac{1}{2}\, \sigma_R^2\, R\, \frac{\partial^2 C}{\partial R^2} + \left[\theta_R - \kappa_R\, R\right] \frac{\partial C}{\partial R} - \frac{\partial C}{\partial \tau} - RP + \lambda\, E\,\{P(t, \tau, S(1+J), R, V) -$$

$$Pt, \tau, S, R, V = 0$$

Additionally, making all the terms involving the stochastic processes for volatility, interest rates, and jumps zero reduces equation (6) to the original BS stochastic differential equation.

This concludes our brief description of the option models of G (equations (2) and (3)), BS (equation (4)), and BCC (equation (6)) [18] to be compared and tested herein. Before going to Section 3 we re-emphasize a few points about the upcoming model comparisons. First, the reader should now appreciate that the simpler but well known BS model compared herein is not a "straw man". Since the important difference between BS and G is the inclusion of debt, if the two models are implemented by exactly the same method, then any difference between the two model values is a direct measure of the leverage effect. Similarly, since the important difference between BS and BCC is the inclusion of stochastic volatility, stochastic interest rates, and jumps, any difference between these two model values

[18] See BCC (1997), p. 2010, for more specific details of equation 5.

(BS and BCC) is a direct measure of BCC's additional parameter effects. This methodology allows a standardized comparison of any relative improvements of the more complex models, G and BCC, to the simpler model of BS which omits these complexities. In other words, how does the inclusion of leverage into BS compare to the inclusion of BCC's additional stochastic processes for stochastic volatility, stochastic interest rates, and jumps. Second, it would always be desirable to implement models in a way that i) is consistent with market data, and ii) does not give any model an unfair parametric advantage. When possible we attempt to follow these implementation guidelines, although BCC's complexity will always result in the advantage of more explanatory parameters.

In the next Section 3 we describe the data necessary for the model implementation techniques used to test for both the presence of leverage effects in index put option prices, and to compare the errors of matched pairs of option model values relative to market prices for the BS, G, and BCC models.

3. Data Measurement and Model Implementation

3a. Data Measurement

The data necessary to compare the three models discussed in Section 2 and to test for the additional effects of leverage on S&P 500 index put options are index option price data, individual equity price data, dividend data, interest rate data, and balance sheet information. We also require the composition of the S&P 500 firms on a daily basis. We collect daily closing stock prices, daily shares outstanding, and the daily composition of the S&P 500 index from CRSP. The interest rate data are daily from the Federal Reserve for government securities with maturities ranging from 1 month to 10 years, which we adjust to use discount factors for the market debt, option strike prices, and dividends.[19]

The index option prices we use are daily put closing prices from Option Metrics from January 4, 1996 through April 30, 2004. This 100 month sample period covers 8 1/3 years and contains about

[19] Option theory requires a risk-less interest rate. However, the over-night call money rate which is used for option market maker margin or the libor rate might also be used. These option prices are not too sensitive to this choice of interest rates.

200,000 index put and call options and 2080 observation days. Option Metrics reports for all options a trade at the close, or the closing *best* bid and *best* ask as a spread, which we average for the closing option price. However, for at-the-money index options (ATM) there is almost always a trade at the close. The nearest to at-the-money (ATM → E=K or K e^{-rt}) S&P 500 index options have the highest daily volume of all traded equity options, trade continuously throughout the day, and thus they should not exhibit much non-synchronicity. [20] However, in order to further minimize non-synchronous problems, first we check to see if there was an option trade on that day. Next we check to see if arbitrage bounds are violated (c.f. $P \leq -E + K e^{-r_T T}$) and if so eliminate these option prices. If non-synchronicity occurred because the stock price moved up after the less liquid in or out-of-the-money put option last traded, then option over-pricing would be observed. If non-synchronicity occurred because the stock price moved down after the less liquid in or out-of-the-money option last traded, then option under-pricing would be observed, and these options would be removed by the above arbitrage check. We also collect the option volume and open interest data and dividend data for the S&P 500 from Option Metrics. Because we cannot perfectly eliminate non-synchronous pricing for the in and out-of-the-money options with this data base, we keep track of the amount of under and over-pricing in order to relate this mis-pricing to the resultant over (under) pricing of in (out-of)-the-money index put options for all models tested.

The balance sheet information for each firm we collect from S&P's annual and quarterly Compustat. Compustat categorizes their data on book value of debt as due in years 1 through 5 (Data 44, 91,92,93,94), and greater than 5 years (Data 9 minus items (91-94)), which we place at 7 years. To these categories we add current liabilities (Data 5), deferred charges (Data 152), accrued expenses (Data 153), short term notes payable, deferred federal, foreign, and state taxes (Data 206,269,270,271), all

[20] Approximately 200,000 contracts traded per day in 2004 on the SPX and Option Metrics takes the *best* bid and ask from the exchange (CBOE, Phlx, Amex, Ise) whose trade is closest to the closing stock price. Non-synchronicity or no trade may occur for away from the money options, but for at-the-money S&P 500 index options this trade will almost always be synchronous. Using the mid-point of the bid/ask avoids bid/ask bounce problems.

payable in year 1. All long-term debt tied to prime (Data 148) and debentures (Data 82), we also place in year 7, respectively.[21] The debt due on each day in each quarter of each year for the S&P 500 firms is the sum of the debt due for all 500 firms for that day in that quarter of that year. This structure of the S&P 500 debt outstanding for each firm permits the computation of the daily duration of the *book value* of aggregate market debt outstanding and the daily amount due at the duration date.

Next we calculate the daily equity market value (cap) of the S&P 500 using the sum of the product of each individual firm's share price times shares outstanding for 500 firms, and we find the factor *f* which is used to normalize the index. We confirm that we match the reported index level each day during our sample. We use this same normalization factor for the daily S&P 500 debt outstanding. This procedure produces daily the exact market value of the aggregate equity and the face value and duration of the aggregate debt outstanding for the 500 S&P corporations.

3b. Model Implementation

Now we have all the data defined on page 12, as P, C, E, F, K_j, r_{FTi}, t, T_i, T_D, ρ, and d, including the extra leverage data necessary for G, and are prepared to implement the models G, BS, and BCC discussed in Section 2. As previously mentioned at the end of the Section 2, we would like to implement the models in a way that is i) consistent with the market data, and ii) does not give any model an unfair parametric advantage. These two implementation goals are related.

First, if the model is intended to value options of different maturities and the implementation is to be consistent with the market data, then the option data, literature, and market practice is unambiguous on the importance of a term structure of volatility.[22] Pan (2002) suggests that "to accommodate a richer term structure of volatility, one solution is to allow for multiple volatility factors",

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[21] Placing long term debt at 7 years follows from Guedes and Opler (JF, v51, 5, 1996), p. 1818, who provide evidence that the mean duration of 7,362 bond issues of long term US corporate debt is 7 years during the 12 year time period 1982-1993.

[22] A term structure of model implied volatility is consistent with the market finding the relevant risk is not the same for options with different expirations. While this idea is intuitive and reasonable, the notion that at risk exposure is different for options on the same underlying and same expiration but different strikes is not either intuitive or reasonable.

which she (and many others, Duffie, et al(2000)) have argued is necessary "if one is trying to price both short and long dated options". [23]

Second, without a term structure of volatility, BS must fit all available options on any specific day for each strike price K_J and time to expiration T_i, with only one unknown parameter estimate, the ATM equity implied volatility, σ_E. Similarly, G must fit all the available option prices with two extrinsic unknown parameter estimates, V(=E+D) and σ_V and one intrinsic transformational parameter, V* . [24] However, the more complex BCC models has the luxury to fit these option prices with either 9 parameter estimates for BCC's best model, SVJ, 8 parameters for SVSI, and 5 parameters for SV. It would be a surprise if a model with far fewer parameters (BS or G) could compete with a model with many more parameters (SVJ). If it were theoretically possible and empirically consistent and practical to implement BCC with a volatility term structure this would be even more unfair to BS and G because it would give the BCC models an even greater parametric advantage. The BS and G models can easily accommodate a term structure of volatility. Many academics have published articles demonstrating its relevance and importance, and the market practice is almost unanimous in using a volatility term structure. However despite the theoretical problems of incorporating a term structure of volatility in BCC models, contemporaneous price data limitations would make it impossible to estimate the required parameters, which would exceed the number of available traded options.

Here the implementation methodology allows the inclusion of a term structure of volatility to accommodate the BS and G models to be consistent with the market data and market practice, but because of data limitations and the large number of required parameters, no volatility term structure can be used for BCC. This methodology actually allows the three models to be implemented in a similar

[23] The numbers of articles are numerous and growing which show that both option price and volatility data suggest the importance of a term structure of volatility. See Pan (2002), p. 32, especially footnote 29, for more references and details regarding the necessity of a term structure of volatility when pricing both short and long dated options. The volatility term structure, like other term structures (c.f. interest rates, default probabilities, etc.), contains important information.

[24] We solve for V or D using the same known E + D =V for all options. V* transforms the debt boundary condition from dependence on E to dependence on V, and is determined by its relation to the known strike prices K_J.

fashion using contemporaneous rather than a larger set of historical data, also consistent with market practice, and renders the models to be more similar in the number of parameters.

Thus, on each day we estimate only four volatilities to accomodate the term structure of volatility, using the ATM options with expiration closest to 15 days, 45 days, 110 days, and the index debt maturity date which is always greater than 365 days. Since index options expire monthly on the third Friday of each month, this generally means the term structure of volatility will be constructed from options that have one month, four months, and six months to expiration, and the insolvency option at the debt maturity will have a consistent maturity. In the matched pair comparisons of the three models each valuing the same matched options, we analyze the pricing errors by grouping all the options into the following five time buckets of days to expiration of 6 – 20, 21 – 72, 73 – 120, 121 – 364, and > 364 days. [25] We use the four volatilities estimated for this term structure for all the options in each relevant group. The volatility term structure becomes flat as the option expiration dates are extended so we value the long dated options in the last two time buckets with the same volatility used for the insolvency option.

Now the importance of including BS in these model comparisons becomes more evident, because this allows us to conclude that it is leverage and not the term structure of volatility that is important for the model differences. We will see this clearly in Section 4 when we examine the value errors relative to the market prices for matched pairs of options for these three models together on the same graphs and in tables. Any observed differences between BS and G cannot be attributed to a term structure of volatility since both BS and G models have the exact same implementation, both using the same term structure buckets for volatility. Thus, the observed differences between BS and G must be attributed to leverage, because if there is no leverage the two models would be identical. In the same graphs, the comparison of BCC and BS shows that BCC is closer on average to the market prices, even

[25] The grouping follows Rubinstein (1985). This implementation with a term structure of volatility for BS and G still gives BCC implemented without a volatility term structure a large parametric advantage

though BS is using a volatility term structure which BCC did not allow in their implementation of BS. So when leverage is added to the BS model already using a volatility term structure, and then BS becomes the G model, we now observe that BS with leverage (i.e. G) is on average much closer to the market prices than BCC. Thus, it cannot be the term structure of volatility that causes the BS model to improve when leverage is added. Instead this improvement must be due to the addition of leverage.

We should also mention that using options prices that are closest to at-the-money (ATM) to imply the volatilities is not a reason for G's relative improvements compared to BCC. Furthermore, using contemporaneous option prices instead of prices lagged a day is not the reason for G's improvement. In the Appendix 1, Tables 7d* and 8d* show that implementing G using the volatility that minimizes the sum of squared pricing errors using all the options with different strikes in the same time to expiration bucket instead of the ATM volatility does not change the results. Furthermore, doing this on a lagged day instead of contemporaneously does not alter our conclusions.

We now proceed to explain how we compute the parameters V = known E + D, σ_{vTi} , σ_{vTD} , and V* using only known contemporaneous equity and option prices and measured leverage parameters F and T_D. First note that V, D, and E must be the same at any point in time, t, for all option expirations and strikes, and σ_{vTi} and σ_{vTD} will be the same for all options expiring in the T_i, expiration bucket, and for the options buckets and debt using the long term volatility, but σ_{vTi} will differ across the expiration buckets as explained. We solve the first option expiration bucket and the debt insolvency bucket together. From the first expiration bucket we use two pairs of put and call option market prices, and the stock price. From the debt insolvency bucket we use equation (5) and the known equity price. We simultaneously solve equations (2), (3), and (5) given market prices for P, C, and E, for parameters V = E+D, σ_{vTi} , σ_{vTD} , and the transformed option exercise strikes, V*. Thus, on any specific day for the first time to option expiration bucket, T_i , we select two pairs or four options, two puts and two calls. All four options have the same actual T_i but each pair has a different strike, K_j , and to minimize non-synchronicity we choose two strike prices, $K_{j=1,2}$, that are nearest to ATM. So our four selected options have the same actual time to expiration, T_i , but one pair of put-call options have strike price K_1 , and the other pair has strike price K_2 . We also use equation (5) to solve for the long term volatility relevant for the debt insolvency condition. We then solve simultaneously these five equations for five unknowns, i) V or D with the substitution (D + E=V), ii) σ_{vTi} , iii) σ_{vTD} , iv) $V_1*(K_1)$, and v) $V_2*(K_2)$. To summarize using our notation, we have market prices for the index equity level, E, and option prices for four options, two puts and two calls, and all four options have the same actual expiration date T_i , but each put-call pair has a different strike K_j chosen to be near to ATM. This is notated as:

Pair #1, strike K_1: $P_1[V=D+E, V_1*(K_1), \sigma_{vTi=1}, \sigma_{vTi}]$ and $C_1 [V=D+E, V_1*(K_1), \sigma_{vTi=1,} \sigma_{vTD}]$ (6)

Pair #2, strike K_2: $P_1[V=D+E, V_2*(K_2), \sigma_{vTi=1}, \sigma_{vTD}]$ and $C_2 [V=D+E, V_2*(K_2), \sigma_{vTi}, \sigma_{vTD}]$ (7)

Equity as option: $E [V, \sigma_{vTD}]$ (8)

At time t, every option in the first expiration bucket is priced with the same total aggregate market values, D from V, using known E, and thus same V=D+E, for all T_i and K_J , and in the first expiration bucket T_i the same volatility, $\sigma_{vTi=1}$, and the same long term debt volatility σ_{vTD} . Each option in the group of options expiring in bucket T_i with different strike price K_j has a $V*(K_J)$ corresponding to its strike price K_j . These critical exercise values, V_J* for each K_J , are known from solving the above five equations represented in expressions (6), (7), and (8). Since V* = E* + D*, we know that options with different strike prices in the same expiration bucket have a different E_J*'s = K_J 's, but have the same D*, while options with the same strike price but in different expiration buckets have the same E* but different $D_{Ti}*$'s. Thus for options in the same expiration bucket with different strike prices, ΔV* is $V_2* - V_1*$ = (E*2 + D*) – (E*1- D*) and thus the known difference in strike prices, $K_2 – K_1$, is simply equal to the differences in the E*'s, or ΔE*. Because we are given the changes in the strike price for all options at any specific expiration date we can use these known strike price changes ΔK to produce the $V*(K_j)$ for each K_j for all option the other options in the first expiration bucket. Then, since we solved for the 5 unknowns D, E, and V, σ_{vTi} , σ_{vTD} and V_J*, with the 5 equations for these two option pairs and the equity as an option, and from this solution we know all the intrinsic $V*(K_J)$'s for the options with different strikes in the first expiration bucket, we can use equations (2) and (3) and the now known parameters to predict the option prices for the rest of the put or call options in the first expiration bucket.

As we move to the second expiration bucket, recall that at any fixed current time t, the values which are remain the same across expiration buckets are implied market values for V_t , D_t , the implied long term volatility σ_{vTD} , and the known equity value E. The parameter values which change across expiration buckets are $V^*(K_J)$ and σ_{vTi} . However, the change in $V^*(K_J)$ for options with the same strike price is because of changes in D_J^* since for the same strike K_J , E_J^* does not change since K_J must equal E_J^*. Thus, at the second expiration bucket there are only 2 unknowns, σ_{vTi} , and D_J^* which changes $V^*(K_J)$ for each option pair. Now we only need to use the two put call option pairs since equation (5) for the equity as a call option on the aggregate market portfolio has the same now known parameters required to produce the known equity index price. Once we solve the four equations for options in the same second expiration bucket, we can use the same procedure described above to predict the values of all the other in and out of the money option in that expiration bucket.

This procedure can be repeated for multiple expiration buckets. However, in order to keep the number of parameters required for accurate option pricing low and not advantageous over other models, we limit the number of options in the term structure of volatility.

In summary, after solving for the required parameters from the equations for the two put call at the money (ATM) option pairs and the equity as an option, we can use these parameters to create the V_J^*'s for all the in the money (ITM) and out of the money (OTM) options in each expiration bucket, and then predict the values of all the other ITM and OTM options in expiration bucket. [26] This implied methodology for finding D_t, and thus V_t, σ_v, and V^* is similar to the BS method for implied equity volatility, σ_{ETi}. It is important to realize that at each time t, the volatility term structure is estimated with the same known aggregate market equity value, E, the same implied aggregate market debt value, D, and

[26] ITM and OTM are abbreviations for in-the-money and out-of-the-money just as ATM abbreviates at-the-money.

the same aggregate market portfolio value, V. Thus, this term structure of volatility is consistent with the contemporaneous known market price of the equity index E, and the observed ATM index option prices at each specific point in time, t.

Next in Section 4 we present the results of comparisons of G to BS and BCC. Section 4 demonstrates that leverage *causes* G to be significantly closer than BS to market prices for 99% of 109,301 matched pair comparisons of OTM options, and to 90% of 50,452 matched pair comparisons of ITM options. Section 4 also shows that G's values are significantly closer to the market prices than BCC's best model, SVJ, for 75% of 139,016 OTM option matched pair comparisons, and to 65% of 59,569 ITM option matched pair comparisons. Thus, BCC's pricing performance is better than BS without leverage, but not as good BS with leverage, which is G.[27]

4. The Results

This section presents the first evidence about the size and variation of the *market value* of aggregate leverage in the S&P 500 firms derived directly from option theory, and details about the model matched pair comparison results for BCC, BS, and G. In addition numerous graphs and detailed tables of each model's pricing errors are presented and discussed. The results illustrate both the statistical and economic significance of the BCC, BS, and G pricing errors, the relation of these errors to the omission of leverage, and relative improvements of leverage with respect to matched pairs of options categorized by time to expiration, leverage, moneyness, and calendar year.

The results demonstrate that changes in the market value of aggregate corporate leverage implied from the most traded ATM index option prices exhibit the anticipated leverage effect on both the underlying equity index and consequently on the price of options on this index. The size of the leverage effect should be less significant when the market value of aggregate leverage is small (less than 1 [D/E <

[27] The number of matched pair comparisons are different between G and BS and G and BCC because G and BS are produce equal values on many more of the comparisons than G and BCC's best SVJ model, and we are mainly interested in when the models are different.

1)], and more significant when aggregate market leverage is large [greater than 1 (D/E > 1)]. However, the tables show that even relatively low leverage ratios have can significant effects on option values.

Section 3 explained our methodology to measure the magnitude of the implied market value of aggregate leverage in the S&P index. Here, we graphically illustrate the variability in the economy's aggregate leverage during our sample period. Then we test whether including a measure of aggregate leverage as an explanatory variable can substantially improve the pricing of equity index options. Similar to earlier option based research findings, we demonstrate highly statistically significant pricing improvements when compared to both simple and more complex models which omit leverage.

4a. Market Leverage

First, using equations (2) and (3) we compute daily the market value of the aggregate debt to equity ratio, D/E, for the 500 firms in the S&P, where, as previously explained, the market value of aggregate debt is derived from the option pricing structure using daily market prices for the equity index and market prices of ATM options on the index. We believe that Figure 1 is the first presentation of implied market value of aggregate debt depicted as a time series of market value of the aggregate D/E ratio for the S&P 500, and we plot D/E along with the level of the S&P 500 equity index.

As expected, the market value of the aggregate debt/equity ratio and the S&P 500 equity index level, are inversely related.[28]

[28] In order to keep the graph uncluttered we do not graph the equity index volatility. However, leverage produces an equity index volatility that is stochastic and inversely related to the equity level and returns. Ito's lemma provides a well known formula showing how leverage makes the equity volatility stochastic and bounded above (greater than) the firm volatility.

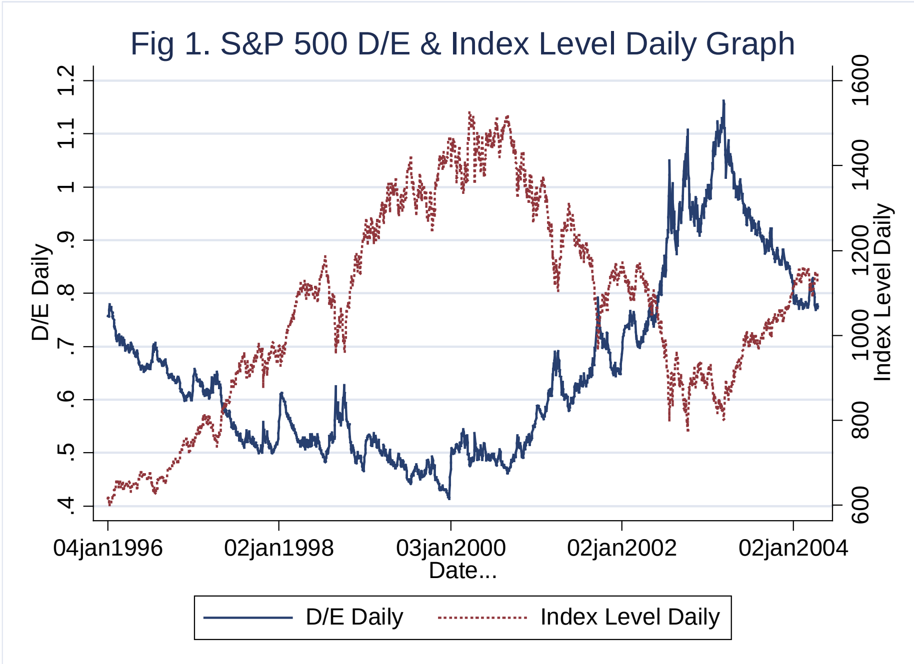


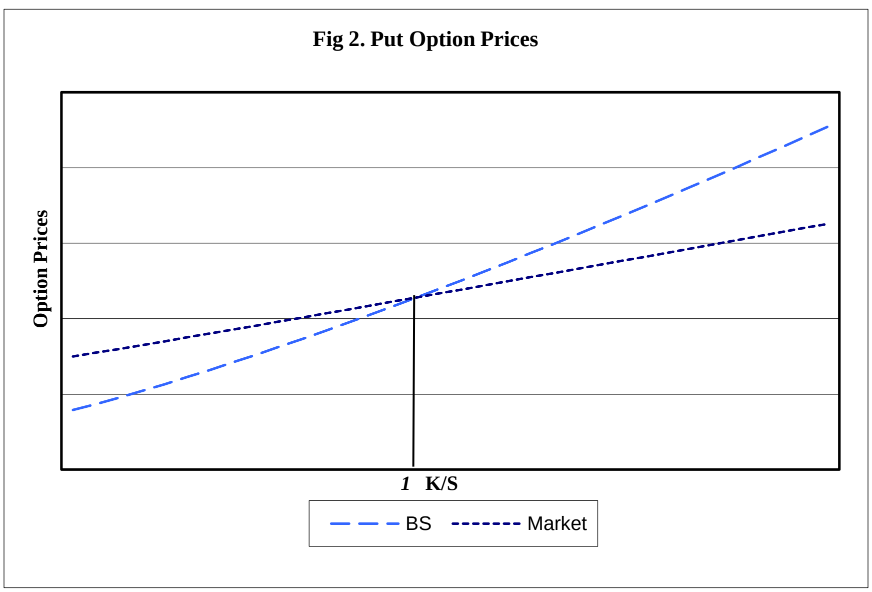
Figure 1 shows that during our sample period January, 1996 to April, 2004, the market value of the aggregate debt/equity ratio for all firms in the S&P 500 has considerable variation, ranging from a minimum of about 0.40 in January 2000, to a maximum of 1.15 in April 2003. This range for the market leverage ratio means that the amount of debt, D, in the corporate economy over this 100 month period, as a percent of the total value of the 500 corporations, is implied to range between 29% and 55%, which is exactly the range of average debt to firm values that Leland (and many others) reported in his optimal capital structure research papers.

Figure 1 also shows the S&P 500 leverage is highest in years 2002 and 2003, and lowest in years 1999 and 2000. Later in this section we demonstrate that the models compared herein which omit leverage, BCC and BS, exhibit greater leverage related valuation errors in 2002 and 2003 when the market value of leverage is larger, and less leverage related valuation errors in 1999 and 2000 when the market value of leverage is smaller. We will refer back to Figure 1 throughout the discussion of results to recall in what years the market leverage was highest or lowest while the market index level was lowest or highest.

Finally, for those not so familiar with the use of the S&P as the market portfolio in asset pricing literature, note that the S&P 500 Stock Index represents about 80% of the market capitalization of all stocks listed on the New York Stock Exchange, and it is often treated as "the market" in the literature on tests of asset pricing models.

4b. Model Pricing Error Comparison for BS and G

First we present separately a "representation" of BS model values relative to the market prices because this characterizes how the literature and later models (BCC, G, et al) evolved to attempt to mitigate the well known BS problems. Figure 2 presents a graph of put option market prices, BS model values, and moneyness, K/S, which is representative of most research findings for the S&P 500 index put options.



Since the equity index level, S in Figure 2, is the same for all K at any point in time during or at the end of any day, as strike prices, K vary, the out-of-the-money (OTM) stock index puts (low K) are shown to be under-valued by BS and the in-the-money (ITM) index puts (high K) are shown to be over-valued by the BS model relative to the market prices.

This BS pricing bias for S&P 500 index put options is probably the most widely known empirical bias in the option pricing literature. However, we will see that when we compare BS to G or BCC for matched pairs of options, there are many different possible outcomes. [29]

We show in Figure 3 that G's option model has the potential to improve or even eliminate these BS valuation errors because of the leverage effect. The reason for this, once again, is the economic effects of leverage create the necessary negative correlation between the index level and the index volatility. This interaction between the index level and index volatility implies that the index volatility is both stochastic and inversely related to the level of the index, and that the resultant implied index return distribution will have a fatter left tail and a thinner right tail than the BS assumption of a normal distribution. Thus, G's option model produces option values that are less (greater) than the BS values for in- (out-of) the-money European index put options. Thus, leverage could potentially completely eliminate this well known BS bias.

[29] Figure 2 represents the most ubiquitous outcome from our data. However, for a very small number of index option matched pairs there are many different model distance comparisons that can be made: both over, both under, one over while the other is under, one equal to the market while the other is either over or under, both equal to each other but either over or under, both equal to each other and equal to the market, and furthermore, there are multiple cases for each situation when the models are not equal to each other. This is discussed in more detail later in this section.

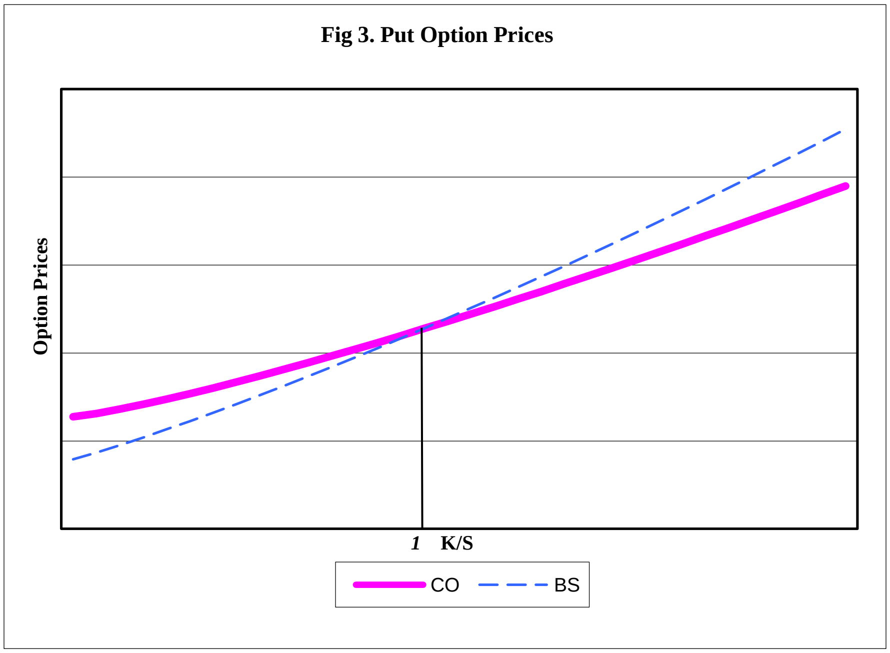
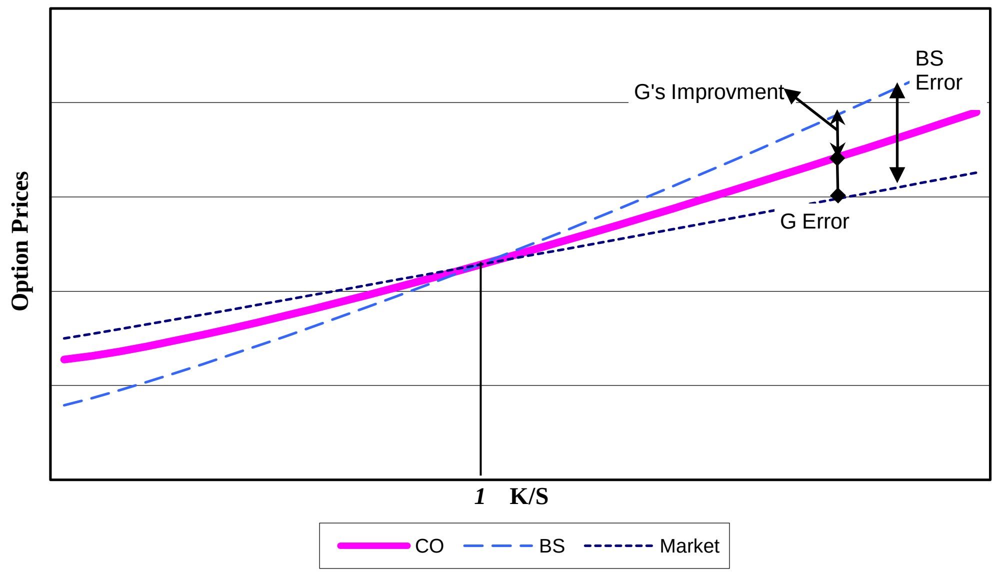


Figure 4 presents how we measure the amount of improvement leverage provides relative to the alternative model (here BS) without leverage for valuing S&P 500 stock index put options during this sample period. During the 2080 day sample period, we examine thousands of matched pairs of all options for each expiration date and each strike price, and we measure the distance between each model's value and the market price. We compare the distance that each model value is from the market price for each matched pair, find the model that produces the closest distance to the market, and we compute the improvement of one model to the other for that pair. We then net these distances for all matched pairs in order to find which model is closest to the market for all matched pairs on average and how much net improvement, if any, is present. We present this analysis for all matched pairs of options for a variety of categories with different times to expiration, different moneyness, and for the different market leverage exhibited during our sample period.

Fig 4. Put Option Prices



The potential improvement represented in Figure 4 with respect to the market price[30] of Geske's compound option model compared to the Black-Scholes model is calculated with the following formula:[31]

$$\frac{\text{BS error} \;-\; \text{G error}}{\text{BS error}} = \frac{(\text{Market - BS}) \;-\; (\text{Market - G})}{(\text{Market - BS})} \qquad (6)$$

The tables that follow demonstrate the importance of leverage by presenting both the statistical significance and the economic significance of G's index option pricing improvements relative to the alternative model, here BS first and then BCC.

[30] If Figure 4's representation turns out to be the most common result from the addition of leverage, then this indicates that there is not enough skewness and excess kurtosis from only leverage, so perhaps adding jumps at the firm level could provide the necessary additional improvement.

[31] Care must obviously be taken with the signs of the variety of matched pair errors explained in footnote 40, especially if one model value distance is above and the other distance is below the market price, when computing the average error across all matched pairs. However, the results depicted in Figures 2, 3, and 4 are found for the vast majority of all option pairs.

4c.1 Tables of BS vs. G by Moneyness, Year, Expiration, and Leverage

Here the focus is on actual rather than representative comparisons of BS to G. Tables 1-4 present a detailed analysis of 57,177 ITM puts and 132,388 OTM put option pricing errors of BS, and G's relative improvements, for different times to expiration by calendar year and by market leverage, using two definitions of ATM. We also present the number of matched pairs of options available in each of these categories during this time period, and we examine both the statistical and economic significance of G's improvements relative to BS.

When the ATM option region is considered to contain strike prices within ±5% of the index level, a large number of options are eliminated. If instead we consider only two options with different strikes per day per time to expiration as the most at-the-money options, defined as *MATM*, then all but two previously eliminated 5% ATM's will now be either in or out-of-the-money and priced with some error. This alternate definition of ATM options (labeled "0%") increases the sample size of mis-priced options. [32]

Table 1 presents details for ITM index put options. Panel A illustrates that if we consider only two options to be ATM each day (the MATM), the sample of in-the-money put index options more than doubles from 23,438 to 57,177 matched pairs. Panel A also shows the most active trading years for ITM put index options during our sample period are 2000, 2001, and 2002, and contain 11,349 of the 23,438 (or 22,381 of the 57,177) total ITM option matched pairs. As expected, Figure 1 comparing market leverage to the market level shows that during these years 2000-2002 the market level was the mostly decreasing while market leverage was increasing. This decreasing market resulted in these years having the largest number of ITM index put options. Also as expected, Table 1 shows that the nearer expiration ITM puts are traded more heavily than the longer expiration puts.

[32] We might also expect this ATM definitional change to reduce the average net valuation error because the ATM options are considered to be more accurately priced. However, if we use the volatility that minimizes the SSE across all options in each term bucket, then perhaps increasing the number of ATM options might increase the average errors. ATM options are the most liquid by measures of volume, open interest, and bid-ask spread, and their implied volatilities are generally the best estimates of future realized volatility. Because of this ATM options are believed to contain the best price information and are considered to be the most accurately priced options.

TABLE 1

PUT ITM

TOTAL NUMBER OF OPTIONS

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	69	329	178	585	388	**1,549**
1997	143	709	306	1,055	403	**2,616**
1998	269	786	383	1,055	473	**2,966**
1999	150	757	354	897	275	**2,433**
2000	259	1,040	386	1,093	377	**3,155**
2001	321	1,183	507	1,221	428	**3,660**
2002	449	1,631	531	1,474	449	**4,534**
2003	247	1,055	352	721	0	**2,375**
2004	31	109	10	0	0	**150**
TOTAL	**1,938**	**7,599**	**3,007**	**8,101**	**2,793**	**23,438**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	629	2,316	905	1,602	733	**6,185**
1997	880	2,866	1,056	2,236	775	**7,813**
1998	976	2,686	957	1,932	824	**7,375**
1999	923	2,657	944	1,809	545	**6,878**
2000	969	2,862	980	1,848	573	**7,232**
2001	888	2,527	944	1,940	635	**6,934**
2002	1,086	3,094	1,048	2,307	680	**8,215**
2003	916	2,701	780	1,354	0	**5,751**
2004	248	510	36	0	0	**794**
TOTAL	**7,515**	**22,219**	**7,650**	**15,028**	**4,765**	**57,177**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	213	943	496	1,280	424	**3,356**
0.5-0.6	540	2,226	821	2,494	967	**7,048**
0.6-0.7	408	1,361	658	1,811	778	**5,016**
0.7-0.8	189	751	248	748	370	**2,306**
0.8-0.9	109	384	174	266	69	**1,002**
0.9-1.0	290	1,150	329	799	152	**2,720**
1.0-1.2	189	784	281	703	33	**1,990**
TOTAL	**1,938**	**7,599**	**3,007**	**8,101**	**2,793**	**23,438**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	1,242	3,555	1,409	2,492	767	**9,465**
0.5-0.6	2,287	6,872	2,241	4,643	1,711	**17,754**
0.6-0.7	1,493	4,590	1,791	3,584	1,332	**12,790**
0.7-0.8	859	2,436	683	1,457	583	**6,018**
0.8-0.9	466	1,267	412	438	90	**2,673**
0.9-1.0	717	2,163	650	1,370	236	**5,136**
1.0-1.2	451	1,336	464	1,044	46	**3,341**
TOTAL	**7,515**	**22,219**	**7,650**	**15,028**	**4,765**	**57,177**

Table 1, Panel B, presents these same ITM index put options by time to expiration and also categorized by their leverage (D/E) ratio. Recall the market D/E ratio during this time period ranges between 40% and 120%, which is depicted in Figure 1. For the 5% ATM sample, Panel B shows that about 25% (5,712/23,438) of ITM put index option matched pairs traded when the market leverage was in its highest range from 80% to 120%.

Table 2 presents the net pricing error improvement of G relative to BS by calendar year and by leverage ratio for the various times to expiration and for the two definitions of ATM for all ITM put index option matched pairs during this sample period. Panel A shows that the improvement of G's model relative to BS with respect to time to expiration varies on average across all years from 12% for the shortest expiration index options (6-20 days) to 45% for longest expirations for the 5% ATM options, and is strictly monotonic across all ranges of expiration. [33] *G's* improvement is greater for the options with longer times to expiration because the leverage has a longer, more correlated effect on their value.

Also note that G's improvement over BS is greatest in the complete years 2002 and 2003 (and the partial year 2004) averaging between 70% and 90% across all times to expiration. This is as expected because Figure 1 (and Table 2, Panel A) shows that market leverage was increasing and highest in 2002 and 2003 (and the part of 2004), with D/E ranging (averaging) between 0.8 and 1.2. Similarly, G's improvement is smallest but still greater than 20% in the low leverage years of 1999 and 2000, when the D/E ratio ranges (averages) between 0.4 and 0.5. Table 2 also illustrates that when ATM is defined as the two most at the money options, the previously excluded but now included ITM options which have smaller pricing errors reduce the net improvement in all years and across all times to expiration except the nearest to expiration, and reduce the total average improvement from 42% to 38%.

Table 2, Panel B, categorizes options by leverage instead of by year. Here it is shown that relative to BS the improvement of G's model increases with the D/E ratio monotonically for every time to expiration bucket, especially when the sample number of options in each category is sufficiently large. This improvement is also monotonic with leverage on average across all times to expiration, and ranges from a low of 24% for the lowest D/E range of 0.4-0.5, to a high of 88% improvement for the highest D/E range of 1.0-1.2. Also ass expected, for the highest leverage categories of 0.9 to 1.2, G's improvement is greatest and averages between 76% and 96% for both 5% and 0% ATM options.

[33] Note that in a few instances the pricing error correction is greater than 100%. This can happen when the two models errors are on opposite sides of the market price.

TABLE 2

PUT ITM

PRICING ERROR IMPROVEMENT

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	***0.68***	-1%	75%	48%	63%	73%	**68%**
1997	***0.57***	-6%	57%	45%	59%	72%	**62%**
1998	***0.54***	133%	23%	21%	30%	40%	**31%**
1999	***0.48***	81%	18%	19%	24%	28%	**24%**
2000	***0.50***	10%	18%	22%	26%	28%	**26%**
2001	***0.60***	-7%	40%	38%	42%	52%	**45%**
2002	***0.72***	28%	71%	71%	80%	74%	**76%**
2003	***1.00***	-28%	78%	63%	96%	N/A	**86%**
2004	***0.80***	-1%	152%	62%	N/A	N/A	**421%**
TOTAL	***0.66***	**12%**	**37%**	**35%**	**42%**	**45%**	**42%**

0 PERCENT

Option Expiration (in Days)

YEAR	D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	***0.68***	-37%	34%	37%	56%	71%	**53%**
1997	***0.57***	15%	33%	38%	56%	71%	**51%**
1998	***0.54***	-10%	19%	21%	30%	39%	**28%**
1999	***0.48***	5%	13%	17%	23%	28%	**20%**
2000	***0.50***	5%	14%	20%	25%	28%	**23%**
2001	***0.60***	76%	35%	37%	40%	52%	**42%**
2002	***0.72***	76%	68%	73%	80%	75%	**75%**
2003	***1.00***	25%	51%	54%	92%	0%	**70%**
2004	***0.80***	-31%	43%	45%	0%	0%	**48%**
TOTAL	***0.66***	**18%**	**28%**	**33%**	**41%**	**45%**	**38%**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	-25%	17%	20%	24%	27%	**24%**
0.5-0.6	12%	24%	24%	32%	39%	**32%**
0.6-0.7	19%	40%	37%	48%	57%	**49%**
0.7-0.8	-8%	67%	47%	66%	65%	**65%**
0.8-0.9	-63%	52%	55%	84%	69%	**71%**
0.9-1.0	29%	74%	67%	85%	88%	**83%**
1.0-1.2	13%	82%	82%	95%	79%	**88%**
TOTAL	**12%**	**37%**	**35%**	**42%**	**45%**	**42%**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	6%	13%	18%	23%	27%	**21%**
0.5-0.6	5%	19%	22%	31%	40%	**29%**
0.6-0.7	14%	29%	35%	46%	56%	**44%**
0.7-0.8	36%	51%	46%	62%	65%	**58%**
0.8-0.9	17%	31%	45%	78%	69%	**54%**
0.9-1.0	295%	61%	62%	83%	91%	**76%**
1.0-1.2	62%	93%	96%	102%	88%	**96%**
TOTAL	**18%**	**28%**	**33%**	**41%**	**45%**	**38%**

Table 3 presents similar data to Table 1 for out-of-the-money (OTM) index put option matched pairs. First consider the number of traded index puts presented in Table 3 for OTM options. In Panel A when ATM is defined as 5% the near expiration index puts are traded much more heavily than the far expiration puts every year. Here the shorter expiration options comprise about 52% (47,605/91,950) of these matched pairs.

Panel A also illustrates that when we consider only two options to be ATM each day (the most at the money options), the sample of OTM put index option increases from 91,950 to 132,388 matched pairs. The most active trading years for OTM put index options during our sample period are 1997, 1998, and 1999, and Figure 1 shows this is the time period when the S&P 500 index level was the mostly increasing, resulting in more OTM options.

TABLE 3

PUT OTM

TOTAL NUMBER OF OPTIONS

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	1,105	3,664	1,626	2,583	661	**9,639**
1997	1,583	5,196	2,152	4,068	882	**13,881**
1998	1,775	5,462	2,054	3,996	948	**14,235**
1999	1,356	5,005	2,119	4,024	722	**13,226**
2000	1,000	4,051	1,643	2,506	366	**9,567**
2001	1,095	4,010	1,459	2,180	404	**9,148**
2002	1,076	3,880	1,531	2,899	583	**9,969**
2003	1,422	4,329	1,928	2,878	0	**10,557**
2004	494	1,102	132	0	0	**1,728**
TOTAL	**10,906**	**36,699**	**14,644**	**25,134**	**4,567**	**91,950**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	1,880	6,223	2,689	3,838	945	**15,575**
1997	2,564	8,081	3,147	5,604	1,191	**20,587**
1998	2,794	8,010	2,825	5,103	1,275	**20,007**
1999	2,460	7,295	2,760	5,128	999	**18,642**
2000	1,830	5,746	2,191	3,323	574	**13,664**
2001	1,759	5,527	1,971	2,965	565	**12,787**
2002	1,678	5,343	2,030	3,728	758	**13,537**
2003	2,357	6,452	2,525	3,608	0	**14,942**
2004	858	1,610	179	0	0	**2,647**
TOTAL	**18,180**	**54,287**	**20,317**	**33,297**	**6,307**	**132,388**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	1,944	6,858	3,174	4,993	948	**17,917**
0.5-0.6	3,485	11,986	4,455	8,601	1,783	**30,310**
0.6-0.7	1,972	7,121	2,941	4,820	1,097	**17,951**
0.7-0.8	1,359	3,845	1,178	2,108	386	**8,876**
0.8-0.9	713	2,030	955	706	52	**4,458**
0.9-1.0	1,061	3,352	1,337	2,584	257	**8,591**
1.0-1.2	372	1,507	602	1,322	44	**3,847**
TOTAL	**10,906**	**36,699**	**14,644**	**25,134**	**4,567**	**91,950**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	3,483	10,032	4,127	6,484	1,340	**25,466**
0.5-0.6	5,701	17,537	6,231	11,133	2,424	**43,026**
0.6-0.7	3,314	11,120	4,449	6,982	1,562	**27,427**
0.7-0.8	2,233	5,736	1,742	2,926	535	**13,172**
0.8-0.9	1,194	3,181	1,285	928	69	**6,657**
0.9-1.0	1,648	4,652	1,703	3,181	320	**11,504**
1.0-1.2	607	2,029	780	1,663	57	**5,136**
TOTAL	**18,180**	**54,287**	**20,317**	**33,297**	**6,307**	**132,388**

Table 3, Panel B, present these same OTM index put options by time to expiration and by debt/equity (D/E) ratio for the same ranges of time to expiration and leverage. Here the higher leverage categories (0.8 to 1.2) comprise about 18% of the OTM matched pairs.

Table 4, similar to Table 2, presents the net pricing error improvement of G relative to BS by year and by D/E ratio for the various times to expiration and for the two definitions of ATM for all OTM put index options matched pairs during this sample period. In Panel A for either ATM definition, the high leverage years 2002 and 2003 again exhibit G's greatest pricing improvement of about 20% relative to BS for these less valuable OTM matched pairs.

TABLE 4

PUT OTM

PRICING ERROR IMPROVEMENT

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	***0.68***	1%	7%	13%	22%	38%	**18%**
1997	***0.57***	2%	8%	14%	25%	40%	**19%**
1998	***0.54***	1%	5%	8%	15%	26%	**13%**
1999	***0.48***	2%	5%	8%	14%	23%	**12%**
2000	***0.50***	0%	6%	9%	15%	23%	**12%**
2001	***0.60***	3%	7%	12%	20%	35%	**15%**
2002	***0.72***	1%	9%	13%	24%	42%	**20%**
2003	***1.00***	3%	10%	14%	29%	N/A	**20%**
2004	***0.80***	1%	5%	8%	N/A	N/A	**5%**
TOTAL	***0.66***	**2%**	**7%**	**11%**	**19%**	**30%**	**15%**

0 PERCENT

Option Expiration (in Days)

YEAR	D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	***0.68***	2%	9%	15%	24%	39%	**18%**
1997	***0.57***	2%	9%	15%	25%	41%	**19%**
1998	***0.54***	2%	6%	9%	15%	27%	**13%**
1999	***0.48***	2%	6%	8%	14%	23%	**12%**
2000	***0.50***	0%	6%	10%	16%	23%	**12%**
2001	***0.60***	4%	8%	12%	21%	35%	**15%**
2002	***0.72***	1%	9%	14%	25%	42%	**19%**
2003	***1.00***	4%	11%	15%	30%	N/A	**20%**
2004	***0.80***	2%	7%	10%	N/A	N/A	**6%**
TOTAL	***0.66***	**2%**	**7%**	**11%**	**19%**	**30%**	**15%**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	1%	5%	8%	14%	22%	**12%**
0.5-0.6	1%	6%	10%	17%	29%	**14%**
0.6-0.7	2%	7%	12%	22%	36%	**17%**
0.7-0.8	2%	7%	12%	22%	40%	**17%**
0.8-0.9	2%	7%	12%	23%	43%	**14%**
0.9-1.0	3%	10%	14%	27%	45%	**20%**
1.0-1.2	1%	14%	17%	32%	42%	**24%**
TOTAL	**2%**	**7%**	**11%**	**19%**	**30%**	**15%**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	2%	5%	8%	14%	22%	**11%**
0.5-0.6	2%	7%	11%	17%	30%	**14%**
0.6-0.7	3%	8%	13%	23%	37%	**17%**
0.7-0.8	3%	8%	13%	23%	40%	**17%**
0.8-0.9	3%	8%	14%	24%	43%	**15%**
0.9-1.0	3%	11%	15%	27%	45%	**20%**
1.0-1.2	0%	14%	17%	32%	42%	**24%**
TOTAL	**2%**	**7%**	**11%**	**19%**	**30%**	**15%**

Also as expected, in the lowest leverage years of 1999 and 2000, G exhibits the smallest pricing error improvement of 12% (excluding the small sample partial year 2004). Table 4 also illustrates that when ATM is defined as as the two most at the money options, the previously excluded but now OTM options which have smaller pricing errors reduce the net pricing improvement in all years.

Table 4, Panel A, again illustrates that G's improvement increases monotonically with option time to expiration, from 2% to 30% on average, again because these options have a longer lasting leverage effect. Table 4, Panel B, demonstrates that G's improvement also increases monotonically with the D/E ratio, for every time to expiration bucket, especially when the sample number of options is sufficiently large. Also, the improvement on average across all times to expiration, increases with leverage monotonically from a low of 12% for the lowest leverage category, 0.4 -0.5, to a high of 24% for the highest leverage category, 1.0-1.2. So, as expected for these OTM matched pairs, the highest (lowest) D/E categories exhibit G's greatest (smallest) improvement over BS. Table 4 also shows that the improvement is greatest in the highest leverage years, 2002 and 2003.

We have also tried a different volatility methodology identical to the implementation by BCC of basing the aggregate net pricing errors in each expiration bucket on the volatility that minimizes the sum of squared errors instead of on the ATM volatility. We find that this does not change any of the conclusions or characteristics of our results. This fact is not so surprising because if the volatility that minimizes the sum of squared pricing errors is moved away from the ATM volatility toward either the ITM or OTM volatilities in order to reduce their errors, then there will be an off-setting effect from the larger errors in the other moneyness direction. This off-setting effect will be present independent of the definition of ATM (%5 or 0% most at the money). Also, we show a one day lag in the volatility estimation does not change any of our conclusions.[34]

4.c.2 Statistical Significance of BS and G Differences in Tables 1-4

Because the differences in each models valuation errors result from bias and are not necessarily normally distributed, we use non-parametric statistics to test the significance of the differences between BS and G's model. This is the same as the significance of the reported improvements, using both the 5% ATM and the 0% (most at the money) ATM definitions.

As can be seen in Table 5 for ITM options and Table 6 for OTM options, we find G's model improvements are all significant at greater than the 99.99% level for every option expiration bucket except the very near expiration options.[35] Near expiration when market option prices are converging to the in or out-of-the-money boundaries there is much more noise in the pricing errors, especially for the out of the money options that are approaching zero.

[34] See Appendix I for these results with implementation identical to BCC (1997).
[35] Furthermore, when using a volatility that is minimizing the sum of squared errors these significance results also hold, and the near expiration options remain significantly different as others have reported (see Heston and Nandi (2000).

TABLE 5

PUT ITM

Rank Sum Test *p* Value

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.8330	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.7702	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.4492	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.6734	0.0000	0.0000	0.0000	0.0000	**0.0000**
2000	0.9489	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.6825	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.7099	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.8178	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.8882	0.0001	0.0072	0.0000	0.0000	**0.0007**
TOTAL	**0.9974**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.5525	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.4056	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.6659	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.3643	0.0000	0.0000	0.0000	0.0006	**0.0000**
2000	0.2872	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.1553	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.0007	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.7271	0.0000	0.0016	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.6345	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.7927	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.7547	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.7005	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.5363	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.2165	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.6204	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.9974**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.3211	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.1387	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.2989	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.2959	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.1855	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.0037	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

TABLE 6

PUT OTM

Rank Sum Test *p* Value

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.8991	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.7483	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.8349	0.0049	0.0000	0.0000	0.0000	**0.0000**
1999	0.8563	0.0016	0.0000	0.0000	0.0000	**0.0000**
2000	0.8385	0.0010	0.0000	0.0000	0.0000	**0.0000**
2001	0.6866	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.7554	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.5709	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.9624	0.3538	0.0299	0.0000	0.0000	**0.4965**
TOTAL	**0.5261**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.5776	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.4204	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.5096	0.0001	0.0000	0.0000	0.0000	**0.0000**
1999	0.4575	0.0001	0.0000	0.0000	0.0000	**0.0000**
2000	0.3697	0.0001	0.0000	0.0000	0.0000	**0.0000**
2001	0.2923	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.6549	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.1099	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.6776	0.0505	0.0437	0.0000	0.0000	**0.1053**
TOTAL	**0.0385**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.8264	0.0018	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.7413	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.7948	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.8772	0.0190	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.8929	0.0088	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.6409	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.6741	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.5261**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

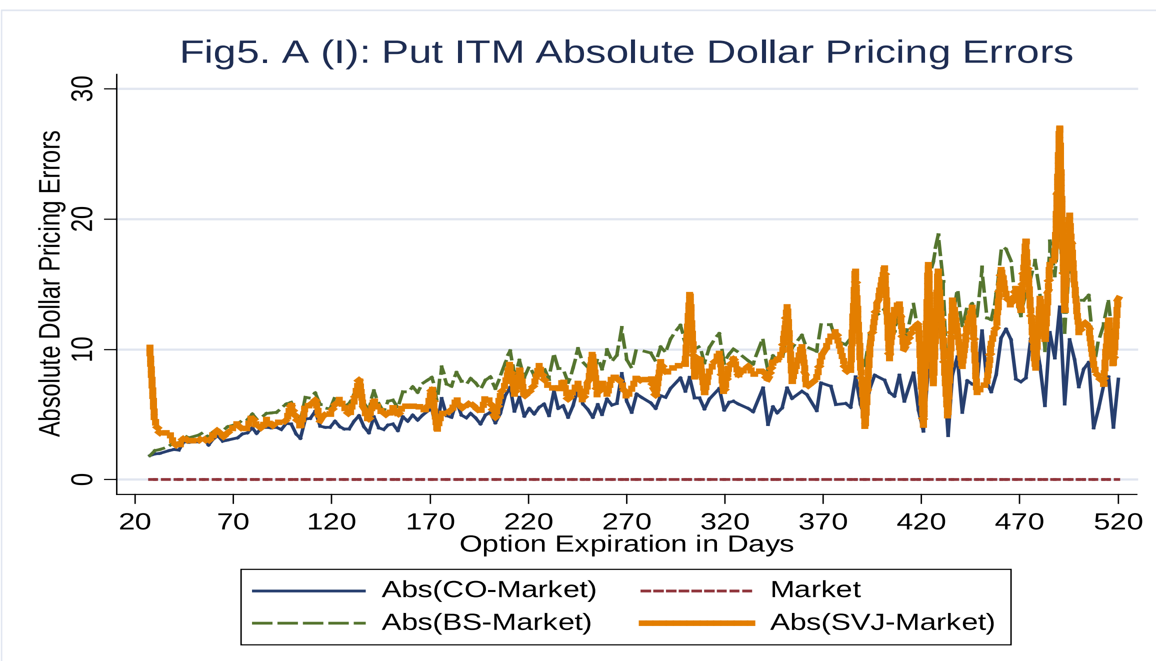
D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.3860	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.2519	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.3628	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.5509	0.0001	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.5244	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.3341	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.4887	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0385**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

4d. Comparisons of Absolute and Relative Distance from the Market for G, BCC, and BS

The previous tables of results in 4c focused on BS biases and G's improvements relative to BS pricing errors. In this section we examine all matched pairs for distance differences to see which model is closest to the market price as measured by the absolute dollar and relative per cent distance from the market price.[36] Now BCC is included in the comparisons with BS and G. Thus, for all matched pairs comparing BCC, BS, and G we determine which model is closest to the market price, and we also compute the relative percent pricing error. First we present Figures for the average distances across both both time to expiration and moneyness.

Time to Expiration Figures

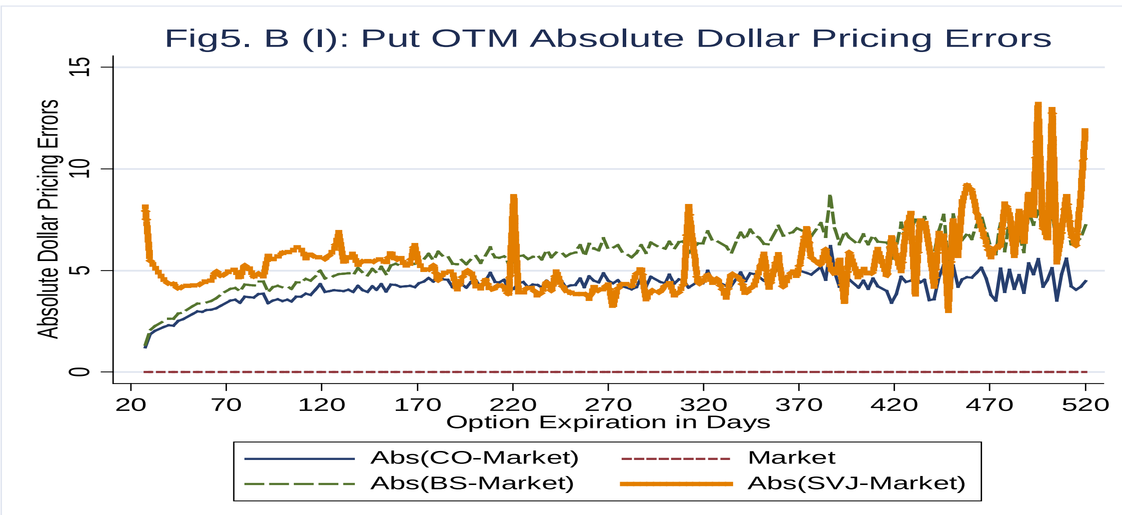
Figure 5A.(I) presents the average ITM absolute dollar pricing errors for matched pairs of index options with different times to expiration for BCC's SVJ, BS, and G. The average is across all strike prices for options having the same number of days expiration and ATM is 5%.



[36] For brevity we leave out Figures 5.A(2) and 5.B(2) of relative percent valuation errors, but these are available by request. However, we report here that G looks even better with the relative percent errors, especially for OTM options whose prices are lower which increases the percent errors.

Figure 5.A(1) shows G is on average always closer to the market price than BS for ITM options with different times to expiration. And similarly, G is on average almost always closer to the market price than BCC's best model, SVJ. Furthermore, BCC's SVJ model is on average almost always closer to the market price than the BS model. [37]

Figure 5B.(I) presents similar average absolute dollar pricing errors comparisons for OTM matched pairs of index options for different times to expiration for BS, BCC's SVJ, and G.

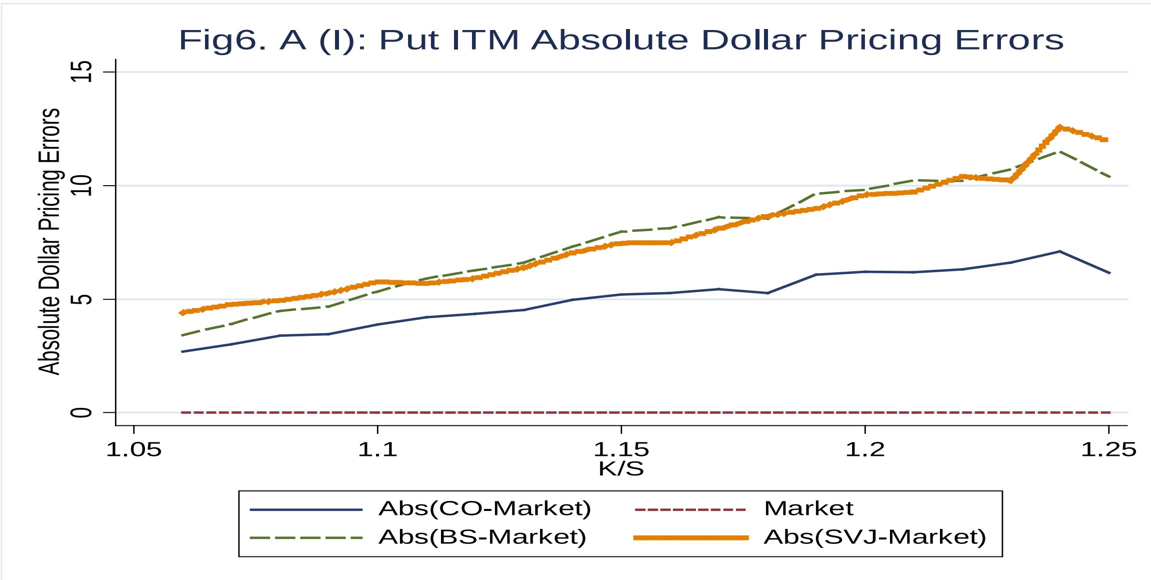


This graph again demonstrates that on average G is always closer to the market price than BS, and G is on average more often closer to the market price than BCC's SVJ model for OTM options with different times to expiration. Likewise, in this OTM comparison BCC's SVJ model again appears to be on average closer to the market price than the BS model.

[37] Here we present only the stochastic volatility with jumps version of BCC (SVJ) because it performs best. The comparisons to BCC's other models, SVSI and SV are available by request. However, we note here that BCC's SVJ and SV are much closer to the market price than the SVSI model.

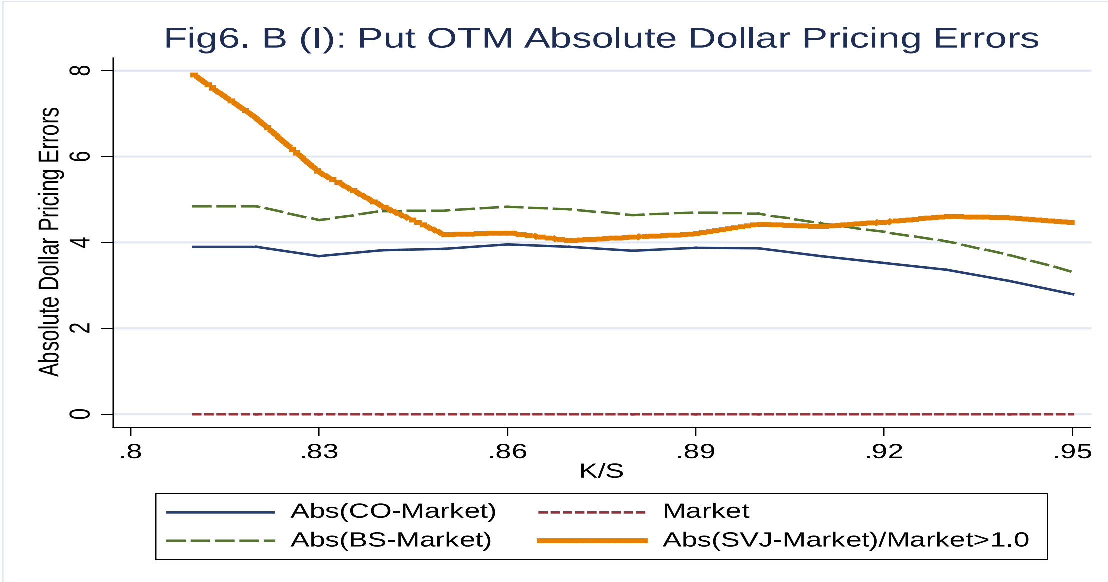
Moneyness Figures

Figure 6.A(I) presents the average ITM absolute dollar pricing errors versus moneyness, where ITM moneyness ranges from 1.05 to 1.25 when ATM is defined as 5%. The average is taken across different expirations for matched pairs of index put options with the same moneyness.



G is again shown to be always closer on average to the market price than both the BS model and BCC's SVJ model for the entire range of ITM amounts. In this Figure 6.A(1) the comparison of BCC's SVJ to BS shows that visually they are somewhat similar, but actually more detail from the tables which follow reveals that BCC is closer to the market for a greater number of matched pairs.

Figure 6B(I) presents the average absolute pricing errors for OTM index options for different moneyness amounts when ATM is defined as 5%.



This graph again demonstrates that G is on average always closer to the market price than both BCC's SVJ model and the BS model for OTM options as index option moneyness varies from 0.95 to 0.80. Here BCC's SVJ is on average sometimes closer to the market price than BS, but sometimes not as close. However, again more detail from the tables which follow shows that BCC is closer to the market price for more matched pairs of OTM options than BS is when both are compared to G.

The above Figures of absolute dollar pricing errors for different option times to expiration and for different option moneyness present a visual image of G, BCC's SVJ, and BS valuation errors in terms of distance from the market price. Each graph is constructed from thousands of matched pairs of options (options with the same strike K and time to expiration T). The superiority of G relative to BCC and BS in terms of closeness to the market price is highly statistically significant in all these comparisons.[38]

The next section, 4e, presents evidence on the economic significance of the differences between BCC's SVJ, BS, and G. However, before we proceed to the discussion of economic significance, now, after examining these Figures portraying each model's distance from the market price, is a good place to

[38] Again for brevity we do not include these tables. However they are available upon request.

recall why the comparison of the G and BCC models with much more complexity to the simpler BS model is important. BS without leverage is a special case of G, or conversely, G is BS with leverage. So the addition of leverage to BS is enough to make G (or BS with leverage) on average *always* closer to the market prices than BCC's best model SVJ for both ITM and OTM options with respect to moneyness shown in Figures 6.A(1) and 6.A(2). Furthermore, Figures 5.A(1) and 5.A(2) show that G (BS with leverage) is on average always closer than BS without leverage for both ITM and OTM options with respect to time to option expiration, and G is almost always closer to the market prices than BCC's SVJ model for both ITM and OTM options with respect to time to option expiration.

This improvement of BS with the inclusion of leverage (i.e. G) cannot be attributed to the term structure of volatility because BS without leverage was implemented with exactly the same volatility term structure buckets using the same ATM option prices to imply the volatilities used for all option valuations in the four time buckets. Thus, the improvement to BS when leverage is included in the model must be attributed to leverage and not the term structure of volatility.

Furthermore, as stated earlier, G, using only 5 explicit parameters (D or V plus the four volatilities in this term structure), is on average closer to the market prices than BCC's best model, SVJ, which is using 9 parameters in this implementation, and so still has a large parametric advantage. However, recall the G's model with leverage implies a stochastic equity volatility process, with negative equity return-volatility correlation, and an altered equity return distribution with a fatter left tail and thinner right tail than the normal distribution, all which change when leverage changes. So, one might reason that G's model implicitly includes the four additional parameters which BCC must assume and estimate for their stochastic volatility process.

4e. Economic Significance of G's Improvements Compared to BS and BCC

To complement the previous figures and tables comparing the pricing errors and distance from the market price of G, BS, and BCC's SVJ, here we report the economic significance. Economic significance as defined here does not mean the market is inefficient or an ability to "beat the market". [39] Instead we compare all matched pairs where one model is closer than the other, and find the total dollar improvements of each model and the net dollar improvement of the better model, and convert that to a basis point per option net improvement for ITM and OTM options. Specifically, Tables 7d and 8d show results when G's model is compared to BCC's best model, SVJ in the following ways: i) by the *number* of matched pairs that G's or BCC's model value is a closer absolute distance to the market price, ii) by the *dollar value* of G's and BCC's improvement when each model is closer to the market price, and iii) by the *net basis points per option* (bp) that the better model's improvement implies for a portfolio of 1 of each option in all matched option pairs. These comparisons are categorized by both calendar year and by leverage.[40]

First, consider Table 7d comparing G's compound option (CO) model and BCC's SVJ model for ITM options when ATM is defined as either 5% (or 0%). In Panel A the columns left to right represent the year(first year is 1996), the present value of all ITM put index option matched pairs for that year for which improvement is measured ($79,072.04), the total number of the matched pairs for that year for which improvement is measured (1,573), the number of those matched pairs where BCC's SVJ is closer to the market price in absolute distance (391), the number of matched pairs where G is similarly closer to the market price (1,182), the dollar value of the SVJ improvement ($392.82), the dollar value of G's

[39] To reinforce this definition, economic improvements are relative to the alternative model, not the market, and in no way do the improvements imply option market inefficiency or arbitrage opportunities. They could be thought of as the advantage one trader using a better model would have over another trader using a lesser model.

[40] For brevity we omit the tables 7a and 8a showing the economic improvements of G relative to BS. We also omit the similar tables 7b, 7c, and 8b, 8c comparing G to BCC's SVSI and SV. However, all these tables are available upon request. We do mention some of the results of the comparison of G to BS because they are somewhat relevant to the comparison of G to BCC's SVJ.

improvement ($4,230.80), and the net basis point advantage (or disadvantage) per option of G's model for that year (493 bp).

The total number of ITM matched pairs of options presented in Table 7d, Panels A and B, comparing BCC's SVJ to G, for the two ATM definitions of 5(0)% is 23,792 (59,569), respectively. G's model is closer to the market price than the BCC's SVJ model for 15,123 (43,733) of these 23,792 (59,569) ITM matched pairs, or about 75% (for the 0% ATM) of the matched pairs, and BCC is closer on 8,669 (15,836) pairs, or about 25% (for the 0% ATM) of the matched pairs. Note that in two of the 8 sample years, 1998 and 1999, when leverage was decreasing and the lowest (0.4 to 0.6), BCC's SVJ had more options closer to the market price than G. However, even in these two years G's model had an economic advantage because when BCC's model mis-valued the matched pairs, the BCC errors were large. Thus, in all the sample years G's model has an economic advantage over BCC's model.

If Table 7a were included here, it would show the total number of ITM matched pairs of options presented in similar Panels A and B, comparing BS to G, for the two ATM definitions of 5% (0%) is 22,853 (50,452). G's model is closer to the market price than the BS model for 19,837 (44,812) of these ITM matched pairs, or about 90% (for the 0% ATM) of the matched pairs, and BS is closer on 3,016 (5,640), or about 10% (for the 0% ATM) of the matched pairs.[41] Thus, BCC's SVJ model is closer to the market price than G's model for greater percentage, 25%, of these ITM matched pair comparisons, than the BS model which is closer to the market price for only 10% of these matched pair comparisons. This confirms the visual conclusions drawn from Figures 5 and 6 that BCC's SVJ is closer to the market than BS for a greater number of matched pair comparisons of ITM options for both the time to expiration and moneyness comparisons.

Table 7d shows that by G being closer to the market price than SVJ on about 75% of the ITM option matched pairs results in a basis point (bp) net improvement on average of 202 bp (298 bp) for ITM options in a portfolio containing one of each option when ATM is defined as 5% (0%),

[41] If the two models have the same value for a matched pair then there is no improvement of one model to the other.

respectively. These numbers are calculated by constructing a portfolio containing one option for each strike price and time to expiration for each day and finding the market value of that option portfolio each day for all days in a year.

In the following we explain in more detail the computation of the dollar and basis point improvement. More specifically, dollar improvement for each model is measured by considering all those matched pairs where a specific model is closer to the market price than the alternative model in absolute distance measured in dollars. The basis point advantage of G's model is then computed by dividing the net dollar improvement for that year or leverage category by the total value of options in that category. For example, in Table 7d, Panel A, across the sample years 1996-2004, for the ±5% ATM definition, G's option model has a total dollar improvement of $86,447.64 and SVJ has a total dollar improvement of $25,614.92. Thus, the net dollar improvement of G's model is $60,832.72, and that divided by the total value of each option in this ITM portfolio, $3,014,765.16, produces the 202 net basis point improvement per option. When the "0%" ATM definition of the two most at-the-money options is used this ITM portfolio value increases to $4,643,089.17 because of the inclusion of previously excluded 5% ATM options. Here, this larger number of near the money options increases the average errors, and the basis point improvement of G relative to SVJ rises to 298 bp per option. While the percent pricing error of G's improvement relative to BCC's SVJ is monotonic in leverage, the basis point improvement need not be since this depends on the dollar value of the options.

In summary, the above discussion showed that BCC's SVJ model is closer to the market price for a greater percentage (25%) of the viable matched pair comparisons than the BS model is for similar comparisons (10%), confirming the visual perspective from Figures 5 and 6. Furthermore, Table 7d also shows the net basis points net improvements per option from using G's model and being closer than BCC's SVJ model to the market price for the two definitions of ATM are on average 202 bp (298 bp), respectively, for ITM options.

TABLE 7d

PUT ITM: CO vs. SVJ

BASIS POINT IMPROVEMENTS

PANEL A

5 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
1996	79,072.04	1,573	391	1,182	329.82	4,230.80	**493**
1997	217,458.61	2,639	586	2,053	757.11	9,266.80	**391**
1998	376,015.80	3,002	1,581	1,421	6,639.27	9,274.37	**70**
1999	357,965.77	2,479	1,273	1,206	4,984.50	9,118.25	**115**
2000	488,897.60	3,226	1,331	1,895	4,798.93	17,685.53	**264**
2001	570,739.54	3,780	1,388	2,392	3,446.13	15,463.91	**211**
2002	660,335.65	4,566	1,668	2,898	4,241.06	13,896.23	**146**
2003	250,669.40	2,372	432	1,940	405.62	6,770.92	**254**
2004	13,610.75	155	19	136	12.50	740.84	**535**
TOTAL	**3,014,765.16**	**23,792**	**8,669**	**15,123**	**25,614.92**	**86,447.64**	**202**

0 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
1996	183,324.83	6,397	1,166	5,231	732.77	10,583.26	**537**
1997	412,350.17	8,095	1,647	6,448	1,564.58	18,309.07	**406**
1998	608,368.59	7,683	2,773	4,910	8,468.84	19,988.20	**189**
1999	641,774.79	7,212	2,674	4,538	7,234.48	20,830.87	**212**
2000	740,235.07	7,578	2,327	5,251	6,245.29	32,654.83	**357**
2001	766,336.47	7,292	2,008	5,284	4,115.08	26,238.94	**289**
2002	861,905.90	8,503	2,387	6,116	4,871.96	24,000.43	**222**
2003	394,330.70	5,971	801	5,170	602.53	16,500.03	**403**
2004	34,462.65	838	53	785	23.61	3,281.54	**945**
TOTAL	**4,643,089.17**	**59,569**	**15,836**	**43,733**	**33,859.14**	**172,387.18**	**298**

PANEL B

5 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
0.4-0.5	420,222.04	3,031	1,392	1,639	4,942.22	12,476.58	**179**
0.5-0.6	951,473.88	7,294	2,997	4,297	10,657.27	30,545.20	**209**
0.6-0.7	559,615.99	4,901	1,680	3,221	4,393.02	16,875.37	**223**
0.7-0.8	320,544.42	2,751	749	2,002	1,302.19	11,198.97	**309**
0.8-0.9	136,328.83	1,108	293	815	376.50	3,837.10	**254**
0.9-1.0	361,897.00	2,702	781	1,921	1,842.41	7,817.97	**165**
1.0-1.2	264,683.00	2,005	777	1,228	2,101.30	3,696.45	**60**
TOTAL	**3,014,765.16**	**23,792**	**8,669**	**15,123**	**25,614.92**	**86,447.64**	**202**

0 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
0.4-0.5	755,611.72	9,117	3,009	6,108	7,442.42	27,686.70	**268**
0.5-0.6	1,533,142.65	18,778	5,724	13,054	14,260.58	59,865.09	**297**
0.6-0.7	839,075.37	12,810	3,117	9,693	5,491.15	31,999.68	**316**
0.7-0.8	489,485.40	7,168	1,335	5,833	1,744.12	22,136.57	**417**
0.8-0.9	204,639.08	2,915	492	2,423	508.94	9,429.56	**436**
0.9-1.0	482,123.65	5,295	1,113	4,182	2,102.89	14,677.66	**261**
1.0-1.2	339,011.30	3,486	1,046	2,440	2,309.04	6,591.93	**126**
TOTAL	**4,643,089.17**	**59,569**	**15,836**	**43,733**	**33,859.14**	**172,387.18**	**298**

Now, consider Table 8d comparing G and BCC's SVJ model for OTM options when ATM is defined as 5% (0%). Table 8d, Panels A and B, incorporates the same format as Table 7d. The number of OTM matched pairs of options compared in Table 8d, Panels A and B, for the two ATM definitions of 5% (0%) is 95,186 (139,016), respectively. G's model is closer to the market price than the SVJ model for 58,466 (90,230) of these matched pairs. So, by comparison we see that G is closer to the OTM put market prices than SVJ on about 65% of the OTM matched pairs.

If Table 8a were included here, it would show the total number of OTM matched pairs of options presented in similar Panels A and B, comparing BS to G, for the two ATM definitions of 5% (0%) is 75,300 (109,301). G's model is closer to the market price than the BS model for 75,052 (108,437) of these OTM matched pairs, or about 99% (for the 0% and ±5% ATM) of the matched pairs, and BS is closer on 3,016 (5,640), or about 1% (for the 0% and ±5% ATM) of the matched pairs.[42] Thus, BCC's SVJ model is closer to the market price than G's model for a greater percentage, 35%, of these OTM matched pair comparisons, than the BS model which is closer to the market price for only 1% of these matched pair comparisons. This confirms the visual conclusions drawn from Figures 5 and 6 that BCC's SVJ is closer to the market than BS for a greater number of matched pair comparisons of OTM options for both the time to expiration and moneyness comparisons.

TABLE 8d

PUT OTM: CO vs. SVJ

BASIS POINT IMPROVEMENTS

PANEL A

5 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
1996	36,385.06	9,833	3,782	6,051	4,622.77	27,018.40	**6155**
1997	99,538.62	14,248	4,893	9,355	8,483.23	51,442.51	**4316**
1998	193,574.20	14,806	6,460	8,346	25,954.88	63,466.75	**1938**
1999	215,100.97	13,608	5,897	7,711	27,879.13	70,066.39	**1961**
2000	113,132.19	9,846	3,143	6,703	9,813.77	63,719.18	**4765**
2001	109,375.51	9,477	3,629	5,848	9,417.92	40,299.83	**2823**
2002	124,487.71	10,424	4,359	6,065	10,210.80	42,162.83	**2567**
2003	75,299.52	11,108	4,156	6,952	8,240.83	42,537.50	**4555**
2004	3,280.94	1,836	401	1,435	594.43	11,080.22	**31960**
TOTAL	**970,174.72**	**95,186**	**36,720**	**58,466**	**105,217.76**	**411,793.60**	**3160**

0 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
1996	97,942.54	16,094	6,011	10,083	6,187.83	34,105.97	**2850**
1997	234,445.61	21,396	6,924	14,472	10,253.84	63,708.62	**2280**
1998	368,363.34	21,044	8,514	12,530	29,326.13	78,609.07	**1338**
1999	422,073.41	19,597	7,584	12,013	31,280.04	90,517.57	**1403**
2000	254,607.18	14,325	4,175	10,150	11,558.33	83,004.16	**2806**
2001	232,489.34	13,433	4,538	8,895	10,589.97	52,836.57	**1817**
2002	240,924.57	14,309	5,269	9,040	11,125.66	53,052.86	**1740**
2003	167,623.06	15,945	5,152	10,793	9,086.26	54,321.52	**2699**
2004	12,176.18	2,873	619	2,254	823.54	14,140.68	**10937**
TOTAL	**2,030,645.23**	**139,016**	**48,786**	**90,230**	**120,231.61**	**524,297.02**	**1990**

PANEL B

5 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
0.4-0.5	211,067.61	17,073	6,597	10,476	27,533.77	98,909.25	**3382**
0.5-0.6	379,758.37	31,832	12,352	19,480	41,819.60	140,692.88	**2604**
0.6-0.7	137,613.54	18,199	6,945	11,254	13,233.11	63,977.76	**3687**
0.7-0.8	79,401.75	10,126	3,703	6,423	7,544.93	46,763.91	**4939**
0.8-0.9	25,226.76	4,562	1,464	3,098	2,826.34	21,138.87	**7259**
0.9-1.0	84,072.19	9,071	3,760	5,311	8,392.22	29,378.59	**2496**
1.0-1.2	53,034.50	4,323	1,899	2,424	3,867.80	10,932.34	**1332**
TOTAL	**970,174.72**	**95,186**	**36,720**	**58,466**	**105,217.76**	**411,793.60**	**3160**

0 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER SVJ	NUMBER CO	SVJ	CO	BP
0.4-0.5	442,620.03	24,904	8,765	16,139	31,689.31	126,167.61	**2135**
0.5-0.6	767,360.30	45,652	16,269	29,383	47,389.56	177,979.29	**1702**
0.6-0.7	319,407.35	28,175	10,110	18,065	16,042.81	81,942.49	**2063**
0.7-0.8	175,064.50	15,210	5,026	10,184	8,685.09	58,856.40	**2866**
0.8-0.9	63,824.64	6,968	2,017	4,951	3,335.82	27,420.64	**3774**
0.9-1.0	164,429.77	12,331	4,422	7,909	8,980.60	37,780.53	**1752**
1.0-1.2	97,938.64	5,776	2,177	3,599	4,108.41	14,150.08	**1025**
TOTAL	**2,030,645.23**	**139,016**	**48,786**	**90,230**	**120,231.61**	**524,297.02**	**1990**

[42] If the two models have the same value for a matched pair then there is no improvement of one model to the other.

The economic improvement of G relative to BCC's SVJ results in a much larger dollar improvement for OTM put options relative to ITM options for the two definitions of ATM of $411,793.60 ($524,297.02), respectively, while BCC has a dollar improvement for OTM index puts of $105,217.76 ($120,231.61).

Thus, the net dollar improvement of G's option model is $306,575.84 ($404,065), and that divided by the total value of each option in the OTM portfolios, $970,174.72 ($2,030,645.23) produces a 3160 (1990) basis point improvement. Here, the basis points improvements of G relative to BCC's SVJ model for OTM index put options for the two ATM comparisons are much greater relative to the ITM basis point improvements. This occurs because BCC's SVJ model produces more extreme values for OTM options.[43]

In this section we have demonstrated considerable economic improvement of G's model with leverage *relative* to the BCC's SVJ or BS models without leverage when pricing the world's most widely traded equity index options on the S&P 500. We have shown that the data necessary to implement G's model for valuing index options are easily available and are the same Compustat, CRSP, and Option Metrics data used in most asset and option pricing research. While G's compound option is often considered to be an exotic option, we have shown here and in another paper that G, as expected clearly dominates the seminal BS model when pricing the world's most common options, those on individual stocks (Geske-Zhou, 2007a) and those on stock indexes. We have also shown that by the inclusion of leverage G's model is able to compete with the more complex BCC's best SVJ model, even though G's model as implemented here uses fewer explicit parameters, 5, compared to BCC's SVJ model, as implemented here with 9 parameters. We have shown that G's improvement must be attributed to leverage and not to the term structure of volatility. We have also shown that BCC's models, if implemented consistently, when data permits, with a term structure of volatility, would have

[43] All of these conclusions are statistically significant at the 99.99 % level as before. The tables are omitted for brevity but are available upon request.

an even more unfair parametric advantage. We have also reasoned why G's model with fewer explicit parameters, but more implicit parameters, is thus more parsimonious and still effective. Finally, we have shown that while leverage is important it is not able to completely close the gap between the market and model prices. Since jumps are shown here and elsewhere, we have stated that both BCC and G could include leverage with jumps at the firm level which would provide a more parsimonious model, since the stochastic volatility process is implicit, and perhaps produce an even better option pricing model.

5. Conclusions

This paper demonstrates the Geske option model can be used effectively to value the world's most widely traded equity index options on the S&P 500 using only contemporaneous market price data. The Geske option model characterizes how the market value of aggregate leverage causes the market equity index risk to change stochastically and inversely with both the implied market value of aggregate leverage and with the return on the index. We believe we are the first to measure and show the importance of the implied market value of debt and stochastic leverage on asset prices. We have shown empirically that both the implied market value of the aggregate debt and the time series variations in this implied market value of aggregate leverage is sufficient to produce very significant statistical and economic improvements by Geske's model compared to models which omit leverage, such as the more complex models of Bakshi, Cao, Chen (SV, SVSI, & SVJ) and the seminal model of Black-Scholes. These improvements are shown to be conclusively and directly related to the leverage. Furthermore, we show why the improvements are greater for options with longer time to expiration because these options are effected by leverage for a longer period. We demonstrate the relative improvements attributable to leverage are both statistically and economically significant for all strikes and all times to expiration. We also demonstrate that these conclusions are independent of the implementation methodologies required for the more complex models discussed. We explain why G's model, which includes implicitly stochastic equity volatility, negative equity return-volatility correlation, and an equity return distribution that has a fatter left tail and thinner right tail parsimonious model, is able to compete with BCC's SVJ model which must explicitly characterize this stochastic equity volatility with many more parameters to estimate. However, we show that after including leverage there is still room for improvement, and perhaps incorporating jumps with leverage at the firm level would result in an even better model.

This research can be further extended to other contracts involving leverage, such as mortgages and cross currency swaps, credit derivatives, such as credit default swaps, and credit spread options. Risk neutral insolvency probabilities could be estimated for the market as measures of country credit rating migrations and systemic credit risk.[44] In addition this paper provides and new methodology for assessing the implied market value of corporate debt which may be more informative than the current use of either i)"matrix pricing" based on matrix interpolations between bond maturities, coupons, and ratings which is done because most corporate bonds to not trade daily like corporate stock, or ii) the book or face value of corporate debt.

[44] See Delianedis and Geske (2000) for evidence of the information in risk neutral default probabilities, and see Chan-Lau and Santos for an approach similar to this for country public debt sustainability and management.

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